

# L2 point vs. geosynchronous orbit for parallax effect by simulations

Lindita Hamolli  
Physics Dept.  
Faculty of Natyral Science  
lindita.hamolli@fshn.edu.al

Mimoza Hafizi  
Physics Dept.  
Faculty of Natyral Science  
mimoza.hafizi@fshn.edu.al

## Abstract

The isolated dark low-mass objects in our Galaxy, such as free-floating planets (FFP), can be detected by microlensing observations. By the light curve can be defined three parameters, but only the Einstein time,  $T_E$  involves the mass, the distance and the transverse velocity of the lens. To break this degeneracy, have to be detected the perturbations in the light curve due to the relative accelerations among the observer, the lens and the source. Recently, toward Galactic bulge are planned space-based microlensing observations by WFIRST, which can be located in L2 point or geosynchronous orbit (GSO). Using the simulations Monte Carlo in C++ we investigate that the better position for the parallax effect detection in microlensing events caused by FFPs is L2 point than GSO.

## 1. Introduction

In the simplest case, when both the lens and the source can be considered as point-like objects and their relative motion with respect to the observer is assumed to be linear, the amplification of the source star follows the Paczynski profile,

$$A_s = [u^2(t) + 2] / [u(t)\sqrt{u^2(t) + 4}]$$

where  $u(t)$  is the separation between the lens and the line of sight in units of Einstein

radius,  $R_E$  and  $u_0$  is the minimum separation obtained at the moment of the peak magnification  $t_0$  [Pac86]. The light curve obtained by above equation is symmetric around the peak and by it can be defined:  $u_0$ ,  $t_0$  and  $T_E$ . By them, only  $T_E$  carries information about the lens. So, the detection of the dark objects by gravitational microlensing is usually limited by a well-known degeneracy in  $T_E$ . In principle, a way to break partially this degeneracy is the parallax effect, which can be detected when (i) the event can be observed during the Earth's orbital motion creating a shift relative to the simple straight motion between the source and lens or (ii) two observers at different locations looking contemporarily towards the same event can compare their observations. The first way (i) is possible in the long microlensing events or in short microlensing events caused by FFPs when they are observed by the space. These objects are discussed more in the scientific community, recently. The formation mechanisms of FFPs remain an open theoretical question in astrophysics. Sumi et al. [Sum11] examined the data of two years microlensing survey from the Microlensing Observations in Astrophysics (MOA) collaboration and found an excess of short timescale events with duration less than 2 days, which is expected to be caused by FFPs with mass range  $10^{-5} \div 10^{-2} M_\odot$ . By ground-based observations, the deviations in the light curve with short timescale, are small and generally undetectable. But, the era of space-based microlensing observations has already

started with Spitzer and Kepler telescopes, which will help to detect the short microlensing events caused by FFPs. Actually two other space-based missions are planned for the future as WIRST and Euclid. In this work we are interested for the parallax effect in microlensing events caused by FFPs. Since WFIRST telescope is going to observe toward the Galactic bulge and its orbit is still under discussion, the main purpose of the paper is to define which location is more suitable for parallax effect, the L2 point or the GSO [Spe13]. The plan of the paper is as follows: in Sec. 2 we present the review of the WFIRST satellite. In Sec. 3 we show the parallax effect induced in light curves. In Sec.4 are shown the main results for the parallax effect detected by WFIRST located in L2 point and GSO and in Sec. 4 we summarize our main conclusions.

## 2. Overview of WFIRST mission

Here we provide an overview of the WFIRST telescope that is potentially relevant for monitoring microlensing events and combined with the results from Kepler, WFIRST will produce the first statistically complete census of exoplanets. WFIRST is planned to launch in the mid-2020s into a Solar orbit at Sun-Earth L2 or in to a GSO. WFIRST will conduct a  $\sim 432$ -day microlensing survey towards the Galactic bulge divided equally between six 72-day seasons. These seasons will be split between the beginning and the end of the mission to maximize the ability of WFIRST to measure the parallax effect for the detected events [Spe13]. In observations towards the Galactic bulge, the Sun angle constraints are the same at GSO and at L2, but WFIRST suffers from

additional observing constraints in GSO due to the Earth and the Moon. The galactic coordinates of the WFIRST line of sight are  $b = -1.6^\circ$ ,  $l = 1.1^\circ$ , the cadency is 15min and the threshold amplification  $A_{th}=1.001$ .

## 3. The parallax effect

The parallax effect is an anomaly in the standard light curve caused due to the acceleration of the observer around the Sun as the L2 point or around the Earth center as the GSO, which can be observable. Since the parameters of these systems and the coordinates of the source star are known, we can calculate precisely the microlensing event light curves caused by a FFP as observed by satellite at GSO or L2 point. Following Dominik [Dom98], the trajectory of the observer in the L2 point and in GSO can be projected onto the lens plane. The coordinates of the L2 point on the lens plane are:

$$x_1(t) = \rho\{-\sin \chi \cos \phi (\cos \xi(t) - \varepsilon) - \sin \chi \sin \phi \sqrt{1 - \varepsilon^2} \sin \xi(t)\}$$

$$x_2(t) = \rho\{-\sin \phi (\cos \xi(t) - \varepsilon) + \cos \phi \sqrt{1 - \varepsilon^2} \sin \xi(t)\}$$

where  $\rho = 1.01a_{\oplus}(1-x)/R_E$  is the length of the L2 point orbit semi-major axis projected onto the lens plane and measured in  $R_E$ . Here,  $a_{\oplus}$  is the semi-major axis of the Earth orbit around the Sun,  $\varepsilon = 0.0167$  is the eccentricity and  $\xi$  is a parameter which is related to the time by  $t = \sqrt{a_{\oplus}^3(\xi - \varepsilon \sin \xi)/GM_{\odot}}$  [Ham13].

The  $\phi, \chi$  characterize the position of the source, which give the longitude measured in the ecliptic plane from the perihelion towards the Earth motion and the latitude measured from the ecliptic plane towards the northern point of the ecliptic. We find the following values for parameters:  $\phi = 167.9^\circ$  and  $\chi = -5.5^\circ$ . The coordinates on the lens plane

of GSO satellite, orbiting in the equatorial plane around the Earth at the distance  $6R_{\oplus}$  and targeting the Galaxy bulge are:

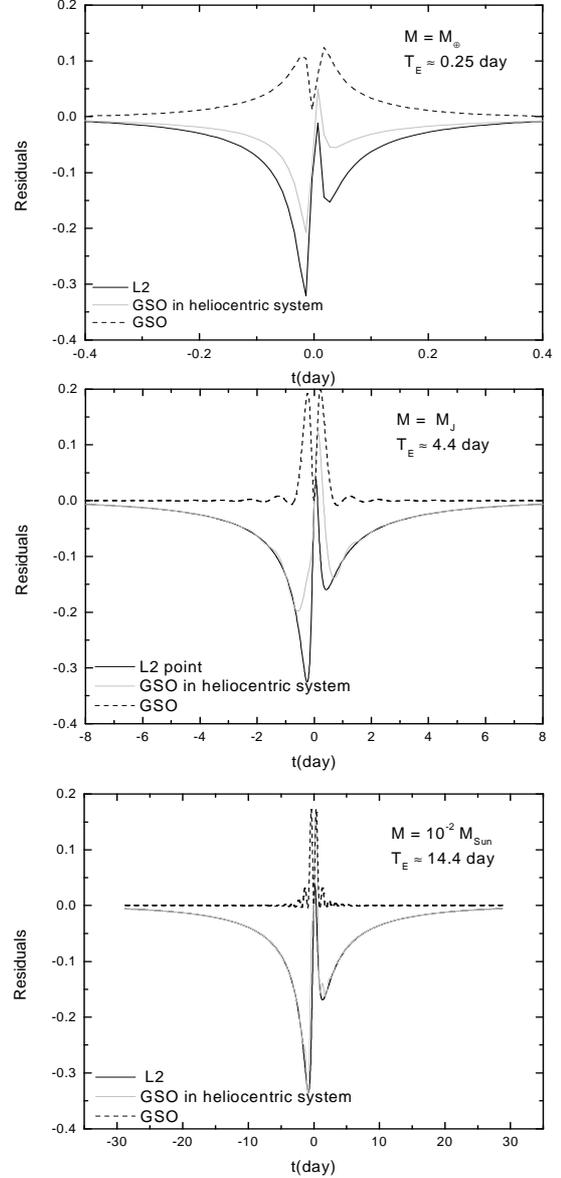
$$x_1(t) = \rho' \{-\sin \delta \cos \alpha \cos(\omega t + \varphi) - \sin \delta \sin \alpha \cos(\omega t + \varphi)\}$$

$$x_2(t) = \rho' \{-\sin \alpha \cos(\omega t + \varphi) + \cos \alpha \sin(\omega t + \varphi)\}$$

where  $\rho' = 6R_{\oplus}(1-x)/R_E$  is the distance of the GSO satellite from center of the Earth projected onto the lens plane and measured in  $R_E$ ,  $\omega = 2\pi/P$  is its angular velocity and  $\varphi$  is the satellite orbital phase with respect to the vernal equinox. The parameters  $\alpha, \delta$  are the right ascension and the declination of the WFIRST line of sight. In fact, the GSO satellite together with the Earth will orbit around the Sun. So, we consider this case calling GSO in heliocentric system (HS). We calculate theoretically the amplification  $A_p(t)$  of the light curve with parallax effect and then defined the residuals between the two curves. The parallax effect is detectable on a light curve when there are at least eight consecutive points with residuals  $Res > 0.001$ .

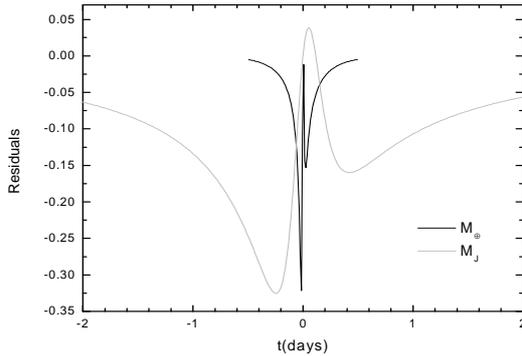
#### 4. Results

The parallax effect depends on the observer orbit, position of it in orbit in conjunction with line of sight [Ham13'] and  $u_0$ . Firstly, we calculate the parallax effect in microlensing events caused by a FFP with mass: Earth mass ( $M_{\oplus}$ ), Jupiter mass ( $M_J$ ) and  $10^2 M_{\odot}$  when WFIRST satellite is in L2 point, GSO and GSO in HS. In fig. 1 we have shown the residuals between the light curve with parallax effect and standard curve for different mass of the lens at the distance  $D_L=4\text{kpc}$ , transit velocity  $v_T=50\text{ km/s}$ , the source distance  $D_S=8.5\text{ kpc}$  and  $u_0=0.05$ . The period and radius of GSO are considered



**Fig. 1.** The residuals between the light curves detectable and standard curves for an microlensing event caused by a lens with mass  $M = M_{\oplus}$  (top),  $M = M_J$  (middle) and  $M = 10^2 M_{\odot}$  (bottom). The curve are calculated when the WFIRST is at the L2 point (black lines), in the GSO (dashed lines) and when the WFIRST is at the GSO in the HS (gray lines).

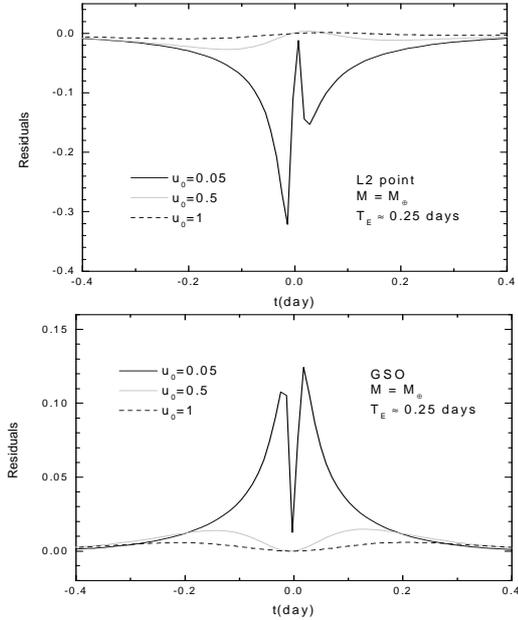
$P=1$  day and  $R=6R_{\oplus}$ . As one can see the residuals is larger for the WFIRST location in L2 point and when the mass of the lens is bigger the oscillations in the residual curves for GSO and GSO in HS are appeared. In figure 2 we have shown the variation of the residuals by the mass of the lens when the WFIRST satellite is positioned at L2 point. We have considered two values of lens mass:  $M=M_{\oplus}$  and  $M=M_J$ . It is clear that the residuals roughly have the same magnitude but the time extensions are different. So, to detect the parallax effect in short events, the cadency has to be smaller. In figure 3 we have shown the residuals between the light curves for the lens mass equal to the Earth mass for



**Fig. 2.** Simulated light curves for a microlensing event caused by a lens with mass  $M=M_{\oplus}$  (black line) and  $M=M_J$  (gray line). The curve are calculated when the WFIRST is in the L2 point.

three different values of the impact parameter,  $u_0$  and the observations are taken by the L2 point and GSO. It is clear that the microlensing events with larger amplification (smaller  $u_0$ ) have bigger residuals and the residuals between curves observed by L2 point is bigger than GSO. Also we calculate the efficiency of the parallax effect in

microlensing events when the WFIRST telescope is positioned at L2 point and GSO in HS using Monte Carlo simulations in  $C_{++}$  [Pre92]. For each event we extract, five parameters: 1)  $D_L$ , based on the disk and bulge FFP spatial distributions, [Gil89], [Pao01], [Haf04] along the WFIRST line of sight; 2)  $M$  from the mass function  $dN/dM \sim M^{-\alpha_{PL}}$ , with  $\alpha_{PL}$  in [0.9-1.6] [Sum11]; 3) FFP relative transverse velocity by the Maxwellian distribution [Han95]; 4)  $u_0$  is randomly extracted from a probability



**Fig. 3.** The residuals between the light curves for a microlensing event caused by a lens with mass  $M=M_{\oplus}$  observed by L2 point (top) and GSO (bottom). The curve are calculated for different value of impact parameter  $u_0=0.05$  (black lines),  $u_0=0.5$  (gray lines) and  $u_0=1$  (dashed lines).

distribution uniformly in the interval [0, 6.54] because  $A_{th}=1.001$ ; 5)  $D_s$  from the Galactic bulge spatial distributions of the stars [Haf04]. We simulate 1000 microlensing

events for the populations of the FFPs in thin disk, thick disk and bulge then calculate theoretically the standard curve, the parallax effect curve and the residuals between them. A microlensing event is detectable if in its light curve there are at least 8 consecutive points with amplification larger  $A_{th} = 1.001$ . We find the parallax efficiency as the ratio of number of events with parallax effect detectable to the number of detected event.

**Table 1.** The parallax efficiency of microlensing events caused by FFPs in the thin disk, thick disk and Galactic bulge when the WFIRST is positioned at L2 point and GSO in HS for  $\alpha_{PL} = 1.3$  and  $\xi_0 = 75^\circ$ .

Our results are shown in table 1.

	Thin disk	Thick disk	Bulge
Parallax Efficiency (L2)	0.141	0.121	0.064
Parallax Efficiency (GSO) in HS	0.123	0.114	0.060

As one can see the efficiency of parallax effect detection is larger for FFPs in the thin disk and lower in the bulge. Also the parallax efficiency is bigger for WFIRST in the L2 point than GSO.

## 5. Conclusions

Taking into account the observing capabilities of the future WFIRST telescope, we have studied the parallax effect in microlensing events caused by a FFP when it is located in L2 point and GSO. We find that the parallax efficiency in microlensing events caused by FFPs is bigger when the WFIRST is positioned at L2 point. Finally, we note that the L2 point is better location for WFIRST satellite than GSO. Since the observations are planed towards the Galactic bulge and the parallax effect depends by the position of the

L2 point around the Sun we remark that the best period for microlensing observation is near the solstice summer [Ham13’].

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