

Cyclic Dominance in the Spatial Coevolutionary Optional Prisoner’s Dilemma Game

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Abstract. This paper studies scenarios of cyclic dominance in a co-evolutionary spatial model in which game strategies and links between agents adaptively evolve over time. The Optional Prisoner’s Dilemma (OPD) game is employed. The OPD is an extended version of the traditional Prisoner’s Dilemma where players have a third option to abstain from playing the game. We adopt an agent-based simulation approach and use Monte Carlo methods to perform the OPD with coevolutionary rules. The necessary conditions to break the scenarios of cyclic dominance are also investigated. This work highlights that cyclic dominance is essential in the sustenance of biodiversity. Moreover, we also discuss the importance of a spatial coevolutionary model in maintaining cyclic dominance in adverse conditions.

1 Introduction

Competition is one of the most fundamental concepts in the study of the interaction between individuals in an ecosystem. Competition occurs when there is a contest for resources, such as food, mates or territories. Competition also favours a selection process, which reaches its peak with the dominance of better-adapted individuals and the extinction of less-adapted individuals. In many cases, rather than one individual dominating, the system can reach an equilibrium where individuals can coexist [8].

Scenarios of coexistence may occur when two or more individuals, or species, form a cycle of dominance. For instance, considering a population of three species: X , Y and Z ; cyclic dominance occurs when X dominates Y , Y dominates Z , and Z dominates X , forming a closed loop, which is also known as an intransitivity [19].

In nature, cyclic dominance plays a key role in the sustenance of biodiversity. For example, the male side-blotched lizard shows an intransitive behaviour when guarding their mates. This kind of lizard can be divided into three categories based on their throat colours:

- Blue-throated males guard small territories with a single female. They are efficient in defending their mate from yellow-throated lizards.
- Yellow-throated males do not guard territories at all, but they move around in search of mates.

- Orange-throated males guard larger territories, keeping harems of females. Consequently, as they have to split their efforts defending several territories, they are less efficient in defending them from yellow-throated lizards. However, they are more aggressive and can steal mates from blue-throated lizards.

Therefore, a cyclic competition exists because the blue-throated males beat the yellow-throated males, the yellow-throated males beat the orange-throated males, and the orange-throated males beat the blue-throated males [16].

It is noteworthy that as the number of species, or the population size, increases, the collective behaviours of a system subject to these sort of scenarios may become much harder to analyse and predict. In this way, frameworks like evolutionary game theory, which employs game theory to evolve populations of rational agents, have been widely applied by researchers as it provides many useful insights to explain such a complex scenario [17].

Particularly, games such as the Rock-Paper-Scissors game [23,12,3,15,10] and the Prisoner’s Dilemma game [22,7,20,18] have been studied in the context of cyclic competition. In these games, a participant’s interactions are generally constrained by particular graph topologies [13], where it has been shown that the spatial organisation of strategies may also affect the outcomes [1]. Recent studies have also explored dynamically weighted networks, where it has been shown that the coevolution of both the game strategies and the spatial environment can further help in understanding real-world systems [2,6,21,4,25].

In this paper, we employ extensive agent-based Monte Carlo simulations to perform the coevolutionary Optional Prisoner’s Dilemma game in a population of agents placed on a lattice grid where, game strategies, and the edges linking agents, adaptively evolve over time. We aim to investigate:

- scenarios of cyclic dominance in the Optional Prisoner’s Dilemma, and
- the necessary conditions needed to break the scenarios of cyclic dominance.

Specifically, the experiments performed are:

- Finding scenarios of cyclic dominance.
- Exploring the sustenance of coexistence after the extinction of one strategy.
- Investigating the robustness of the coexistence of three strategies when some portion of the strategies mutate into another strategy.
- Investigating the impact of the coevolutionary rules in the sustenance of coexistence.

The paper outline is as follows: Section 2 presents a brief overview of the previous work in cyclic dominance in the context of evolutionary and coevolutionary game theory. Section 3 introduces the Optional Prisoner’s Dilemma game, describes the coevolutionary model adopted, and outlines the experimental set-up. In Section 4, some scenarios of cyclic competition between three strategies are investigated. Section 5 explores the necessary conditions to break the cyclic competition. Lastly, Section 6 summarizes the main results and outlines future work.

2 Related Work

Started by John Maynard Smith, evolutionary game theory has been studied since the 1980s where ideas from evolutionary theory have been applied to game theory. Game theory models situations of conflict between rational agents, i.e., individual players make decisions, in which the outcome will depend on the other players' decisions [17].

Evolutionary game theory has been used as an important framework to explore and study the phenomena of cyclic competition, or intransitivity, which can be found in many real-world systems in different domains such as biology [15,10] and physics [11,3]. Moreover, it has been widely studied at a higher level of abstraction, providing insights into the understanding of oscillatory and stochastic systems [23,12].

Despite the fact that cyclic competition has been observed in two-strategy games [20], it is more likely to happen in games involving three or more strategies [19]. The rock-paper-scissors (RPS) game remains one of the most oft-studied games in scenarios of cyclic dominance due to its intransitive nature, in which the loop of preference between pairs of strategies is very straightforward — paper covers rock, rock crushes scissors and scissors cuts paper [24,9]. It is noteworthy that such an intransitive behaviour has also been noticed in other evolutionary games such as the Optional Prisoner's Dilemma game [7,18] and the voluntary public goods game [5]. However, they have been much less explored in this specific context of cyclic dominance.

For instance, Yu et al. [22] proposed a study of the influence of the population size and the level of individual rationality on the evolutionary dynamics of the Voluntary Prisoner's Dilemma (VPD) game, which is very similar to the Optional Prisoner's Dilemma (OPD) game, in which a third type of strategy is considered. In their paper, scenarios of cyclic dominance in the VPD game are discussed. It was shown that these scenarios prevent the full dominance of a specific strategy in the population.

Recent studies have also explored the use of coevolutionary rules in game theory. First introduced by Zimmermann et al. [25], those rules propose a new model in which agents can adapt their neighbourhood during a dynamical evolution of graph topology and game strategy. As discussed by Perc and Szolnoki [14], the coevolutionary games constitute a natural upgrade of the well-know spatial evolutionary games [13], where dynamic spatial environments are taken into consideration. In fact, the coevolution of strategies and spatial environment has given ground to a new trend in evolutionary game theory due to its wider applicability in the understanding of more realistic scenarios [6,21].

The inclusion of coevolutionary rules in the Optional Prisoner's Dilemma game has been recently proposed by Cardinot et al. [2], who identified that coevolutionary rules may favour the emergence of cyclic competition. However, many questions remain unanswered, such as its robustness against frozen states, i.e., when strategies become extinct because of some disturbance in the system.

3 Methodology

In this section, we will describe the Optional Prisoner’s Dilemma game, the Monte Carlo methods and the coevolutionary rules adopted. Finally, the spatial environment and the experimental set-up are outlined.

3.1 The Optional Prisoner’s Dilemma

The Optional Prisoner’s Dilemma (OPD) game is an extension of the classical version of the Prisoner’s Dilemma (PD) game. This extension incorporates the concept of abstinence, where agents can abstain from playing the game. It leads to a three-strategy game in which agents can not only defect or cooperate, as in the classical PD, but can also choose to abstain from a game interaction. Consequently, there are nine payoffs associated with each pairwise interaction between strategies. However, as defined in other studies [1,5], in this work we also assume that if one or both players abstain, both will obtain the same payoff, which is called the loner’s payoff (L). Hence, as illustrated in Table 1, the OPD game is actually characterised by five payoffs, where the other four payoffs are known as the reward for mutual cooperation (R), punishment for mutual defection (P), sucker’s payoff (S) and temptation to defect (T).

Table 1: The Optional Prisoner’s Dilemma game matrix.

	C	D	A
C	R,R	S,T	L
D	T,S	P,P	L
A	L	L	L

In order to establish the dilemma of the OPD, it is important to consider that the loner’s payoff (L) obtained by abstainers is greater than P and less than R , and that the traditional constraints of the Prisoner’s Dilemma still hold, i.e., $T > R > P > S$. Thus, in this extension the dilemma arises when the payoff values are ordered such that $T > R > L > P > S$. In consonance with common practice [22,6,13], as the evolutionary rule depends on the payoff differences between agents, the payoff values can be rescaled to $R = 1$, $P = 0$, $S = 0$, $T = b$ and $L = l$, where $1 < b < 2$ and $0 < l < 1$, which, in turn, maintain the dilemma.

3.2 Monte Carlo Simulation

This work considers a population of N agents placed on a square lattice with periodic boundary conditions, i.e., a torus topology (upper-bottom and left-right borders must match each other exactly). In this lattice, each agent interacts only with its eight immediate neighbours (Moore neighbourhood) by playing the Optional Prisoner’s Dilemma game with coevolutionary rules. In our experiments, initially, each agent is designated as an abstainer (A), cooperator (C) or defector (D) with equal probability. Each edge linking agents has the same weight $w = 1$, which will adaptively change in accordance with the agents’ interactions.

Monte Carlo simulations are performed to investigate the dynamics of the coevolution of both game strategy and link weights. In one Monte Carlo (MC) step, each player is selected once on average, that is, one MC step comprises N inner steps where the following calculations and updates occur: an agent (x) is randomly selected from the population; its utility $u_{xy} = w_{xy}P_{xy}$ is calculated for each of its eight neighbours (represented as y), where w_{xy} is the link weight between agents x and y , and P_{xy} corresponds to the payoff obtained by agent x on playing the game with agent y ; the average accumulated utility, i.e. $\bar{U}_x = \sum u_{xy}/8$, is calculated and used to update the link weights (Eq. 1); as the link weights have been updated, the utilities must be recalculated; finally, strategies are updated based on the comparison of the accumulated utilities U_x and U_y (obtained from a randomly selected neighbour) (Eq. 2).

As shown in Equation 1, the link weight (w_{xy}) between agents is updated by comparing the utility (u_{xy}) and the average accumulated utility (\bar{U}_x),

$$w_{xy} = \begin{cases} w_{xy} + \Delta & \text{if } u_{xy} > \bar{U}_x \\ w_{xy} - \Delta & \text{if } u_{xy} < \bar{U}_x \\ w_{xy} & \text{otherwise} \end{cases}, \quad (1)$$

where Δ is a constant such that $0 \leq \Delta \leq \delta$, where δ ($0 < \delta \leq 1$) defines the weight heterogeneity. Moreover, as done in previous research [2,6,21], the link weight w_{xy} is also adjusted to be within the range of $1 - \delta$ to $1 + \delta$. In this way, when $\Delta = 0$ or $\delta = 0$, the link weight keeps constant ($w = 1$), which results in the traditional scenario where only the strategies evolve.

In order to update the strategy of the agent x , the accumulated utilities U_x and U_y are compared such that, if $U_y > U_x$, agent x will copy the strategy of agent y with a probability proportional to the utility difference (Eq. 2), otherwise, agent x will keep its strategy for the next step.

$$p(s_x = s_y) = \frac{U_y - U_x}{8(T - P)}, \quad (2)$$

where T is the temptation to defect and P is the punishment for mutual defection [2,6].

3.3 Experimental Set-up

In this work, the population size is constant, $N = 102 \times 102$, in all simulations, which are run for 10^6 Monte Carlo steps. In order to alleviate the effect of randomness in the approach, each specific experimental set-up is run 10 times.

Initially, we identify scenarios of intransitivity in the Optional Prisoner's Dilemma game, i.e., the values of b , l and Δ which promote the coexistence of the three strategies (Sec. 4). After that, the stable population is tested to find the necessary conditions to break the equilibrium. To do this, the following experiments are performed: investigating the outcomes of populations with only two types of strategies (Sec. 5.1); exploring the effects of different mutation rates (Sec. 5.2); and investigating the importance of the coevolutionary model in the sustenance of cyclic competition (Sec. 5.3).

4 Cyclic Competition with Three Strategies

Given an initial population with the same number of abstainers, cooperators and defectors uniformly distributed, we start by investigating some parameter settings in which a state of cyclic competition can be observed. Specifically, we look for combinations of the loner’s payoff l , the temptation to defect b , and the link amplitude Δ/δ , which promote an equilibrium between the three strategies. This experiment is based on the work described by Huang et al. [6], where it is shown that some parameters settings may promote cyclic dominance. However, it is noteworthy that this phenomenon is not discussed in their paper and their methods consider only the classical Prisoner’s Dilemma.

In the traditional case of the Optional Prisoner’s Dilemma game, i.e., for a static and unweighted network ($\Delta = 0.0$), results show that abstainers or defectors dominate in most scenarios and the dominance is closely related to the payoff values.

In comparison, the population rarely reaches a state of cyclic competition between the three strategies. In fact, it was only noticed when the temptation to defect is in the range $b = [1.1, 1.2]$. Certainly, it is more likely to happen in this scenario because there is no big advantage in choosing a specific strategy and consequently the strategies tend to remain in equilibrium. However, this behaviour is not very stable.

When it comes to the cases of $\Delta > 0.0$, results show that a wide number of different parameter settings can spontaneously promote the intransitive behaviour, in which cooperators, defectors and abstainers remain in equilibrium. For instance, Figure 1 shows the progress, over the Monte Carlo time steps, of each strategy for $\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ and $b = 1.9$. We can observe that agents quickly organise in the population in a way that the fraction of each strategy remains about 33% ($\pm 7\%$).

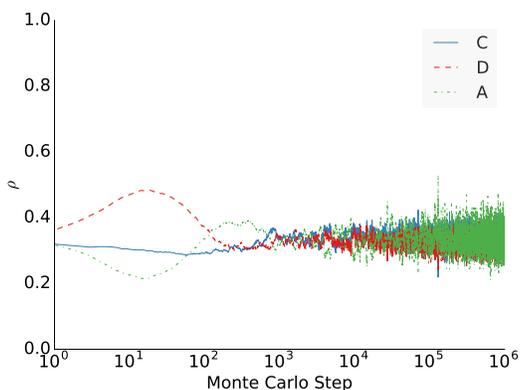


Fig. 1: Progress of the fraction of cooperators, defectors and abstainers during a Monte Carlo simulation for $\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ and $b = 1.9$.

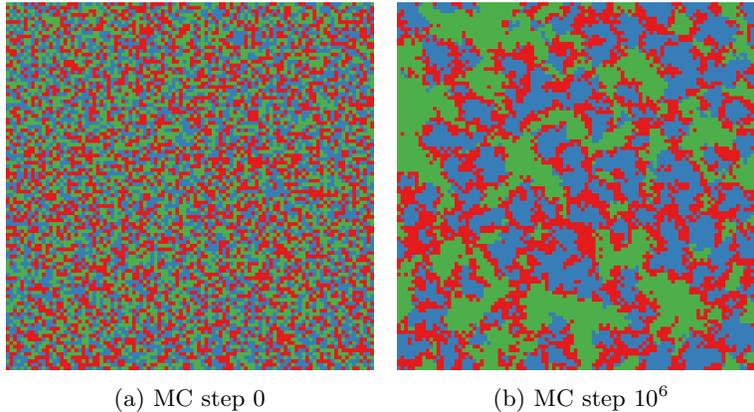


Fig. 2: Snapshots of the distribution of the strategy in the Monte Carlo simulation for $\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ and $b = 1.9$. In this Figure, cooperators, defectors and abstainers are represented by the colours blue, red and green respectively.

In order to investigate what spatial patterns emerge in this scenario, we also took some snapshots of the population at different Monte Carlo steps. Particularly, the MC steps 0 and 10^6 are illustrated in Figure 2.

An interesting phenomenon in this simulation is that abstainers tend to form bigger clusters by dominating defectors. However, abstainers ensure that a small fraction of defectors remain in its surrounding area as a mechanism to protect them against an invasion of cooperators. Defectors, in turn, attempt to encircle cooperators, disconnecting them from abstainers, as a way to isolate and easily dominate them. These dynamics explain the reason why the population is never fully dominated by any one strategy.

5 Breaking the Cyclic Competition: Extinction of Species

In real-world systems, species may become extinct due to a variety of causes such as climate change, habitat degradation, diseases, genetic factors, etc. This sort of scenario is also present in many other domains, for example, in business with the extinction of companies caused by a pricing war.

In other cases, species may mutate in order to avoid extinction. For example, in the case of the common side-blotched lizards, it is known that yellow-throated males can, in specific instances, mutate into blue-throated males and once they transform, it cannot be reverted. Moreover, the yellow-throated male is the only species able to undergo mutation [16].

Inspired by the behaviour of some species in nature such as the side-blotched lizards and the *Escherichia Coli* bacteria, we investigate scenarios in which a species can mutate and we investigate the effect that this phenomenon can have on the population and its evolutionary mechanisms. These set of experiments aim

to explore what are the necessary conditions to maintain the cyclic competition between the three strategies, even in adverse scenarios in which the fraction of agents of a given strategy is reduced in the population, up to its complete extinction.

5.1 Two Species

For the experiments involving only two strategies, or species, we use the evolved population (strategies and spatial structure) obtained in our first experiment (Sec. 4), i.e., the outcome of 10^6 Monte Carlo steps. All other parameter settings are kept the same ($\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ and $b = 1.9$).

Before the simulation, we replace all individuals of a strategy by another, obtaining a population of two strategies. It was found that a state of cyclic competition between only two strategies cannot be reached and that the outcomes are always a full dominance of a specific strategy. The results can be summarised as follows:

- With an initial population of C and A , C will dominate.
- With an initial population of D and A , A will dominate.
- With an initial population of C and D , D will dominate.

As well as mimicking outcomes that we observe in nature, these results highlight the importance of cyclic competition in the sustenance of biodiversity. For instance, in our abstract model, the state of complete dominance of a strategy does not necessarily incur advantages. In other words, some species may prefer to live in smaller numbers in the environment in order to give more opportunity for prey to develop. Thus, although defectors prefer to stay away from abstainers, a population fully occupied by defectors lacks resources ($P = 0$). Thus, defectors need abstainers to keep the cooperators alive, which in turn will enable defectors to increase their payoffs.

5.2 Three Species

Following the same procedures performed for the pairwise simulation (Sec. 5.1), and for the same parameter settings, we now explore the behaviour of a population with three strategies, or species, after an adverse scenario occurs in which most of the agents of a particular strategy undergo mutation.

This analysis is important in order to understand the necessary conditions for a population to maintain the stable coexistence of strategies in cyclic competition. Thus, considering that the initial population has about 33% of each strategy, we run several Monte Carlo simulations varying the mutation rate from 1 to 99 percent. Surprisingly, results show that the state of cyclic competition is very robust and for all simulations the population quickly returns to the equilibrium of about 33% of each strategy with the same spatial pattern as shown in Figure 2.

In order to further explain the results witnessed in these experiments, in which the vast majority of our simulations with extremely high mutation rates,

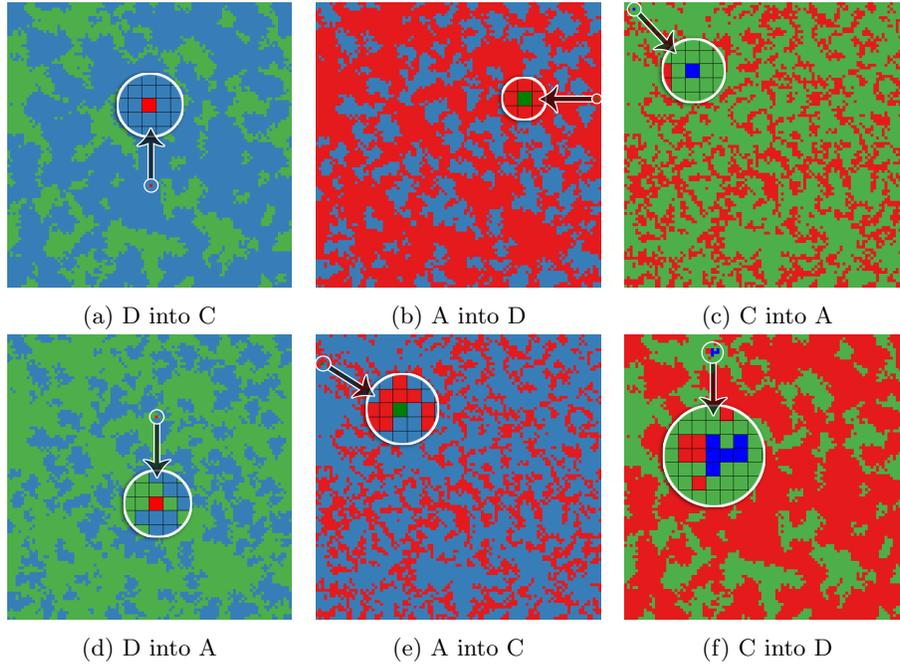


Fig. 3: Initial population after mutation of strategies. All scenarios recover the balance between the three strategies, showing a spatial pattern similar to the one illustrated in Figure 2. In this Figure, cooperators, defectors and abstainers are represented by the colours blue, red and green respectively.

i.e. 99.9%, converged to the equilibrium; we decided to analyse cases where only one agent of a specific strategy does not mutate in the population (i.e., maximum mutation rate). This scenario, in turn, is illustrated in Figure 3, which shows the snapshots of the mutated population at the initial Monte Carlo step.

Results show that only one individual of a species needs to be kept in the population in order for the cyclic competition to remain. All scenarios in Figure 3 quickly reverted back to the spatial pattern illustrated in Figure 2. However, it is only possible if the single agent is linked with a sufficient number of subordinate agents. Namely, a cooperator must have abstainers in its neighbourhood; similarly abstainers must have defectors, and defectors must have cooperators. This condition explains the robustness of our previous experiment, in which the balance was recovered even when 99% of the agents of a strategy were mutated. In this situation, as we have a population of 10404 agents, the chances of at least one of 104 agents being connected to a satisfactory number of subordinated strategies is very high.

It can be seen that when mutating *Ds* into *Cs*, we unbalance the final population to about 66.666% *Cs* and 33.333% *As*. We observe that this difference is not too relevant for the final outcome and that the key role in sustaining the

cyclic competition is in the hands of the single agent (D in this example). Results show that balance between the three strategies is always recovered when the single agent is completely surrounded by its subordinate strategy. For instance, a single defector surrounded by cooperators will never be replaced by a cooperator because its utility will be greater than the utility of any neighbour.

When the mutation occurs from a dominant to a subordinate strategy such as, from defector to cooperator, abstainer to defector or cooperator to abstainer, the chances of the single agent being kept in the middle of a suitable cluster (subordinate strategy) are very high. This scenario can be observed in Figures 3a, 3b and 3c. However, in the opposite case it is not generally possible because, during the evolutionary dynamics, a subordinate strategy will almost never survive alone in the middle of a cluster of a dominant strategy.

Therefore, the special aforementioned cases are not as stable as the case in which the single agent is surrounded by subordinated neighbours. Its stability will depend on the number of links with subordinated strategies and the respective values of the link weights. In our experiments, it was observed that when the single agent is a defector, the cyclic competition is recovered more often when the number of cooperators in the neighbourhood is greater than or equal to four (Fig. 3d). When the single agent is an abstainer, at least five of the eight neighbours need to be defectors (Fig. 3e).

Finally, as shown in Figure 3f, it is usually impossible to sustain cyclic competition when the single agent kept after mutation is a cooperator. It happens because cooperators are very sensitive to the presence of defectors and, in this sort of scenario, they need at least one partner to be able to be rewarded for mutual cooperation (R). Thus, better results are obtained as the number of cooperators kept together increases.

5.3 Destroying the Environment

Previous experiments discussed in Sections 5.1 and 5.2 have considered the mutation of species (strategies) applied to an already coevolved population (species and spatial environment). Namely, strategies are transformed, but the spatial environment is still the same.

In this section we are interested in exploring the impacts of changing the spatial environment at the mutation step such that all link weights between agents are reset back to $w = 1$ as existed in the initial settings of the first experiment (Sec. 4).

Results show that the robustness witnessed in previous simulations, in which only one species was needed to recover the balance between the three species, is actually only possible because the spatial environment is kept unchanged. The coevolutionary rules adopted enables agents to constantly strengthen beneficial connections and weaken harmful ones, adapting the environment to fit individual needs. Hence, in the previous experiments, as the non-mutated agent is probably the most adapted in the neighbourhood, its strategy spreads quickly, allowing the population to recover the balance.

This finding highlights the importance of the coevolutionary model in allowing agents to adapt the environment to sustain the diversity of strategies.

6 Conclusions and Future Work

In this paper, we have investigated the phenomenon of cyclic dominance in a coevolutionary Optional Prisoner's Dilemma, in which both game strategies and edges linking agents adaptively evolve over time. An agent-based Monte Carlo simulation approach was adopted to perform the evolutionary game in a population of agents placed on a lattice grid with a Moore neighbourhood.

Despite the fact that the rock-paper-scissors game remains one of the most oft-studied games in scenarios of cyclic dominance, in this paper, we showed that the same behaviour can also be noticed in the Optional Prisoner's Dilemma game, which allows the investigation of more complex scenarios that may lead to a variety of outcomes.

Results show that cyclic dominance between the three strategies can emerge spontaneously in a wide range of parameter settings, i.e., l , b , Δ and δ , including the traditional case ($\Delta = 0.0$) for an unweighted and static network. However, it was observed that populations of only two strategies can quickly lead to dominance of one strategy, which may lead to a much lower performance.

Experiments revealed that the equilibrium between the three strategies is maintained even in adverse scenarios, in which the mutation rate is extremely high. It was observed that having only one agent of a strategy is often enough to enable the population to revert back to a balanced state. However, this single agent must be surrounded by subordinated agents, i.e., cooperators surrounded by abstainers, abstainers surrounded by defectors and defectors surrounded by cooperators.

Moreover, it was shown that the coevolutionary spatial method adopted plays a key role in the sustenance of coexistence because it allows agents to also adapt the environment, which is reasonable in more realistic scenarios. For instance, in real life, a population is often changing the environment over time in order to improve their performance and welfare. Thus, in adverse scenarios, it is much easier for an individual to overcome and survive in such an evolved environment.

Future work will involve the mathematical analysis of the necessary conditions to sustain the coexistence of three competing strategies of the Optional Prisoner's Dilemma game, allowing us to further explain the results obtained by Monte Carlo simulations.

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