Diagrammatic Autarchy.

Linear diagrams in the 17th and 18th centuries.

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***Abstract*—This paper explores the notion of autarchy of diagrammatic notations for logic debated in the German-speaking world of the 18th-century, especially as applied to linear diagrams invented by G. W. Leibniz and J. H. Lambert.**

Keywords— linear diagrams; autarchy; Leibniz; Lambert; Ploucquet

INTRODUCTION

In this paper we explore the notion of autarchy of diagrammatic representation that was debated in the German-speaking world in the 18th century. What is diagrammatic autarchy? In one of his writings, Leibniz claimed that one of the aims of the *characteristica universalis* (his big project of a general formal and deductive method for science) is to find "autarchic" (αυτάρκεις) characters: "One must know that characters are more perfect the more they are *autarchic*, in such a way that all the consequences can be derived from them" [1, pp. 800-801]. We will use this Leibnizian term to indicate an important property of some diagrammatic representations, and we will try to show that much of the debate about diagrams and iconic representations in the 18th century, largely reported in [2], may be considered as a debate about the notion of autarchy. Of course, there is much more in that debate than the discussion of diagrams for syllogistic [3; 4, ix]. However, we believe that the notion of autarchy is able to capture an important aspect of that debate. We will focus on the linear diagrams invented by Leibniz and Lambert and discussed in the German logical panorama of that time.

We do not attempt to answer the question whether or not the notion of autarchy might be re-phrased or explained in the terms of some contemporary theory of diagrammatic reasoning or read through a more sophisticated logical-philosophical conception, although we will mention a couple of interesting parallels in the last section. Our principal aim here is to understand what these thinkers thought about diagrammatic representations, and especially what their criteria were to believe that one system of diagrammatic representation is better than, or preferable to, another with respect to its being more or less autarchic. To be precise, therefore, our reconstruction contributes not so much to the history of logic diagrams, but to the history of the *ideas* about logic diagrams, or to the history of the *philosophy* of diagrams.

I. LEIBNIZ

In some of his writings, Leibniz (1646-1716) claimed that the aim of the characteristic is to find (*adhibire*) characters such that all the consequences can be derived from them. Such characters are "autarchic" (αυτάρκεις). Paraphrasing Heinrich Hertz's famous maxim, Leibniz's ideal of diagrammatic autarchy amounts to this, that the necessary logical consequences of the diagram are always the diagram of the natural necessary consequences of imagined object [5, p. 75].

In his mathematical and logical works, Leibniz worked out different examples of "autarchic" systems of symbols. For example, the binary notation is said to be more autarchic than the decimal in that in the binary "all that can be affirmed about numbers can be demonstrated from their characters" [1, p. 800], which is not true for the decimal. Further, Leibniz considered algebra as an imperfect instrument for treating geometry; algebra is only the characteristic of indeterminate numbers or magnitudes (grandeurs), but does not express places, angles and motion. A more perfect system of geometrical notation (*characteristica geometrica*) is therefore imaginable in which the simple enunciation of the problem is already its solution, or one in which the enunciation, the construction and the demonstration are one and the same thing [1, p. 910; 6, II, pp. 20-21, 228-229; 6, V, pp. 141ff).

The notion of autarchy also applies to logical notations. In his 1686 "Generales Inquisitiones de Analysi Notionum et Veritatum" [7, pp. 356-399] and in other writings of roughly the same period [7, pp. 206-210, 247-249, 292-321], Leibniz proposes a system for representing propositions and syllogisms by means of linear diagrams. Such diagrams, as one of these writings says, are expressly intended as a "demonstration of the logical form" (*de forma logicae comprobatione per linearum ductus* [7, p. 292].

In Leibniz's linear system (Figures 1-4) the extension of concepts is represented by parallel straight lines, while the dotted vertical lines indicate the relation of inclusion or exclusion among concepts: when the vertical lines cut off real segments on each parallel, the proposition is affirmative, when they pass entirely outside of one or both the parallels the proposition is negative (cf. [8; 4, viii]). Leibniz claims that this system is capable of showing which of the four propositional forms are convertible and which are not. The diagrams of the universal negative and of the particular affirmative (Figures 2, 3) are symmetrical, and therefore these propositions are convertible (*conversio simplex*: "No B is C" is convertible into "No C is B"; the same applies to the particular: "Some B is C" is convertible into "Some C is B"); The diagrams of the universal affirmative and of the particular negative (Figures 1, 4) are not symmetrical, and therefore these propositions are not convertible. Of course the universal affirmative is convertible into a particular (*per accidens*: "All A are B", then "Some B is A").

It is important to note that, besides these linear diagrams, Leibniz draws the correspondent circular diagrams in the way Euler would do later. To differentiate the circular diagram of the particular affirmative from that of the particular negative, he uses letters (Figures 3, 4). In the circular diagrams the letters are placed in such a way as to indicate the nature of the proposition, whether affirmative or negative. In the linear diagrams this expedient is not necessary, for the figure shows by itself whether the particular proposition is affirmative or negative. Therefore, Leibniz believes, the linear are more autarchic than the circular diagrams, for in the latter the figure is not self-sufficient in determining whether the proposition is affirmative or not: we must use a conventional or symbolical devise in order to differentiate the two forms.

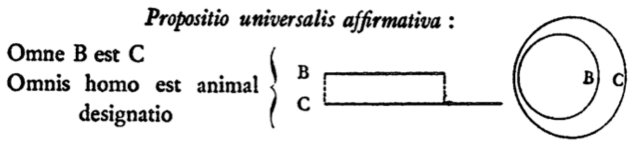

Fig. 1. [7], p. 292

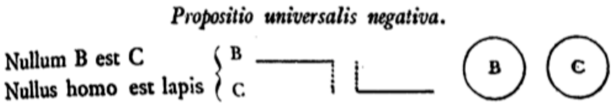

Fig. 2. [7], p. 293

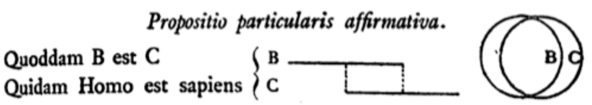

Fig. 3. [7], p. 293

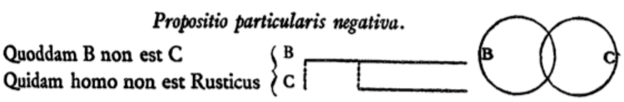

Fig. 4. [7], p. 293

Leibniz also proposes a version of these diagrams in which the part of the line which is relevant for the affirmation or negation is doubled [7, pp. 311-312] (see Figure 5). This method – that is, to double the part of the line which is affirmed or denied of the other – is important because it represents visually what Leibniz calls the distribution or non-distribution of the terms, that is their quantity. A term is universal if its line is completely doubled; it is particular if its line is only partially doubled. In the universal affirmative, for example, the line of the subject is completely doubled, and so the subject is universal, while in the particular the line of the subject is only partly doubled, and so the subject is particular.

In order to construct the diagram of the syllogism, Leibniz draws the major premise and then, using the line of the middle term already drawn, adds the minor premise. To obtain the conclusion, he draws two continuous vertical lines starting from the double part of minor term towards the major term. If these continuous verticals cut off a real segment of the other extremes, then the conclusion is affirmative. If they fall outside it, the conclusion is negative. For example in *Barbara* (Figure 6), the two continuous vertical lines from D fall entirely on B, and so the conclusion is affirmative. Further, all D is taken into consideration – its line is completely doubled - and so the conclusion is universal: "All D are B". In *Camestres* (Figure 7) the two continuous lines are again drawn from D to B, but they fall outside B, and therefore the conclusion is negative. Further, all D is again taken into consideration, so the conclusion is universal: "No D is B".

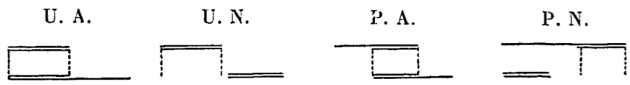

Fig. 5. [7], p. 311-12

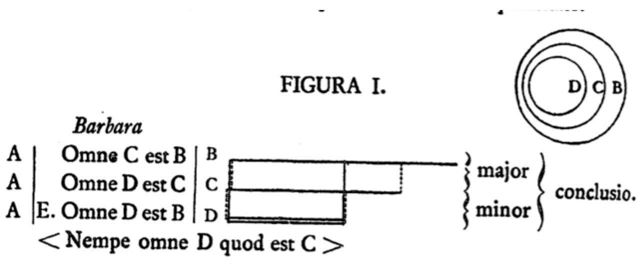

Fig. 6. [7], p. 294

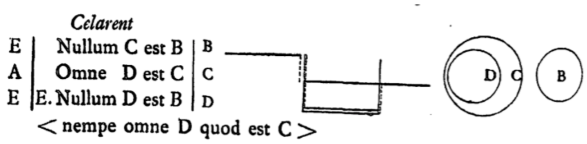

Fig. 7. [7], p. 295

II. LAMBERT

Johann H. Lambert (1728-1777) calls "scientific" those signs that are so constructed as to serve as perfect substitutes for their objects. The more a system of signs can be made object of reasoning according to simple rules, the more scientific it will be: "The signs of concepts and things are scientific in the stricter sense if they not only represent in general those concepts and things, but also indicate relationships such that the theory of the object and the theory of its signs can be interchanged" [9, III, § 23].

Lambert's *Zeichnungsart*, his system of linear diagrams, is quite similar to Leibniz's. It is not clear whether Lambert knew Leibniz's diagrams, as most of the relevant texts have been published later. In his *Neues Organon* [9, I, §§ 173-194], Lambert represents concepts by means of lines, propositions as relations between two lines, and syllogisms as relations between three lines. Lines may be either closed or open (having dotted extremities), depending on the certainty or uncertainty of the distribution of the terms represented by them (i.e. depending on the quantity of these terms). The four traditional propositional forms are represented as in Figures 8-11. The use of uppercase and lowercase letters at the extremities of the continuous segments is of no use at all, and may be easily ignored.

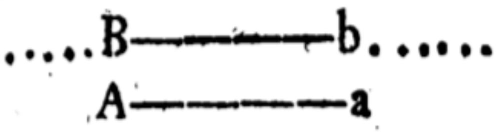

Fig 8. [9], I, § 181

Fig 9. [9], I, § 183

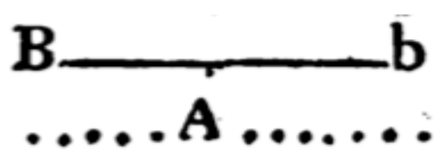

Fig 10. [9], I, § 184

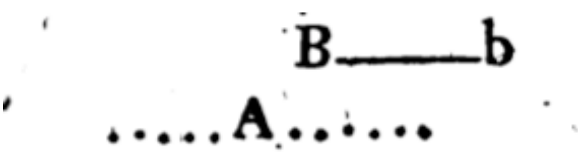

Fig. 11. [9], I, § 184

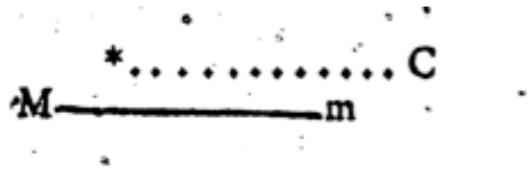

Fig. 12. [2], p. 218

Lambert claims that his *Zeichnungsart* not only shows what relations obtain among concepts, but also shows what other relations may be deduced therefrom by the mere observation of the figures [8, I, §§ 191, 194]. Like Leibniz, Lambert claims that his diagrams are capable of distinguishing the different propositional forms one from another; further, each of the four propositional forms has its own diagram, which is different from the others, so that there is no risk that different propositions might be represented by the same diagram, or that different diagrams represent the same proposition.

But further, Lambert claims that these diagrams always and necessarily indicate what parts of a concept are undetermined, that is, they express our imperfect knowledge about a concept's extension, therefore showing whether or not a proposition is convertible. For example, take the universal affirmative (Figure 8); we may convert it *per accidens* into the particular affirmative "Some B are A" simply by reading the diagram top-down instead of bottom-up. The dotted part of the line indicates that, when converted, the corresponding term is to be taken particularly (Some B). Likewise, the universal negative (see Figure 9) may be simply converted (*conversio simplex*) by reading it from the right to the left ("No B is A"). This means that the same diagram can express different propositions depending on the way we read it.

It is not clear how things stand with the particular affirmative (Figure 10). Reading it top-down, as we do for the universal affirmative, would not give us the converted proposition. We would like to read it top-down as "All B are some A", which introduces the quantification of the predicate, but Lambert would not have been happy with that (he famously opposed the quantification of the predicate maintained by G. Ploucquet).

If we compare Lambert's diagrams to Leibniz's, we see that while Leibniz's diagram for the particular affirmative is symmetrical, thus suggesting simple conversion (Figure 3) Lambert's diagram, on the contrary, is not symmetrical, and does not show whether and how the proposition can be converted (Figure 10).

It has further to be noted that Lambert proposes different ways to draw these linear diagrams. Figure 12 represents an alternative way of diagramming the particular negative Some M are not C. Lambert marks by an asterisk the limit of the extension, that is, the point beyond which the extension of a term cannot go without invalidating the proposition. For example, if we allow the dotted line of C to surpass the asterisk, the line C would extend to cover completely the line M, and the proposition "Some M are not C" would be false [2, p. 218]

Lambert however insisted on a point that was of crucial importance for him. The idea is that those premises from which something follows should be capable of being diagrammed, while those from which nothing follows should not: "I begin by drawing the middle term, and then I draw either of the other two terms. If the third is capable of being drawn, then the representation gives me anything that follows immediately from the premises. If the third term cannot be drawn, then nothing follows therefrom" [2, p. 152].

Let us take the two negative premises "No M is P" and "No S is M". I begin by drawing the middle term M (Figure 13). Then I draw the major term P (Figure 14) so as to place it completely outside M (for no M is P). Now I should represent that "No S is M". So I have to represent the third term, the minor term S, so as to exclude it from M, too. There are at least two geometrical possibilities here, for I can draw S either below P or not (see Figure 15). Since I am not entitled to choose between these possibilities, no conclusion follows from these two premises.

If, on the other hand, one of the premises were either a universal affirmative or a particular, things would be different. For example, if the second premise were "All S are M", it could well be represented, for there is just one possible place to draw the line of S (see Figure 16). This is the valid syllogistic form of first figure *Celarent*.

Figures 17 and 18 represent the diagrams of the first and second figure according to Lambert [9, I, § 219]. In his *Zeichnungsart*, Lambert argues, everything that is relevant for the syllogistic calculation is represented; once a couple of propositions is diagrammed, one immediately sees whether something follows from it or not, and this is all that is required to have a scientific or autarchic system of notation.

While Leibniz's linear diagrams were not known in his times, Lambert's method was much debated in the scientific community of 18^{th} century German-speaking world. Georg Jonathan Holland (1742–1784), in the *Anhang* to his *Abhandlung über die Mathematik* [2, pp. 95-108], compared Lambert's logical calculus to that of his Tübingen professor Gottfried Ploucquet. Holland claims that Lambert's system of linear diagrams is not a real characteristic, as it is possible in it to represent premises from which false conclusions follow.

Let us take the premises: "All P is O", and "No A is P". If we represent them as Holland does in the *Anhang* (see Figure 19), then the conclusion seems to follow that "No O is A", which is a false conclusion. Lambert's method of diagrams seems therefore imperfect, for in it it is possible to infer a false conclusion. But Lambert replies that Holland's diagram for this syllogism is wrong: "The extension of the line O is greater than P, but *indeterminately greater*. And therefore it must in this case be dotted" [2, p. 151]. When the proposition "All P is O" is represented as in Figure 20, we see that it is not the entire line O which is excluded from the line A, but only the continuous part of it that coincides with P. So we must conclude *not* that "No O is A", but only that "Some O are not A", which is the right conclusion and which gives us the valid syllogistic form *Fesapo* of the fourth figure. This indicates why Lambert attaches so much importance to the expression of the quantification of concepts by means of dotted lines. Without this graphic device, the system may yield false conclusions.

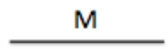

Fig. 13. Middle term "M"

Fig. 14. "No M is P"

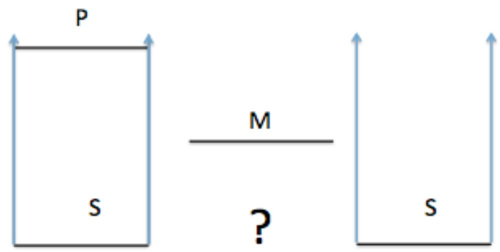

Fig. 15. Two geometrical possibilities for "No S is M"

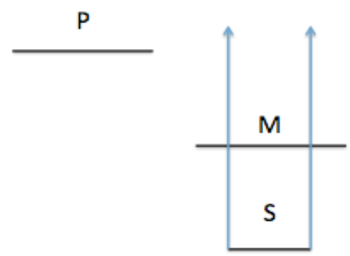

Fig. 16. One geometrical possibility for "All S are M"

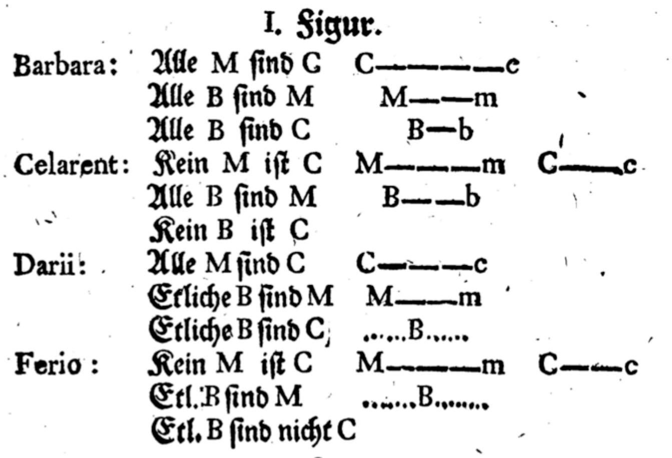

Fig. 17. [9], I, § 219

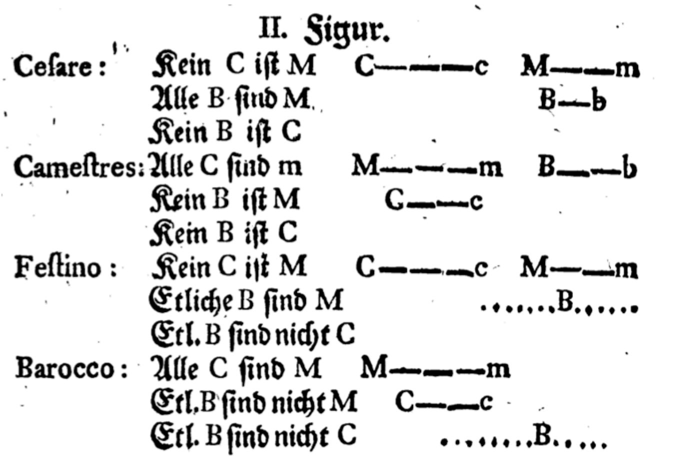

Fig. 18. [9], I, § 219

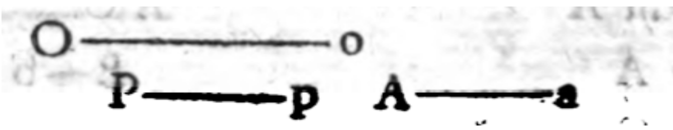

Fig. 19. [2] p. 104

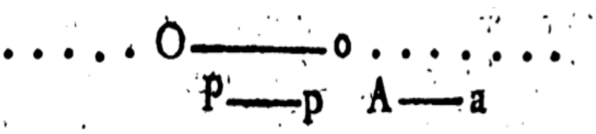

Fig. 20. [2] p. 106

III. Ploucquet

Mention has to be made in this context of Gottfried Ploucquet (1716-1790), professor of philosophy at Tübingen and famous for having introduced in logic the quantification of the predicate. Although he was somehow skeptical about the idea of a universal characteristic both in the sense of a *universal calculus* and in the sense of a *universal language*, he nonetheless invented different systems of logical representation, including graphical and algebraical. His diagrams for syllogism are quite similar to Euler's circles [2, pp. 6-8, 157-158] (Figures 21, 22).

However, Ploucquet's main interests lie in symbolic notations. His fundamental idea is that every affirmative proposition states an identity between subject and predicate: "The judgment is not the cognition of two things, but of just one; and the affirmative proposition reflects this by expressing one thing by different signs" [2, p. 52]. The theory of the identity of subject and predicate in an affirmative proposition is the ground of Ploucquet's much discussed "quantification of the predicate": not only the subject but also the predicate of a categorical form is qualified by means of a quantifier expression 'omne' (all) or 'quoddam' (some). If I affirm, "All men are animal", animal is here taken particularly, that is, as "some animal", so that the proposition actually affirms that "all men are some animal". As a consequence, Ploucquet claims that each categorical form can be converted: since conversion consists in nothing else but exchanging subject and predicate, each categorical form is convertible, provided that the quantity of the predicate is made explicit by adding the "quantifiers". In his symbolic notation, he uses uppercase letters for universally quantified terms, lowercase letters for particularly quantified terms, the symbol > for negation, and juxtaposition for affirmation (see Figure 23).

In the debate with Lambert, Ploucquet moves several objections to Lambert's system of diagrams. First, he claims that Lambert's system has no specific sign to show whether a term is universal or particular (as he does in his own symbolic notation) [2, pp. 166-167]. Secondly, the diagram in Figure 24 can be read either as "All A are B" or as "Some B are A", which latter is the former proposition converted *per accidens*. Since Ploucquet does not accept the traditional version of the doctrine of conversion, these are two different propositions for him, and each has to have its own diagram. This can be done, he claims, if we mark graphically whether a term is universal or particular.

Thirdly, Ploucquet observes that the representation of our imperfect knowledge about a concept's extension by means of dots is of no use at all [2, p. 170]. Again, in the diagram in Figure 24, the dotted part represents that we do not know whether there are B that are not A, and that the only relevant part of the assertion is that all A are B. Since Ploucquet believes that in this proposition subject and predicate should be identical, he needs not employ the dots to represent our imperfect knowledge about B. For him, there is no such a thing as imperfect knowledge about a concept's extension.

Lambert's reply is that that which Ploucquet considers as a fault of the linear system - representing undetermined concepts by means of open or dotted lines – is on the contrary a virtue of it. For if we agree that "All A are B" may cover both the case in which B is greater than A (B>A) and the case in which B is identical with A (B = A), then the use of the dotted lines is of the utmost importance: we are obliged to represent both the determined and the undetermined part of a concept's extension. By the device of the dotted lines, this indetermination is appropriately "made intuitive" (*diese Unbestimmtheiten recht augenscheinlich zu machen*) [2, p. 215].

Ploucquet in its turn proposes an amendment of Lambert's linear diagrams (Figures 25-27). In these diagrams any concept is expressed by a straight line as in Lambert's system, but the quantity of the terms is not expressed by continuous or dotted lines, but by uppercase letters for universal concepts and lowercase letters for particular concepts [2, pp.179-181]. As one can easily perceive, Ploucquet's system is a in fact a sort of mixture of algebraical notation (the representation of universal/particular terms with uppercase and lowercase letters) and geometrical notation (the lines one above the other to indicate affirmation of identity, and one external and separated from another to indicate negation). In other words, this system is neither completely diagrammatic, nor completely symbolic, but uses both algebraical and geometrical structures in order to express propositions and syllogisms.

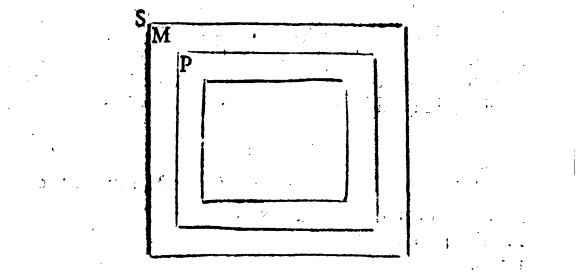

Fig. 21. [2] p. 6.

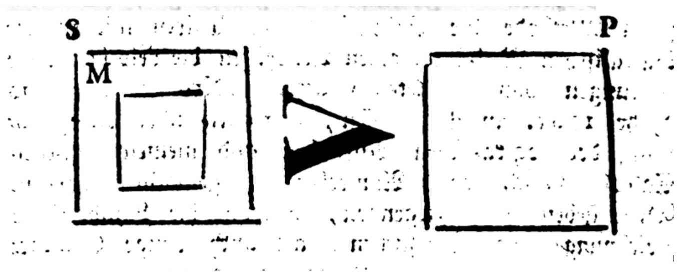

Fig. 22. [2] p. 258

Universal affirmative	**Universal negative**
Sp	***S>P***
All S are some P	No S is all P
Particular affirmative	**Particular negative**
sp	***s>P***
Some S are some P	Some S are not all P

Fig. 23. The 4 standard propositional forms according to Ploucquet

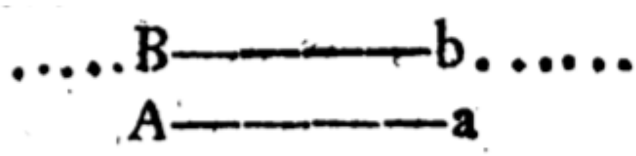

Fig. 24. [9], I, § 181

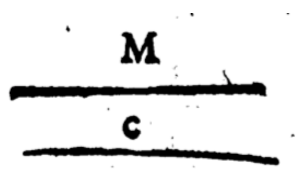

Fig. 25. [2] p. 179

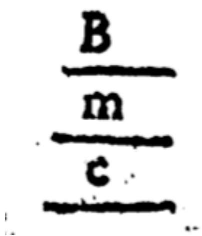

Fig. 26. [2] p. 179

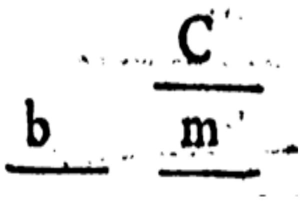

Fig. 27. [2] p. 180

IV. CONCLUSION

In his correspondence with Holland, Lambert states that in Ploucquet's symbolism it is on the basis of "external" information (i.e., syllogistic rules) that it is found e.g. that from a given formula nothing follows. It would be better, according to Lambert, if this "not following" could be shown by the diagram itself [10, pp. 192-193]. Lambert believes he has provided a rule to detect invalid syllogistic forms simply by the rules of construction of their diagram**.** As he declares: "Ploucquet calculates, while I construct or draw" [2, p. 151].

A couple of points may here be mentioned which indicate directions for further research, both historical and theoretical. One century after the debate, the ideal of an autarchic system of signs is still at work in the philosophy of notation of Gottlob Frege (1848-1925). The aim of the *Begriffsschrift* (1879) is expressly that of preventing anything intuitive or extra-logical from penetrating unnoticed in the chain of reasoning. Accordingly, all that is necessary to deduction has to be appropriately represented, so that the inferential chain is kept free of gaps, and at the same time anything without significance for the inferential sequence has to be omitted [11].

However, we believe that the closest explanation available of the notion of autarchy is the conception of corollarial reasoning, which is due to Charles S. Peirce (1839-1914). Peirce distinguishes between two kinds of deductive reasoning, which he calls theorematic and corollarial: "every Deduction involves the observation of a Diagram (whether Optical, Tactical, or Acoustic) and having drawn the diagram (for I myself always work with Optical Diagrams) one finds the conclusion to be represented by it. [...] My two genera of Deductions are 1st those in which any Diagram of a state of things in which the premisses are true represents the conclusion to be true and such reasoning I call *Corollarial* because all the corollaries that different editors have added to Euclid's elements are of this nature. 2nd Kind. To the Diagram of the truth of the Premisses something else has to be added, which is usually a mere May-be and then the conclusion appears. I call this *Theorematic* because all the most important theorems are of this nature" [12, pp. 869-870]. In corollarial reasoning, the diagram of the premises *already represents* the conclusion; in theorematic reasoning, by contrast, the diagram of the premises must be transformed and experimented upon – in geometry, for example, subsidiary lines or figures are drawn - in order for it to represent the conclusion [13, 2.267]. Against Kant, Peirce maintains that *all* deductive reasoning, not just mathematics, is diagrammatic (constructive in Kant's sense). We have to distinguish not between constructive and non-constructive forms of reasoning, but among different forms of constructive thinking according to the complexity of the construction (i.e., diagrammatization) involved therein [13, 3.560].

In Peirce's terms, an autarchic system of diagrams is one in which any reasoning that can be performed is of the corollarial kind. In corollarial reasoning neither auxiliary constructions nor the appeal to "extra-diagrammatical" logical rules is needed in order to draw the conclusion desired. All that which is necessary to reasoning must be expressed diagrammatically in such a way as to enable the diagram of the premises to be, at once, also the diagram of the conclusion. In Lambert's terms, a corollarial reasoning is one in which either the following or the not-following of a conclusion is shown by the diagram itself.

The second point worth mentioning concerns current diagram research. What we call "autarchy of diagrammatic representations" seems to correspond to the notion of "free ride", or information which arises in a diagram as a by-product of its syntax. Already Jon Barwise and John Etchemendy observed that "Diagrams are physical situations. They must be, since we can see them. As such they obey their own set of constraints. [...] By choosing a representational scheme appropriately, so that the constraints on the diagrams have a good match with the constraints on the described situation, the diagram can generate a lot of information that the user never need infer. Rather, the user can simply read off facts from the diagram as needed" [14, p. 23]. As explained by Atsushi Shimojima, in any system of diagrams whatsoever there exists a set of operational constraints which may or may not intervene in the process of encoding and extracting information [15, p. 28]. Under certain conditions, some operational constraints will give rise to a *free ride*: "a free ride is where a reasoner attains a semantically significant fact in a diagram site, while the instructions of operations that the reasoner has followed do not entail the realization of it. Thus, we can view the process as one in which the reasoner has attained the fact without taking any step specifically designed for it" [15, p. 32]. Under different conditions, the operational constraints will produce "overdetermined alternatives" [15, p. 33], that is, will produce pieces of information which do not follow from the diagram of the premises.

In contemporary terms, then, the debate on logic diagrams pictured above may be taken as a debate on operational constraints. When Leibniz claimed that the most perfect systems of representations are those that are autarchic he was maintaining that those systems of logical or mathematical notation must be preferred in which the operational constraints always give rise to free rides. In his system of linear diagrams, the drawing of the conclusion from the premises is always a free ride because the conclusion is obtained directly from the diagram of the premises, without being necessary that any specific step designed for it be taken.

Likewise, Lambert's idea that in an adequate system of representation those premises from which nothing follows should not be capable of being diagrammed is captured by the notion of overdetermined alternative. A system which, given certain operational constraints, may produce overdetermined alternatives is one in which, in Lambert's terms, the following or not-following of a proposition upon another is not a consequence of those constraints, but is the effect of "external" (non-diagrammatical) logical rules. In other words, if a system of diagrammatic representation is capable of producing overdetermined alternatives (as in the case of *Bocardo* in Figure 13), then that system is not autarchic in the

Leibnzian and Lambertian sense. On the contrary, if the system is capable of producing all the consequences as free rides, then that system is autarchic. An autarchic system of diagrammatic representation is therefore one in which a certain set of operational constraints always gives rise to free rides (corollarial reasoning) and never to overdetermined alternatives.

The picture is no doubt more complicated than that, and new problems may arise which might contribute drawing parallels between old and new problems in logic, and building new bridges between the history of logic diagrams and current trends in diagrams research in computing and cognitive sciences.

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