

Use of Neural Networks for Monitoring Beam Spectrum of Industrial Electron Accelerators

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Abstract. This paper investigates technique for solving spectrometry inverse problem the neural network as method for reconstruction of electron beam spectrum using depth-charge curve. The inverse problem turned into multivariable optimization and the form of spectrum is based on proposed three-parameter model. Radial basis function network calculates the parameters of this model. We developed computational experiment using Monte-Carlo technique to evaluate strengths and weaknesses of proposed approach and compare neural networks with conventional data evaluation methods.

Keywords. Neural nets, Inverse problems, Monte Carlo, Radiation technologies, Depth-charge curve

Key terms. ComputerSimulation, Methodology, MachineIntelligence

1 Introduction

One of the main characteristic of the irradiation processes is an energy of beam. This parameter influences on absorbed dose in target. Therefore, standards for radiation technologies [1, 2] predetermine the upper bound of beam energy to prevent ionization of the object under irradiation. Because of accelerator features, electrons in beam have different energy. Thus, the beam energy represented by some function, which shows relations between particles number and their energies. This function called beam spectrum. In practice at least three parameters define the spectrum: average (E_{av}) and probably (E_p) energies and full width on half maximum (E_w). In order to measure beam energy dosimetric wedge and stack are widely used in centers of radiation technologies. These devices allow to determine only average and probable energies of beam [1-6]. Of course, these two parameters does not allow to reconstruct full energy distribution. Thereby developing of new instruments and methods of dosimetric measurements is actual problem.

Mentioned devices intend to measure distributions of absorbed dose or charge [5, 6]. The measured depth-dose (depth-charge) curves relate to beam spectrum

through Fredholm integral equation and finding exact spectrum is an ill-posed inverse problem [7]. This means that evaluated spectrum obtained by conventional mathematical methods can differ with true energy distribution. There are, for example, method of least squares (MLS) or method of Tikhonov regularization (MTR). Above all, important disadvantage of the MLS and MTR is impossibility to include additional solution conditions, for example, correlations between parameters, positivity and other. This lack can bring to violation of conditions, given by physical laws. It should be mentioned that in common case the neural networks (NN) solve approximation tasks and find solutions based on existing precedents after supervised training [8–12]. So the one of the ways of improving dosimetry effectiveness is developing of methods for measurement results evaluation based on neural networks. In order to apply NN for dosimetric data processing it is necessary to solve next problems: select networks topology, obtaining data for NN training, developing methods for data preprocessing and interpretation, system for evaluation network effectiveness.

So current research is about feasibility of using neural networks for developing system of measurement results evaluation for beam spectrum monitoring of industrial electron accelerators. We will discuss mathematical model of measurement process, which was built in order to compile training set for network learning procedure (Section 2). Section 3 describes methods under investigation. In section 4, we will show approach for methods evaluation, which contains computational experiment and comparison criteria. In section 5 given comparison results of neural networks and conventional methods testing.

2 Physical process and mathematical model

In order to calculate radiation energy, it is a common practice in field of radiation technologies to measure depth-dose curve by dosimetric wedge. However, the works of recent years propose new devices based on measurement of depth-charge curve that can realize on-line energy monitoring [3–6]. In this work, we will consider mathematical abstraction of these devices and will build method for beam spectrum controlling using depth-charge curve.

2.1 Devices

Device [5] consists of two plates only and intend to calculate probable energy as a value which linearly depends on charge in first plate to sum charge ratio. Measurer in [6] contains 10 absorbers. But in order to simplify average energy calculation the plates were combined and authors use similar to [5] dependency.

Fig. 1 shows principal schema of measurer. Dosimetric stack consists of set of plates - absorbers. The absorbers material is often aluminum, because of radiation ruggedness. The electron beam falls on the sequence of plates. Electrons stop at different depths depending on their energy. Thus, absorbers collect some charge which can be measured by current integrators connected to corresponding plate. The set of measured values represents the depth-charge curve.

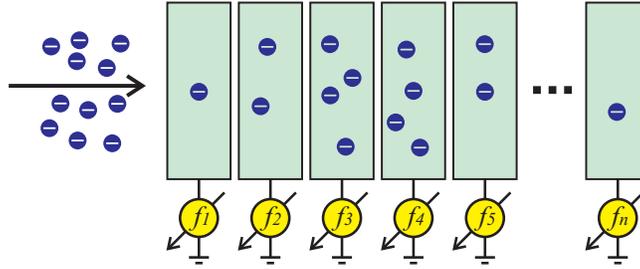


Fig. 1. Common schema of stack for depth-charge measurement

Mathematical model of the measurement process is based on a semi-empirical model of the depth-charge distribution for monoenergetic electrons and model of charge measurement uncertainty. Direct problem describes relation between known beam spectrum and depth-charge curve through equation:

$$f(x) = \int_{E_L}^{E_R} Q(x, E)y(E) dE, \quad x \in [0, x_R], \quad (1)$$

where $y(E)$ - describes relation between number of particles and their energy (electrons spectrum), $f(x)$ - describes depth distribution of charge, x_R - measurer full width, $[E_L, E_R]$ - operating energy range of accelerator, integral kernel $Q(x, E)$ corresponds to radiation type (α , β , γ) and measurer internal characteristics (including absorbers material). Works [13, 14] describe appropriate relations for monoenergetic beam and depth-charge curve.

In the research we neglect charge leakage and suppose that distance between absorbers is neglectfully small. It means that each particle from initial beam can stops in absorbers and pass through current integrator or can pass through whole device with no impact in depth-charge curve.

The measurement results of charge distribution in absorbers is set $f = \{f_1, f_2, \dots, f_n\}$ (see Fig. 1), where n - number of absorbers, f_i - integral of $f(x)$ over the depth for i -th absorber:

$$f_i = \int_{x_i}^{x_i + \Delta x} \int_{E_L}^{E_R} Q(x, E)y(E) dE dx, \quad (2)$$

where Δx - absorbers width. Equation (2) can be approximated as:

$$f_i = \frac{\Delta x}{2} \sum_j p_j^E y_j [Q(x_k + (i-1)\Delta x, E_j) + Q(x_k + i\Delta x, E_j)], \quad (3)$$

where $i = \overline{1, n}$, $j = \overline{0, m}$, $m = (E_R - E_L)/\Delta E$ - number of steps of function $y(E)$ discretization over energy axis, ΔE - step of spectrum energy discretization, y_j - value of $y(E)$ in approximation nodes, coefficient p_j^E defines method and step

of function $y(E)$ approximation. Then the measurement process can be shown as system of linear equations:

$$Ay = f \Leftrightarrow \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}, \quad (4)$$

where elements of matrix A are:

$$a_{i,j} = \frac{\Delta x}{2} p_j^E [Q(x_k + (i-1)\Delta x, E_j) + Q(x_k + i\Delta x, E_j)]. \quad (5)$$

In order to approximate $y(E)$ by method of trapezoids, coefficients p_j^E are:

$$p_j^E = \begin{cases} \Delta E/2 & j = 0 \vee j = m \\ \Delta E & otherwise \end{cases}. \quad (6)$$

It's obvious that complexity of spectrum reconstruction grows with increasing of m (dimension of vector y). In order to reduce problem the we used parameterization of $y(E)$. As mentioned above, the general practice is denoting spectrum by parameters: E_p , E_{av} , E_w . Therefore, it is reasonable to make model of the beam spectrum, which use three parameters.

2.2 Model of electrons spectrum

Fig. 2 shows geometrical interpretation of electrons spectrum model considered in the present work. The graph of spectrum consists of two part: left exponential and right linear slopes. The parameters of this model are:

- E_{max} - maximal particles energy in the beam,
- E_p - most probable energy,
- E_s - energy of 10 times decreasing of the intensity compared to E_p electrons along left slope.

In the future discussion the Π will denotes set of spectrum parameters, i.e. $\Pi = \{E_s, E_p, E_{max}\}$.

Parameters of the model correspond to characteristics of beam used in practice according to:

$$\begin{aligned} E_p &= E_p, \\ E_w &= \frac{\ln 0.5}{\ln 0.1} (E_p - E_s) + \frac{E_{max} - E_p}{2} \\ E_{av} &= E_s + \ln \left(\frac{E_{max} - E_p}{4} + \frac{0.45(E_s - E_p)}{\ln 0.1} \right) \end{aligned} \quad (7)$$

and mathematical expression for spectrum is:

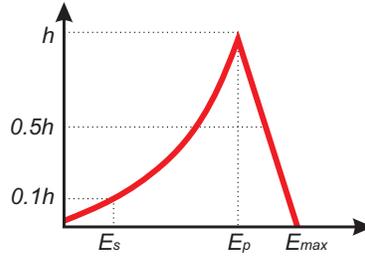


Fig. 2. Model of electron beam spectrum

$$y(E) = \begin{cases} he^{\mu(E-E_p)}, & 0 < E \leq E_p \\ k_1E + k_2, & E_p < E \leq E_{max} \\ 0, & E_{max} < E \end{cases}, \quad (8)$$

$$\mu = \frac{\ln(0.1)}{E_s - E_p}, k_1 = \frac{h}{E_p - E_{max}}, k_2 = \frac{hE_{max}}{E_{max} - E_p}, \quad (9)$$

where $E \in [0; \infty]$, $h = y(E_p)$ - maximum of function $y(E)$ and was obtained with supposition of

$$\int_{E_s}^{E_{max}} y(E) dE = 1. \quad (10)$$

Therefore, maximum of energy distribution is:

$$h = y(E_p) = [0.9 \frac{E_s - E_p}{\ln(0.1)} + 0.5(E_{max} - E_p)]^{-1}. \quad (11)$$

It should be mention, that in accordance to physical laws the function $y(E)$ is positive or equal zero for all accepted E and parameters correlates as:

$$0 < E_s < E_p \leq E_{max}. \quad (12)$$

2.3 Model of measurement

In the real experiment measured f_i differ with its real value. This error grounded on weaknesses of measurer and external influence. We will mark set of true values of $f(x)$ as f , and use \tilde{f} to mark set of values complemented with measurement uncertainty:

$$\tilde{f} = (1 + \varepsilon\xi)f, \quad (13)$$

where ε - value of standard deviation of measurement error, ξ - random variable distributed in accordance to standard normal distribution:

$$\xi = \cos(2\pi r_1) \sqrt{-2 \ln(r_2)}, \quad (14)$$

where r_1, r_2 - random variables which are distributed in accordance with standard uniform distribution.

We will use similar signature to denote evaluated parameters $\tilde{\Pi}$, \tilde{E}_s , \tilde{E}_p , and \tilde{E}_{max} reconstructed spectrum \tilde{y} instead their true values without tilde.

3 Methods for spectrum reconstruction

3.1 Neural networks

In order to apply NN for solving spectrometry inverse problem reconstruction of spectrum can be represented as multivariable function fitting. Suppose that function ϕ implements measurement process of depth-charge curve, i.e. $\tilde{f} = \phi(\Pi)$. Therefore, inverse function $\tilde{\Pi} = \phi^{-1}(\tilde{f})$ realizes transformation from depth-charge curve to beam spectrum. So approximation of ϕ^{-1} can be used to get spectrum using depth-charge curve. In the work we used general regression neural network (GRNN) [15] to fit ϕ^{-1} . This network needs set of precedence for supervised learning. Consider algorithm of training set creation.

Implemented measurement models allow to create pairs $s = (\tilde{f}, \Pi)$, where \tilde{f} calculates from parameters set Π . The collection of s is based on different Π and represents a reference points for ϕ^{-1} fitting:

$$(s_1 \cdots s_N) = \begin{pmatrix} \tilde{f}_1 & \tilde{f}_2 & \cdots & \tilde{f}_N \\ \Pi_1 & \Pi_2 & \cdots & \Pi_N \end{pmatrix} \quad (15)$$

where N - number of elements in training set. For future discussion, we will denote each unique Π in training and testing sets as reference spectrum. Note, that each values of parameters for all Π from training set was normalized in accordance to $[E_L; E_R] \rightarrow [0; 1]$. Of course, outputs of network were scaled back during testing.

3.2 Conventional methods

Consider methods, which is traditionally used for measurement results evaluation. The data which were obtained by these methods is a base level to determine NN effectiveness for solving spectrum reconstruction problem. The method of least squares calculates parameters Π as:

$$\tilde{\Pi}_{MLS} = \arg \min_{\Pi} \left\| A\tilde{y}_{\Pi} - \tilde{f} \right\|, \quad (16)$$

where $\|\cdot\|$ - Euclidian norm. Method of Tikhonov regularization expands MLS through additional stabilizer function:

$$\tilde{\Pi}_{MTR} = \arg \min_{\Pi} \left(\left\| A\tilde{y}_{\Pi} - \tilde{f} \right\| + \alpha \|\tilde{y}_{\Pi}\| \right), \quad (17)$$

where $\alpha > 0$ - regularization parameter. It should be remind that using of mathematical model of measurement process gives true values of electrons spectrum. So α can be calculated from [7]:

$$\alpha^* = \arg \min_{\alpha} \left(\frac{\|y - \tilde{y}_{\alpha}\|}{\|y\|} \right). \quad (18)$$

In the work we applied Nelder-Mid simplex method numerical solution of (16), (17) and (18).

4 Algorithm for evaluation methods preparing and testing

4.1 Comparison approach

Implemented models of spectrum, measurement process and methods for data evaluation compose computational experiment (Fig. 3 shows sequential diagram). The experiment aim is comparison of methods for spectrum reconstruction. The approach which was used to build experiment uses Monte-Carlo technique: system generate measurement results, each methods reconstruct spectra using samples of depth-charge cure, system calculates statistical characteristics of reconstruction error. Computational experiment consists of three steps: preparation, main part (loop Common) and results interpretation.

Preparation of an experiment includes setting parameters of models and methods. Main part is a series of subexperiments with varied measurement uncertainty ε . Each of them contains two steps: training of NN and selected methods comparison. Both processes include generation of pairs $s = (\tilde{f}, \Pi)$ which is based on predefined set of Π . But these sets of reference spectrum are different. Testing procedure (loop Data Evaluation) repeats sampling of \tilde{f} , evaluates appropriate Π by each method and collects reconstruction error based on truth and calculated spectra based on proposed set of indicators. The results processing step aims to build relationships that show correlations between accuracy of spectrum reconstruction and varied error of measurement.

Software for experiment execution implemented in MATLAB with Neural Network Toolbox (function newgrnn as NN), Optimization Toolbox (function fminsearch as MLS and MTR). In order to speed up computational experiment, software was executed on high performance cluster [16] with Distributed Computing Toolbox.

4.2 Comparison indicators

In order to assess the effectiveness of methods for reconstruction of beam energy characteristics we suggested set of indicators. The set consists of the standard statistical estimates of data evaluation error and indicator of methods reliability. There are two indicators type: mismatch along energy axis (estimate shift of reconstructed spectrum along horizontal axis) and common indicator. Consider details of each indicators.

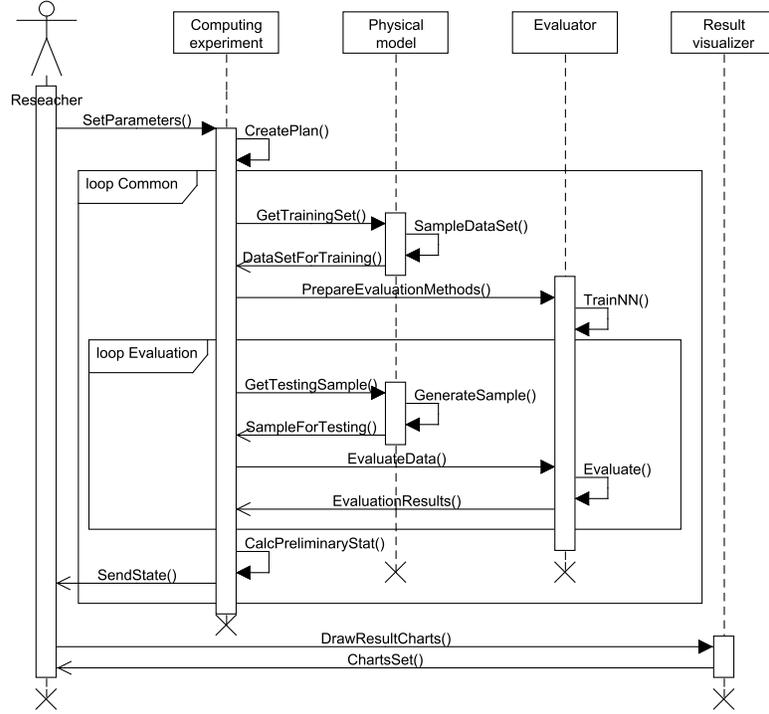


Fig. 3. Sequential diagram of computer experiment

1. *Mismatch along energy axis.* Average $M(r)$ and standard deviation σ_r of distance along intensity axis between reconstructed and true spectra are based on:

$$r = \frac{1}{n} (y - \tilde{y})^2; \quad (19)$$

2. *Common characteristics.* Probability of method failure P . We suppose that the method failure is a case when applying mathematical methods leads to impossible (due to physical laws) solution, i.e. the solution breaks condition (12). It is obvious that value $1 - P$ characterize method reliability.

5 Results and discussions

5.1 Parameters of computation experiment

In order to evaluate methods effectiveness with suggested indicators we made computational experiment with parameters shown in Table 1.

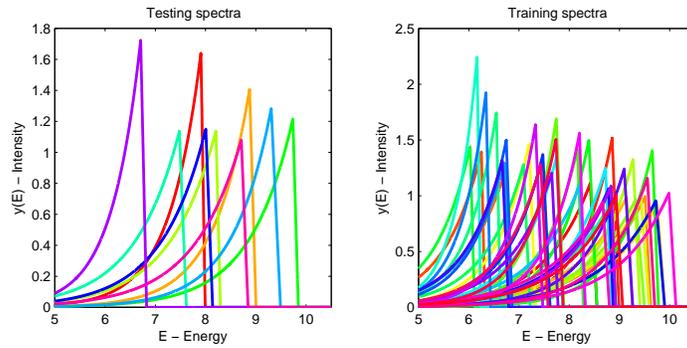
The training and testing sets include reference spectra with parameters:

Table 1. Common experiment parameters

Parameter	Value
Characteristic for measurement	depth-charge curve
Absorbers material	Aluminum ($Z = 13$, $Am = 27$)
Absorber's width (Δx)	$0.4 g/cm^2$
Device total width (x_R)	$6 g/cm^2$
Uncertainty (ε)	Varied from 0% to 30%, step 1%
$[E_L; E_R]$	$[0MeV; 10.2MeV]$
E_w of reference spectra	Randomly from 2% to 10% of E_p
$y(E)$ discretization step (ΔE)	$0.05MeV$
Number of reference spectra	9000 (training) and 41000 (testing)

$$\begin{aligned}
E_p &= r_1, & r_1 &\sim U[E_L, E_R], \\
E_{max} &= E_p(1 + 2r_2), & r_2 &\sim U[0.01, 0.02], \\
E_s &= E_p - \frac{\ln 0.1}{\ln 0.5} r_3, & r_3 &\sim U[0.01, 0.08].
\end{aligned} \tag{20}$$

Fig. 4 shows examples of sampled reference spectra. Number of the spectra for training and testing sets is reduced, but proportion saved. As shown on Fig. 4 and in Table 1 the testing set is bigger than training set. It is necessary to get appropriate assessment of method based on NN with influence of retraining.

**Fig. 4.** Reference spectra for a) NN training and b) methods testing

As shown in Table 1 the device consists of 15 absorbers. This configuration is chosen based on previous research [18] which was aimed to find optimal discretization step of depth-charge curve for spectrum reconstruction by NN. It should be mention that in works [17, 18] sets for methods testing and preparation based on reference spectra with fixed E_w parameter and same maximum $h = 1$. Therefore, seeking of optimal absorbers width is open for future research.

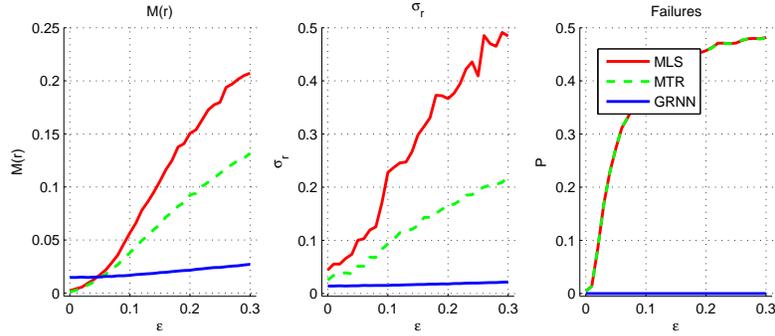


Fig. 5. Results of methods comparison

5.2 Results and discussion

Fig. 5 contains obtained dependencies, which describe relation between methods evaluation error and measurement uncertainty. The charts 5a and 5b based on indicator (19). Chart 5c shows probability of method failure.

For MLS and MTR experiment proves expected results. Methods are sensitive to uncertainty in input data. Fig. 5a and 5b show that error of MLS and MTR solutions rapidly grows with increasing of ϵ . With respect to probability of failure, both methods demonstrated almost equal inefficiency. It can be mean that stabilizing additions in MTR does not affect to the method reliability. It should be mention that the reason of MLS and MTR error for $\epsilon = 0\%$ is discretization inaccuracy which appears when transforming integral (1) to system (4).

As an opposite to conventional methods, the solutions obtained by NN have smaller dependency between evaluation error and input data uncertainty. Furthermore as shown on Fig. 5a, 5b the GRNN evaluates spectra more accurate than MLS and MTR for measurement uncertainty more than 5-7%. The main advantage of NN method is that GRNN reconstruct beam spectrum parameters with no failures (see Fig 5c), i.e. all obtained solutions are compliance with physical laws.

6 Conclusion

The work shows GRNN method effectiveness for solving inverse dosimetry problem of electron spectrum reconstruction using depth-charge curve. The main advantages of proposed technique compared to conventional methods is allowance to apply additional solutions conditions. It lids to getting robust evaluation method. As shown in the work methods based on NN can be used for building on-line energy monitoring systems in centers of radiation technologies.

Furthermore, we proposed comparison approach based on Monte-Carlo technique and set of effectiveness indicators. The approach allows testing different

types of evaluation methods and can be used for methods optimization in order to select or apply technique for industrial problems solving.

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