

Clocks Model for Specification and Analysis of Timing in Real-Time Embedded Systems

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Abstract. Problems concerning formal semantics for Clock Constraint Specification Language (CCSL) are considered in the paper. CCSL is intended for describing logical time models for real-time embedded systems and the language is a part of UML profile for MARTE. There exist two approaches to introduce a denotational semantics for CCSL. A pure relational subset of CCSL is defined in the paper. The notion of time structure with clocks is introduced to refine describing denotational semantics for this CCSL subset, which authors called RCCSL. Semantic properties of RCCSL have been studied. Theorem about coincidence semantics of RCCSL for the two approaches is proved.

Keywords. Embedded system, real-time system, time modelling, time structure, clock constraint, formal specification

Key terms. ConcurrentComputation, FormalMethod, SpecificationProcess, VerificationProcess, MathematicalModeling

1 Introduction

Nowadays, the growth of using distributed real-time systems (including embedded systems) [4] is the developing trend for Information and Communication Technology. There are two reasons for such growth: first, the physical limit for processor acceleration is reached, and, second, using mobile and cloud technologies are explosively expanded. The impossibility to continue over-clocking of a processor leads to using a multi-core system, which is parallel and distributed. A complex consisting of a computational cloud and an ensemble of mobile devices is a parallel and distributed system too. Moreover its structure is not fixed.

Each of the cases requires using different kinds of multiprocessing architectural and software solutions [3]. Therefore, providing correct working of such systems requires more research in the area.

Mathematical modelling of systems makes possible to develop formal specifications and methods of their analysis as a base for trustworthy system constructing. There are a lot of approaches to modelling multiprocessor systems. First of all, the following ones should be noticed: CSP of C.A.R. Hoare [8], π -calculus of R. Milner [11], abstract state machine model [5], and processing algebra [14].

This paper is devoted to formal methods for an important subclass of multiprocessing distributed systems, namely, real-time embedded (RTE) systems. These methods are closely connected with the UML profile for MARTE (Modelling and Analysis of Real-Time and Embedded systems) [2, 15]. In the context of the MARTE approach UML [16, 17] is used to build engineering models of a developing system. But the UML notation does not support detailed description of interactions for joining components into a united RTE system. A very common way to specify conditions for the system integrity is through the Object Constraint Language (OCL) [12]. However, no facilities for specifying temporal constraints are provided by the OCL standard. The Clock Constraint Specification Language (CCSL) [2] was defined in an annex of MARTE as a way to build logical and temporal constraints on model elements.

CCSL is intended to describe the temporal ordering of interactions between components of a distributed software system. It focuses on the ordering of event occurrences, but not on their chronometric characteristics. It relies on a logical time model inspired by the work on synchronous systems and their polychronous extensions.

The denotational semantics for basic constructions of CCSL is given in [10]. It is based on the notion of a time structure with clocks, other approach [1] defines an operational way to compute runs for CCSL specifications. The main contribution of this paper is a demonstration that the relationship of semantics consequence based on time structures as models of constraints and semantics consequence based on time structures associated with runs are only equivalent for a subset of CCSL, which we call RCCSL.

2 Syntax of Pure Relational CCSL

In the paper we restrict ourself to a very simple sublanguage of CCSL, which we call the pure relational CCSL (RCCSL). Syntax of this subset is given here using EBNF [6].

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clock constraint =
  clock relation, {'', '}, clock relation};
clock relation =
  clock reference, sign of clock relation, clock reference;
sign of clock relation =
  'subclocking' |

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'exclusion'           |
'coincidence'        |
'cause'              |
'precedence';
clock reference =
? any element of clock set ?;

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Below we use the next notation for symbols of clock relations (see Table 1).

Table 1. Symbols of clock relations

Relation name	Relation symbol
subclocking	\sqsubset
exclusion	$\#$
coincidence	\equiv
cause	\prec
precedence	\prec

These five binary relations on a clock set \mathcal{C} are determined as logical primitives for CCSL in [1].

Defining semantics for RCCSL is one of the paper objectives. Following the paper [10], we define the denotational meaning for a set of clock constraints as some class of time structures expanded by a classification for event occurrences. The next section is devoted to describing such structures.

3 Time Structure with Clocks

Let consider a set of event occurrences, which is below denoted by \mathcal{I} . Elements of the set \mathcal{I} are called instants. Some pairs of instants denotes instant pairs, whose elements are ordered in time: $i_1 \prec i_2$ is denoted the fact "an instant i_1 causes an instant i_2 " or equivalently "an instant i_1 cannot occur later than an instant i_2 ", where $i_1, i_2 \in \mathcal{I}$. This relation is called 'cause'. It is naturally to suppose that cause is a pre-order.

As known [7, section 1.3], each pre-order can be decomposed uniquely into the union of two relations such that the former is a strict order (it is denoted below by ' $<$ ' and called a precedence) and the latter is an equivalence (it is denoted below by ' \equiv ' and called a coincidence). These relations are connected by the next property:

$$\begin{aligned}
 &\text{for any instants } i_1, i'_1, i_2, i'_2 \in \mathcal{I} \\
 &\text{the validity of } i_1 \equiv i'_1, i_2 \equiv i'_2, \text{ and } i_1 < i_2 \text{ implies} \\
 &\text{truth of } i'_1 < i'_2.
 \end{aligned} \tag{1}$$

Moreover, if we have a strict order and an equivalence on the same set and these relations satisfy (1) then their union is a pre-order.

Now, we can introduce the notion of a time structure for formalising our understanding a set of instants.

Definition 1. Let (\mathcal{I}, \preceq) be a pair of a set and a pre-order on this set respectively. Denote by \prec the strict order corresponding to the pre-order \preceq . The pair (\mathcal{I}, \preceq) is called a time structure if the next property (the property of cause finiteness [13]) holds:

$$\text{the set } \{i' \in \mathcal{I} \mid i' \prec i\} \text{ is finite for all } i \in \mathcal{I}. \quad (2)$$

Definition 1 is based on the corresponding definition in [10]. One can compare them with the definition of a time structure in [13]. Difference consists in a possibility of modelling an instant coincidence.

Note that Definition 1 specifies the set of instants and some time relations on it but it does not determine any classification of instants in compliance with their sources. Therefore, in the following [2] we introduce such a classification by adding a finite set of instant sources called clocks and by mapping the set of instants into this clock set.

Definition 2. Let (\mathcal{I}, \preceq) be a time structure, \mathcal{C} be a finite set of clocks, and $\pi : \mathcal{I} \rightarrow \mathcal{C}$ be a map then the quadruple $(\mathcal{I}, \preceq, \mathcal{C}, \pi)$ is called a time structure with clocks if the next property holds:

$$\begin{aligned} &\text{for any clock } c \in \mathcal{C} \text{ and } i_1, i_2 \in \pi^{-1}(c) \\ &\text{the validity of } i_1 \neq i_2 \text{ implies truth of } i_1 \prec i_2 \vee i_2 \prec i_1, \\ &\text{i.e. } \pi^{-1}(c) \text{ is linearly ordered by the restriction of the cause.} \end{aligned} \quad (3)$$

If $c \in \mathcal{C}$ then the set $\pi^{-1}(c)$ is usually denoted by \mathcal{I}_c . It can be considered as an event stream generated by the source associated with the clock c .

From Definition 1 and Definition 2 the next fact follows immediately.

Proposition 1. Let $(\mathcal{I}, \preceq, \mathcal{C}, \pi)$ be a time structure with clocks then

1. \mathcal{I}_c is well-ordered by the strict order \prec for all $c \in \mathcal{C}$;
2. ordinal type of \mathcal{I}_c for any $c \in \mathcal{C}$ is less or equal to ω , where ω is the first infinite ordinal.

Proof. Firstly note that property (3) implies linear ordering \mathcal{I}_c for an arbitrary $c \in \mathcal{C}$.

Further, suppose that A is some non-empty subset of \mathcal{I}_c for an arbitrary $c \in \mathcal{C}$, i is some element of A .

If for all $i' \in A$ the statement $i \prec i' \vee i = i'$ is true then $\inf A = i \in A$.

If there exists $i_0 \in A$ such that $i_0 \prec i$ then the set $A(i) = \{i' \in A \mid i' \prec i\}$ is not empty. It is evident that $A(i) = A \cap \{i' \in \mathcal{I}_c \mid i' \prec i\}$. This equality and the property of cause finiteness (2) imply finiteness of $A(i)$. So, taking into account

property (3) we can conclude that $A(i)$ is a finite linearly ordered set. Hence, there exists $i_* \in A(i)$ such that $i_* = \inf A(i)$. It is evident that

$$\inf A = \inf A(i) = i_* \in A(i) \subset A.$$

Thus, $\inf A \in A$ and \mathcal{I}_c is well-ordered.

The supposition that ordinal type of \mathcal{I}_c for some $c \in \mathcal{C}$ is greater than ω is inconsistent with the property of cause finiteness (2). \square

Corollary 1. *Any instant $i \in \mathcal{I}$ is uniquely determined by the pair $(\pi(i), \text{idx}(i))$, which is an element of the set $\mathcal{C} \times \mathbb{N}$. Here, idx is a map from \mathcal{I} into \mathbb{N} such that*

$$\text{idx}(i) = |\{i' \in \mathcal{I}_{\pi(i)} \mid i' \prec i\}| + 1,$$

where the number of elements in a set A is denoted by $|A|$.

The designation $\mathfrak{T}^{\mathcal{C}}$ is used below to refer to the class of time structures with \mathcal{C} as a set of clocks.

Remark 1. One can show that this class is a set but we do not do it in the paper.

4 Denotational Semantics for RCCSL

Usually, a denotational semantics can be considered as the theory of models for the corresponding language. We shall use time structures with clocks as models for describing meaning of clock constraints.

4.1 Some General Notes

One can identify a class of event occurrences of the same type with a set of instants for some clock in the process of specifying interactions between components of distributed parallel systems. Such an identification is provided by fixing a set of clocks \mathcal{C} and describing rules of interacting system components. These rules divide the set $\mathfrak{T}^{\mathcal{C}}$ into two subsets: the subset of time structures satisfying the constraints and the set of time structures contradicting them. Taking into account the specification of RCCSL one can say that a clock constraint is a finite set of clock relations. If the set of clock relations determining the constraint is denoted by \mathfrak{C} then the fact "the time structure $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ satisfies the constraint \mathfrak{C} " can be written as $\mathcal{T} \models \mathfrak{C}$. More precisely, $\mathcal{T} \models \mathfrak{C}$ means that for each $C \in \mathfrak{C}$ the clause $\mathcal{T} \models C$ is true.

Further, for a constraint \mathfrak{C} , $\llbracket \mathfrak{C} \rrbracket$ denote the following set $\{\mathcal{T} \in \mathfrak{T}^{\mathcal{C}} \mid \mathcal{T} \models \mathfrak{C}\}$.

The first important problem is the consistency problem for the constraint. The rigorous problem formulation has usually the form:

Problem 1 (Consistency Problem). For a constraint \mathfrak{C} check that the set $\llbracket \mathfrak{C} \rrbracket$ is not empty.

The second important problem is the semantic consequence for the constraints. The rigorous problem formulation has the next form:

Problem 2 (Semantic Consequence Problem). For a constraint \mathfrak{C} and a clock relation τ check that $\llbracket \mathfrak{C} \rrbracket \subset \llbracket \tau \rrbracket$ (or in the another notation $\mathfrak{C} \Vdash \tau$).

Below we use the notation $\{\mathfrak{C}\}$ for the set of clock relations that form the constraint \mathfrak{C} . It is easy to see that the next properties of the relationship \Vdash are true.

Proposition 2. *The next properties are satisfied:*

1. if a constraint \mathfrak{C} and a clock relation τ satisfy the condition $\tau \in \{\mathfrak{C}\}$ then $\mathfrak{C} \Vdash \tau$;
2. if constraints \mathfrak{C}_1 and \mathfrak{C}_2 and a clock constraint τ satisfy the next condition $\mathfrak{C}_1 \Vdash \tau'$ for all $\tau' \in \{\mathfrak{C}_2\}$ and $\mathfrak{C}_2 \Vdash \tau$ are true then $\mathfrak{C}_1 \Vdash \tau$ is true.

Proof is omitted □

To complete defining the denotational semantics for RCCSL we should determine the meaning of basic clock relations.

4.2 Subclocking

This relation is intended for specifying a requirement to synchronize each instant of one clock with some instant of another clock. In this case the first clock is called a subclock of the second clock.

More precisely, let $c', c'' \in \mathcal{C}$ and $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ then $\mathcal{T} \models c' \sqsubset c''$ means that there exists a strict monotonic map $h : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c''}$ such that $i \equiv h(i)$ for any $i \in \mathcal{I}_{c'}$.

Proposition 3 (Trivial Subclocking). *For each $c \in \mathcal{C}$ the clause $\Vdash c \sqsubset c$ is true.*

Proof is trivial □

Proposition 4 (Transitivity Law for Subclocking). *For each $c', c'', c''' \in \mathcal{C}$ the clause $c' \sqsubset c'', c'' \sqsubset c''' \Vdash c' \sqsubset c'''$ is true.*

Proof. Let $h_{c'' c'} : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c''}$, $h_{c''' c''} : \mathcal{I}_{c''} \rightarrow \mathcal{I}_{c'''}$ be strict monotonic maps providing the validity of the clauses $\mathcal{T} \models c' \sqsubset c''$ and $\mathcal{T} \models c'' \sqsubset c'''$ respectively for some \mathcal{T} . It is easy to see that the map $h_{c''' c''} \circ h_{c'' c'}$ provides the validity of the clause $\mathcal{T} \models c' \sqsubset c'''$ □

4.3 Exclusion

This relation is used for specifying the mutual exclusion for two events.

More formally, let $c', c'' \in \mathcal{C}$ and $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ then $\mathcal{T} \models c' \# c''$ means that for any $i' \in \mathcal{I}_{c'}$, $i'' \in \mathcal{I}_{c''}$ the coincidence $i' \equiv i''$ is false.

Proposition 5 (Irreflexivity Law for Exclusion). *For each $c \in \mathcal{C}$ the equality $\llbracket c \boxminus c \rrbracket = \emptyset$ is true.*

Proof is trivial □

Proposition 6 (Symmetry Law for Exclusion). *For each $c', c'' \in \mathcal{C}$ the clause $c' \boxminus c'' \Vdash c'' \boxminus c'$ is true.*

Proof is trivial □

4.4 Coincidence

This relation describes synchronization of two event sources.

More precisely, let $c', c'' \in \mathcal{C}$ and $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ then $\mathcal{T} \models c' \boxplus c''$ means that there exists a strict monotonic bijection $h : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c''}$ such that $i \equiv h(i)$ for any $i \in \mathcal{I}_{c'}$.

Proposition 7 (Trivial Coincidence). *For each $c \in \mathcal{C}$ the clause $\Vdash c \boxplus c$ is true.*

Proof is trivial □

Proposition 8 (Symmetry Law for Coincidence). *For each $c', c'' \in \mathcal{C}$ the clause $c' \boxplus c'' \Vdash c'' \boxplus c'$ is true.*

Proof. Let $h : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c''}$ be a strict monotonic bijection providing the validity of the clause $\mathcal{T} \models c' \boxplus c''$ for some \mathcal{T} and h^{-1} be its inverse map. Suppose that $i', i'' \in \mathcal{I}_{c''}$, $i' \prec i''$, and $h^{-1}(i') \not\prec h^{-1}(i'')$ then either $h^{-1}(i') = h^{-1}(i'')$ or $h^{-1}(i'') \prec h^{-1}(i')$. But the first alternative contradicts to bijectivity of h , and the second alternative and strict monotonicity of h implies $i'' \prec i'$. The last clause contradicts to irreflexivity of the precedence relation. These contradictions show that h^{-1} is a strict monotonic map.

Further, for any $i \in \mathcal{I}_{c''}$ we have that $h^{-1}(i) \in \mathcal{I}_{c'}$ and $h^{-1}(i) \equiv h(h^{-1}(i)) = i$. Thus, the clause $\mathcal{T} \models c'' \boxplus c'$ is true □

Proposition 9 (Transitivity Law for Coincidence). *For each $c', c'', c''' \in \mathcal{C}$ the clause $c' \boxplus c'', c'' \boxplus c''' \Vdash c' \boxplus c'''$ is true.*

Proof is similar to proof of Proposition 4 □

4.5 Cause

This relation is intended for specifying that each instant of one clock is caused by an instant in another clock.

More precisely, let $c', c'' \in \mathcal{C}$ and $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ then $\mathcal{T} \models c' \boxdot c''$ means that there exists a strict monotonic map $h : \mathcal{I}_{c''} \rightarrow \mathcal{I}_{c'}$ such that $h(i) \preceq i$ for any $i \in \mathcal{I}_{c''}$.

Proposition 10 (Trivial Cause). *For each $c \in \mathcal{C}$ the clause $\Vdash c \boxdot c$ is true.*

Proof is trivial □

Proposition 11 (Transitivity Law for Cause). *For each $c', c'', c''' \in \mathcal{C}$ the clause $c' \boxed{\preceq} c'', c'' \boxed{\preceq} c''' \Vdash c' \boxed{\preceq} c'''$ is true.*

Proof is similar to proof of Proposition 4 □

4.6 Precedence

This relation is a stronger variant of the cause relation.

Namely, let $c', c'' \in \mathcal{C}$ and $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ then $\mathcal{T} \models c' \boxed{\prec} c''$ means that there exists a strict monotonic map $h : \mathcal{I}_{c''} \rightarrow \mathcal{I}_{c'}$ such that $h(i) \prec i$ for any $i \in \mathcal{I}_{c''}$.

Proposition 12 (Irreflexivity Law for Precedence). *For each $c \in \mathcal{C}$ the equality $\llbracket c \boxed{\prec} c \rrbracket = \emptyset$ is true.*

Proof is trivial □

Proposition 13 (Transitivity Law for Precedence). *For each $c', c'', c''' \in \mathcal{C}$ the clause $c' \boxed{\prec} c'', c'' \boxed{\prec} c''' \Vdash c' \boxed{\prec} c'''$ is true.*

Proof is similar to proof of Proposition 4 □

4.7 Interdependencies Laws for the Basic Relations

Above we considered properties of each basic relation but interdependencies between these relations were not in our focus. Thus, such interdependencies are considered below. The next lemma is needed to ground these dependencies.

Lemma 1. *Let (X, \leq) be a well-ordered set and $\phi : X \rightarrow X$ be a strict monotonic map such that for all $x \in X$ the assertion $\phi(x) \leq x$ is true then ϕ is the identity map.*

Proof. One can prove the lemma by using the transfinite induction □

Proposition 14 (Interdependencies Laws for the Basic Relations).

1. *For each $c', c'' \in \mathcal{C}$ the clause $c' \boxed{\sqsubset} c'', c'' \boxed{\sqsubset} c' \Vdash c' \boxed{\equiv} c''$ is true.*
2. *For each $c', c'' \in \mathcal{C}$ the clock relations $c' \boxed{\sqsubset} c''$ and $c' \boxed{\#} c''$ are inconsistent, i.e. $\llbracket c' \boxed{\sqsubset} c'', c' \boxed{\#} c'' \rrbracket = \emptyset$.*
3. *For each $c', c'' \in \mathcal{C}$ the clause $c' \boxed{\sqsubset} c'' \Vdash c'' \boxed{\preceq} c'$ is true.*
4. *For each $c', c'' \in \mathcal{C}$ the clause $c' \boxed{\preceq} c'', c'' \boxed{\preceq} c' \Vdash c' \boxed{\equiv} c''$ is true.*

Proof. 1) For any $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ the validity of the assertion " $\mathcal{T} \models c' \boxed{\equiv} c''$ " implies $\mathcal{T} \models c' \boxed{\sqsubset} c''$ is evident.

Let's check the validity of the inverse assertion. Denote the strict monotonic maps that provide for some $\mathcal{T} \in \mathfrak{T}^{\mathcal{C}}$ the validity of $\mathcal{T} \models c' \boxed{\sqsubset} c''$ and $\mathcal{T} \models c'' \boxed{\sqsubset} c'$

by $h_{c'' c'} : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c''}$ and $h_{c' c''} : \mathcal{I}_{c''} \rightarrow \mathcal{I}_{c'}$ respectively. We claim that they are mutually inverse.

Indeed, for any $i \in \mathcal{I}_{c'}$ we have the next coincidences: $i \equiv h_{c'' c'}(i)$ and $h_{c'' c'}(i) \equiv h_{c' c''}(h_{c'' c'}(i))$. These coincidences and the Transitivity Law for Coincidence (see Proposition 9) provide the validity of the coincidence $i \equiv h_{c' c''}(h_{c'' c'}(i))$. Taking into account that both i and $h_{c' c''}(h_{c'' c'}(i))$ are elements of $\mathcal{I}_{c'}$ and the fact that restriction of \preceq on $\mathcal{I}_{c'}$ is a strict order (see Proposition 1) one can derive the equality $i = h_{c' c''}(h_{c'' c'}(i))$.

The equality $i = h_{c'' c'}(h_{c' c''}(i))$ for all $i \in \mathcal{I}_{c''}$ is derived similarly. Thus, $h_{c'' c'}$ is a bijection.

2) Proof is trivial.

3) Proof is trivial.

4) Really, let $h_{c'' c'} : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c''}$ and $h_{c' c''} : \mathcal{I}_{c''} \rightarrow \mathcal{I}_{c'}$ be strict monotonic maps provided for some $\mathcal{T} \in \mathfrak{T}^{\mathcal{T}}$ the validity of the clauses $\mathcal{T} \models c'' \sqsubseteq c'$ and $\mathcal{T} \models c' \sqsubseteq c''$ respectively. Then the map $\phi = h_{c' c''} \circ h_{c'' c'} : \mathcal{I}_{c'} \rightarrow \mathcal{I}_{c'}$ is strict monotonic and it satisfies the condition $\phi(i) \preceq i$. Therefore, applying the Lemma 1 allows to conclude that ϕ and the identity map are equal \square

5 Runs and Chronometers

Following [1], in this section we introduce the notion of a run for a set of clocks. We use this notion to define a behavioural model for the set of clocks.

Definition 3 (see [1]). *Let \mathcal{C} be a finite set of clock then any map $\mathbf{r} : \mathbb{N} \rightarrow \mathbf{2}^{\mathcal{C}}$ such that $\mathbf{r}(t) = \emptyset$ implies $\mathbf{r}(t') = \emptyset$ for all $t' > t$ is called a run for \mathcal{C} .*

This definition means that if \mathbf{r} is a run then at the (global) time t all clocks of the set $\mathbf{r}(t)$ and only them are triggered.

For each run \mathbf{r} one can construct a quadruple $\mathcal{T}[\mathbf{r}] = (\mathcal{I}_{\mathbf{r}}, \preceq, \mathcal{C}, \pi_{\mathbf{r}})$ by the following way:

- $\mathcal{I}_{\mathbf{r}} = \{(c, t) \in \mathcal{C} \times \mathbb{N} \mid c \in \mathbf{r}(t)\}$;
- $(c', t') \preceq (c'', t'')$ if and only if $t' \leq t''$;
- $\pi_{\mathbf{r}}(c, t) = c$ for all $(c, t) \in \mathcal{I}_{\mathbf{r}}$.

Proposition 15. $\mathcal{T}[\mathbf{r}]$ is a time structure with clocks for given run \mathbf{r} .

Proof. It is proved by trivial checking properties (2) and (3) \square

Hence, we can define the semantic relationship between a run \mathbf{r} and a constraint \mathfrak{C} by the next way: $\mathbf{r} \models \mathfrak{C}$ if and only if the clause $\mathcal{T}[\mathbf{r}] \models \mathfrak{C}$ is true. Also, we can introduce the relationship $\mathfrak{C}_1 \Vdash_{run} \mathfrak{C}_2$ as an abbreviation of the sentence "for any \mathbf{r} such that $\mathbf{r} \models \mathfrak{C}_1$ the next relationship $\mathbf{r} \models \mathfrak{C}_2$ is valid".

Proposition 15 allows to suggest that a run carries more information than a time structure because a run depends on global time. A refinement and a substantiation of this hypothesis is discussed below.

The notion of chronometer is introduced to specify dependences between time structures and runs.

Definition 4. Let $\mathcal{T} = (\mathcal{C}, \mathcal{I}, \preceq, \pi)$ be a time structure with clocks and $\chi : \mathcal{I} \rightarrow \mathbb{N}$ be a map such that the next assertions are true:

$$\text{for any } i', i'' \in \mathcal{I} \text{ the coincidence } i' \equiv i'' \text{ implies } \chi(i') = \chi(i'') \quad (4)$$

$$\text{for any } i', i'' \in \mathcal{I} \text{ the strict precedence } i' \prec i'' \text{ implies } \chi(i') < \chi(i'') \quad (5)$$

$$\text{for any } t, t' \in \mathbb{N} \text{ the validity of the clauses } t \in \chi(\mathcal{I}) \text{ and } t' < t \text{ implies truth of the clause } t' \in \chi(\mathcal{I}) \quad (6)$$

then χ is called a chronometer on \mathcal{T} [9].

Example 1. Let \mathcal{C} be a finite set of clocks, \mathbf{r} be a run for \mathcal{C} . Then it is evident that the map $\chi_* : \mathcal{I}_{\mathbf{r}} \rightarrow \mathbb{N}$ determined by the equality $\chi_*(c, t) = t$ is a chronometer.

Hence, Example 1 shows that each time structure generated by a run has a native chronometer χ_* .

Proposition 16. Let \mathcal{T} be a time structure with clocks and $\chi : \mathcal{I} \rightarrow \mathbb{N}$ be a chronometer then the map $\mathbf{r}[\mathcal{T}, \chi] : \mathbb{N} \rightarrow \mathbf{2}^{\mathcal{C}}$ defined by the next formula

$$\mathbf{r}[\mathcal{T}, \chi](t) = \pi(\chi^{-1}(t)) \quad (7)$$

is a run.

Proof. To prove the proposition we should show that $\mathbf{r}[\mathcal{T}, \chi](t) = \emptyset$ for some $t \in \mathbb{N}$ implies $\mathbf{r}[\mathcal{T}, \chi](t') = \emptyset$ for any $t' \geq t$.

Suppose existence of t_1 and t_2 such that $t_1 < t_2$, $\pi(\chi^{-1}(t_1)) = \emptyset$, but $\pi(\chi^{-1}(t_2)) \neq \emptyset$. Taking into account this assumption one can derive that $\chi^{-1}(t_1) = \emptyset$ and $\chi^{-1}(t_2) \neq \emptyset$. Hence, $t_1 \notin \chi(\mathcal{I})$ and $t_2 \in \chi(\mathcal{I})$. We have obtained the contradiction to condition (6) of Definition 4 \square

The next property for the chronometer χ_* from Example 1 holds.

Proposition 17. Let \mathbf{r} be a run for a clock set \mathcal{C} then the next equality holds

$$\mathbf{r}[\mathcal{T}[\mathbf{r}], \chi_*] = \mathbf{r}. \quad (8)$$

Let $\mathcal{T} = (\mathcal{C}, \mathcal{I}, \preceq, \pi)$ be a time structure with clocks and $\chi : \mathcal{I} \rightarrow \mathbb{N}$ be a chronometer on \mathcal{T} then the map $\widehat{\chi} : \mathcal{I} \rightarrow \mathcal{C} \times \mathbb{N}$ defined in the next way $\widehat{\chi}(i) = (\pi(i), \chi(i))$ is a map onto $\mathcal{I}_{\mathbf{r}[\mathcal{T}, \chi]}$ such that any coincidence $i' \equiv i''$ implies the coincidence $\widehat{\chi}(i') \equiv \widehat{\chi}(i'')$ in $\mathcal{T}[\mathbf{r}]$ and any precedence $i' \prec i''$ implies the precedence $\widehat{\chi}(i') \prec \widehat{\chi}(i'')$ in $\mathcal{T}[\mathbf{r}]$.

Proof. Really,

$$\begin{aligned} \mathbf{r}[\mathcal{T}[\mathbf{r}], \chi_*](t) &= \pi_{\mathbf{r}}(\chi_*^{-1}(t)) = \\ &= \pi_{\mathbf{r}}(\{(c, t) \in \mathcal{I}_{\mathbf{r}}\}) = \pi_{\mathbf{r}}(\{(c, t) \in \mathcal{C} \times \mathbb{N} \mid c \in \mathbf{r}(t)\}) = \mathbf{r}(t). \end{aligned}$$

Further, $(c, t) \in \mathcal{I}_{\mathbf{r}[\mathcal{T}, \chi]}$ if and only if $c \in \mathbf{r}[\mathcal{T}, \chi](t)$. It is easy to see that the last clause is equivalent to existence of $i \in \mathcal{I}$ such that $c = \pi(i)$ and $t = \chi(i)$, i.e. it is equivalent to $(c, t) = \widehat{\chi}(i)$.

If $i' \equiv i''$ then $\chi(i') = \chi(i'')$ by definition of a chronometer, hence $\widehat{\chi}(i') \equiv \widehat{\chi}(i'')$. Similarly, if $i' \prec i''$ then $\chi(i') < \chi(i'')$, therefore $\widehat{\chi}(i') \prec \widehat{\chi}(i'')$ \square

Proposition 18. *There exists only one chronometer on $\mathcal{T}[\mathbf{r}]$ for any run \mathbf{r} .*

Proof. For any run \mathbf{r} there exists the chronometer χ_* on $\mathcal{T}[\mathbf{r}]$. Let χ be an other chronometer on $\mathcal{T}[\mathbf{r}]$. For $(c', t), (c'', t) \in \mathcal{I}[\mathbf{r}]$ using (4) we have $\chi(c', t) = \chi(c'', t)$. Hence, taking into account Definition 3 one can obtain that $\chi(c, t) = \tau(t)$ where τ is strict monotonic function from α into α for some cardinal $\alpha \leq \omega$. Thus, τ is the identity function and $\chi = \chi_*$ \square

Hence, a chronometer exists on a time structure associated with a run. We claim that a chronometer exists on any time structure with clocks.

The next binary relation \triangleleft on a time structure with clocks will be used for describing an algorithm that calculates timestamps for instants. More precisely, if $i', i'' \in \mathcal{I}$ then $i' \triangleleft i''$ means that for all $i \in \mathcal{I}$ the validity of the next clause $i \prec i'' \ \& \ i' \preceq i$ implies truth of the coincidence $i \equiv i'$. It is easy seen that if $i_1 \equiv i'_1, i_2 \equiv i'_2$, and $i_1 \triangleleft i_2$ then $i'_1 \triangleleft i'_2$.

Now we can construct the algorithm that allows to calculate timestamps for instants on an arbitrary time structure with clocks. This Algorithm 1 is a generalization of Lamport's algorithm [9].

Algorithm 1: Computing timestamp for an instant

```

input :  $\mathcal{T} = (\mathcal{C}, \mathcal{I}, \preceq, \pi)$  is a time structure with clocks,
          $i$  is an element of  $\mathcal{I}$ 
output: timestamp for the instant  $i$ 

1 begin
2   count  $\leftarrow$  1;  $D \leftarrow \emptyset$ ;  $W \leftarrow \emptyset$ ;      // -- initializing work variables --
3   while  $i \notin D$  do                                     // -- main loop -----
4      $W_+ \leftarrow \{j \in \mathcal{I} \mid j \notin D \ \& \ \text{idx}(j) = \text{count}\}$ ;
5      $W_+ \leftarrow W_+ \cup \{j \in \mathcal{I} \mid j \notin D \ \& \ (\exists j' \in W_+) j' \equiv j\}$ ;
6      $W \leftarrow W \cup W_+$ ;
7      $D_+ \leftarrow \{j \in W \mid (\forall j' \in \mathcal{I})(j' \triangleleft j \Rightarrow j' \in D)\}$ ;
8      $D \leftarrow D \cup D_+$ ;
9      $W \leftarrow W \setminus D_+$ ;
10    count  $\leftarrow$  count + 1;
11  end
12  return count;
13 end

```

Theorem 1 (existence of a chronometer). *Let \mathcal{T} be a time structure with clocks and $\chi_0 : \mathcal{I} \rightarrow \mathbb{N}$ be the function calculated by Algorithm 1 then χ_0 is a chronometer on \mathcal{T} .*

Proof. One can see that Algorithm 1 builds two sequences of sets

$$D_0 \subset D_1 \subset D_2 \subset \dots \subset D_n \subset \dots$$

$$W_0, W_1, W_2, \dots, W_n, \dots$$

in accordance to the following computational scheme:

$$\begin{cases} W_0 &= \emptyset \\ D_0 &= \emptyset \\ W_{n+1} &= (W_n \cup \{j \in \mathcal{I} \mid (\exists j' \in \mathcal{I})(j' \equiv j \ \& \ \text{idx}(j') = n + 1)\}) \setminus D_n \\ D_{n+1} &= D_n \cup \{j \in W_{n+1} \mid (\forall j' \in \mathcal{I})(j' \triangleleft j \Rightarrow j' \in D_n)\} \end{cases}$$

and maps an instant $i \in \mathcal{I}$ into $\chi_0(i) = \inf\{n \in \mathbb{N} \mid i \in D_n\}$.

Firstly, note that supposition about partial definiteness of χ_0 implies existence of an infinite sequence $i_1 \triangleright i_2 \triangleright \dots$. But it contradicts the causes finiteness property (2).

Secondly, it is true by the construction of D_n that the validity of $i' \equiv i''$ implies the truth of the following statement: $i' \in D_n$ if and only if $i'' \in D_n$. Hence, we obtain that $i' \equiv i''$ implies $\chi_0(i') = \chi_0(i'')$.

Further, similar reasoning provides the validity of the following statement: $i' \prec i''$ implies $\chi_0(i') < \chi_0(i'')$.

Finally, the simple inequality $\text{idx}(i) \leq \chi(i)$, which is correct for any $i \in \mathcal{I}$ and any chronometer χ on \mathcal{T} , provides the validity of property (6) \square

Corollary 2. *There exists a chronometer on an arbitrary time structure with clocks.*

6 Equivalence of Semantics for RCCSL Determined by Relations \Vdash and \Vdash_{run}

In the section the notion of a chronometer is used to prove the theorem about equivalence of the relationships \Vdash and \Vdash_{run} . The theorem is the main result of the paper. Taking into account the theorem one can confine himself to checking semantic consequence by using runs. This opens a way to constructing an operational semantics of RCCSL so that it is equivalent to the denotational semantics defined above.

We need two lemmas to prove the main theorem.

Let's use the notation $i_1 \parallel i_2$ for instants i_1 and i_2 such that $i_1 \not\prec i_2$ & $i_2 \not\prec i_1$.

Lemma 2. *Let $\mathcal{T} = (\mathcal{C}, \mathcal{I}, \preceq, \pi)$ be a time structure with clocks and i_1, i_2 be instants such that the clause $i_1 \parallel i_2$ is true then there exists a chronometer χ on \mathcal{T} satisfied the following condition $\chi(i_1) < \chi(i_2)$.*

Proof. Let's consider the quadruple $\mathcal{T}' = (\mathcal{C}, \mathcal{I}, \preceq', \pi)$ such that $i' \prec' i''$ is valid if one of the next conditions is true

1. $i' = i_1$ and $i'' = i_2$;
2. $i' \prec i''$;
3. $i' \prec i_1$ and $i_2 \prec i''$;

and $i' \preceq' i''$ if and only if $i' \equiv i''$ or $i' \prec' i''$. It is easy seen that the relation \preceq' is a pre-order. More over, it satisfies properties (2) and (3). Hence, \mathcal{T}' is

a time structure with clocks. Using Corollary 2 we obtain that there exists a chronometer χ on \mathcal{T}' . But then χ is a chronometer on \mathcal{T} and $\chi(i_1) < \chi(i_2)$ is true \square

Corollary 3. *Let $\mathcal{T} = (\mathcal{C}, \mathcal{I}, \preceq, \pi)$ be a time structure with clocks, $i', i'' \in \mathcal{I}$ be instants, then*

1. $i' \prec i''$ is valid if and only if for any chronometer χ on \mathcal{T} the inequality $\chi(i') < \chi(i'')$ is true;
2. $i' \equiv i''$ is valid if and only if for any chronometer χ on \mathcal{T} the equality $\chi(i') = \chi(i'')$ is true.

Lemma 3. *Let $\mathcal{T} = (\mathcal{C}, \mathcal{I}, \preceq, \pi)$ be a time structure with clocks, $\boxed{*}$ be an arbitrary sign of a clock relation, c' and c'' be clocks then $\mathcal{T} \models c' \boxed{*} c''$ if and only if $\mathbf{r}[\mathcal{T}, \chi] \models c' \boxed{*} c''$ for any chronometer χ on \mathcal{T} .*

Proof. It is evident that $\mathcal{T} \models c' \boxed{*} c''$ implies $\mathbf{r}[\mathcal{T}, \chi] \models c' \boxed{*} c''$ for any chronometer χ on \mathcal{T} . Hence, we need to prove the inverse statement.

1) Suppose that $\mathbf{r}[\mathcal{T}, \chi] \models c' \boxed{\subset} c''$ for any chronometer χ on \mathcal{T} . Then for any $i \in \mathcal{I}_{c'}$ and for each chronometer χ there exists an instant $i^\chi \in \mathcal{I}_{c''}$ such that $\chi(i) = \chi(i^\chi)$. Denote by X the set formed all i^χ . It is a nonempty subset of $\mathcal{I}_{c''}$. Suppose that there exists at least two different elements in the set X . Let's denote them by i^{χ_1} and i^{χ_2} . Taking in account linearity of the order on $\mathcal{I}_{c'}$ and $i^{\chi_1} \neq i^{\chi_2}$ one can suppose that $i^{\chi_1} \prec i^{\chi_2}$. Therefore $\chi_1(i) = \chi_1(i^{\chi_1}) < \chi_1(i^{\chi_2})$. Thus, one of the two cases is realised: $i \prec i^{\chi_2}$ or $i \parallel i^{\chi_2}$. But in the first case we obtain the inequality $\chi_2(i) < \chi_2(i^{\chi_2})$, which contradicts to the choice of i^{χ_2} . Hence, $i \parallel i^{\chi_2}$ is true. Similarly, one can obtain that $i \parallel i^{\chi_1}$ is true. Therefore, we proved that $|X| > 1$ implies $i \parallel i^\chi$ for all $i \in \mathcal{I}_{c'}$ and any chronometer χ .

Let $i^* = \inf_{\chi \in X} i^\chi$ then $i \parallel i^*$ and $\chi(i^*) \leq \chi(i^\chi) = \chi(i)$. This is a contradiction because Lemma 2 provides existence of some chronometer χ_0 such that $\chi_0(i^*) > \chi_0(i)$. Hence, X contains only one element, which we denote by $h(i)$. By construction we have $\chi(i) = \chi(h(i))$ for any chronometer χ . The last property implies strict monotonicity of h and the coincidence $i \equiv h(i)$. Therefore, $\mathcal{T} \models c' \boxed{\subset} c''$.

2 and 3) Suppose that $\mathbf{r}[\mathcal{T}, \chi] \models c' \boxed{*} c''$ for any chronometer χ on \mathcal{T} then it is evident that $\mathcal{T} \models c' \boxed{*} c''$ where $\boxed{*}$ equals to $\boxed{\#}$ or $\boxed{=}$.

4 and 5) Suppose that $\mathbf{r}[\mathcal{T}, \chi] \models c' \boxed{*} c''$ for any chronometer χ on \mathcal{T} where $\boxed{*}$ equals to $\boxed{\preceq}$ or $\boxed{\prec}$. Similarly, in the first case one can derive that $\mathcal{T} \models c' \boxed{*} c''$ is true \square

Theorem 2 (about equivalence of semantics). *Let \mathcal{C} be an arbitrary finite set of clocks, \mathfrak{C}_1 and \mathfrak{C}_2 be RCCSL constraints then the $\mathfrak{C}_1 \Vdash \mathfrak{C}_2$ is true if and only if $\mathfrak{C}_1 \Vdash_{run} \mathfrak{C}_2$ is true.*

Proof. One can easily see that the Theorem is the direct consequence of the Lemma 3 \square

7 Conclusion

In the paper we have considered the pure relational subset of CCSL (RCCSL) and have introduced semantics for it by using a class of mathematical objects called by authors time structures with clocks.

We have studied semantic properties of RCCSL (see Propositions 3 – 9). We hope that these properties can be a background of an axiomatic basis for analysing relational clock constraints.

Further we have introduced the notions "a run" and "a chronometer". It allowed us to study interrelations between time structures and runs, to introduce the alternative semantics closer to the operational approach than the denotational semantics discussed earlier.

Finally, the main theorem about equivalence of these two semantics (see Theorem 2) has been proved.

We are planning to continue our research in the next areas:

- building an axiomatic theory of the semantic consequence for RCCSL constraints;
- extending results on complete CCSL;
- studying an operational semantics of CCSL and specifying its interrelations to the denotational semantics.

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