Mathematical Modeling of Artificial Mitral Heart Valve

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Abstract. The research shows the mathematical model, describing the dynamics of the artificial aortic heart valve and the model of blood thrombus moving in large vessels, as well as the method of numerical calculation of these models. There are represented numerical modelling results of the tricuspid valve operation and the blood thrombus moving in large vessels.

Keywords: Mathematical modeling, artificial mitral heart valve, aneurysm, immersed boundary method

1 Introduction

The research of heart and blood-vascular system diseases is a task that has extremely high socio-economic importance and a long history. The knowledge of the human cardiovascular system is actual as never before and during the last few decades the methods of mathematical modeling are widely used in their accumulation.

In the world 80-90% of non-traumatic subarachnoid hemorrhages are occurred due to the bursting of intracranial aneurysm [10]. The aneurysm rupture leads to neurological deficits related to brain tissue damage or even death. Approximately 50% of patients with aortic aneurysm bursting die before the hospitalization [11]. Each year, about 250 thousand heart valve restoration or surrogation operations are carried out in the world and the number of these operations and their necessity is increasing year by year [9].

The cardiovascular system is extremely complex. The heart is a complex multi-valve muscular organ. Blood vessels are multilayered structure that substantially differs from each other, depending on the type of vessel and its position. Blood has a heterogeneous structure including formed elements. Some authors consider the blood as an incompressible viscous Newtonian fluid [13] including formed blood elements [8]. Sometimes the blood circulation is presented as non-Newtonian fluid flow [3].

This research presents a mathematical model of blood circulation in large blood vessels, suitable for modelling of vascular malformations and operation of artificial heart valves. Artificial heart valves are one of the most complex prosthetic devices in cardio surgery. They enable you to deal effectively with diseases and injuries of natural valves, but their operation time is much less than a human life, which means that the patients need to be re-prosthesis every few years. Mechanical valves have high reliability and durability, but can lead to a serious deformation of the blood circulation, the formation of the blood cell clot and as a result to the thrombus formation. Biomaterial valves don't have this drawback, but they are less durable, and their production is a difficult technical problem, which is not completely resolved at the present time.

Also in this research we investigate the mechanism of the formation of blood aneurysms in the large blood vessels. The research [10], [16] shows that the formation of blood aneurysms is caused by swelling one of the layers of its wall, of intima, which is the most subtle and least durable. It defects the vessel shape and the average blood circulation in it.

The mathematical models, written in the form of quite complex differential equations that can be solved by various numerical methods, are often used as a modeling tool of the blood circulation. One of the most commonly used methods is the finite element method (FEM) [6], [17]. FEM is widely tested on the problems of elasticity theory and hydrodynamics. Furthermore there is a sufficient number of sets, realizing this method. FEM enables to take into account the complex shape of the solution field for the deflection of the blood vessel walls and valve leaflets, but the need to take into account the interaction between the fluid and flexible walls leads to a constant reconfiguration of the analysis grid to comply the changing shape of the object, consequently the finite element method has a high temporal and spatial complexity. There is another approach to solving problem of blood circulation in vessels based on the applying of the lattice Boltzmann method [15], [1]. This approach uses the methods of statistical mechanics, numerically solving Boltzmann discrete equation, the direction of the fluid flow is defined in the lattice sites and the fluid flow is possible only in the directions of the lattice. Another common method for researching of the hemodynamics, vascular structures and heart valves problems is the method of the immersed boundary. The immersed boundary method is a relatively new technique that was proposed for the modeling of the heart valves operation [7], [12]. It enables you to simulate the deformation of arbitrarily thin valve leaflets.

In this research we consider the blood circulation in the large elastic blood vessels and artificial heart valve as a three-dimensional variable flow of the incompressible fluid with variable density and viscosity [4], [5]. The resulting system of the differential equations with appropriate boundary and initial conditions describes the artificial tricuspid aortic valve operation, the formation and growth of the blood aneurysm in large blood vessels. The immersed boundary method is used in conjunction with the method of nets to solve the differential problems.

2 Problem definition

The blood consists of plasma and measured formed elements, which account for about 45% of the total volume. The size of the formed elements is small compared to the size of a large vessel: for example, aortic diameter is $3 * 10^{-2}$m, and the diameter of an erythrocyte is $6 * 10^{-9}$m. The research [14] shows that the blood plasma behaves like a Newtonian fluid, which makes the blood an incompressible inhomogeneous two-component liquid with variable viscosity and density. The blood vessel walls and valve leaflets we will consider as fluid-tight surfaces that have certain rigidity. Also the blood vessel walls and valve leaflets can be deformed under the influence of the fluid pressure.

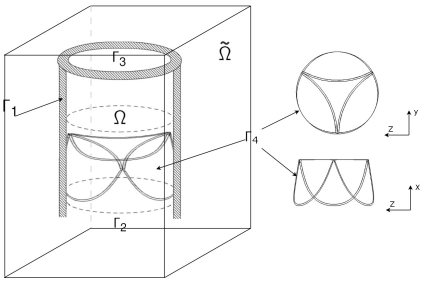

Fig. 1. The solution field.

The source of the fluid flow in vessels and heart valves is the pressure generated by contraction of the heart muscle. Therefore, we will describe its flow in the blood vessels and heart valves by Navier-Stokes equations system with variable density and viscosity and the pressure drop is set up at the input-output of the solution field, depending on the time:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nabla\sigma + \mathbf{f} \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{2}$$

with the initial conditions and the boundary conditions

$$\mathbf{u}(\bar{x}, 0) = \mathbf{u}_0 \qquad \mathbf{u}|_{\Gamma_1, \Gamma_4} = \mathbf{u}_b \qquad v, w|_{\Gamma_2, \Gamma 3} = 0 \tag{3}$$

$$p_{\Gamma_2} = p_{in} \qquad p_{\Gamma_3} = p_{out} \tag{4}$$

$\mathbf{x}(x, y, z) \in \tilde{\Omega}$ are the points of the solution field, $\mathbf{u}(u, v, w)$ is the vector of the velocity field, $p(\mathbf{x}, t)$ is the pressure, $\sigma = \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$ is the viscous stress tensor, $\mu(\mathbf{x}, t)$ is the fluid viscosity, $f(\mathbf{x}, t)$ is the vector of the body forces. The fluid density is defined by the formula

$$\rho = c(\rho_2 - \rho_1) + \rho_1 \tag{5}$$

and the viscosity is a function of the flow rate, its density and their gradients. In this research we use a simple linear relationship between fluid density and viscosity:

$$\mu = c(\mu_2 - \mu_1) + \mu_1 \tag{6}$$

Here, ρ_1, μ_1 is the density and viscosity of the carrier fluid (blood plasma in this case), ρ_2, μ_2 is the density and viscosity of the admixtures (formed elements), $c(\mathbf{x}, t)$ is an admixtures concentration determined from the transfer equation

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0 \tag{7}$$

with the initial conditions

$$c(\mathbf{x}, 0) = c_b(\mathbf{x}), \ x \in \tilde{\Omega} \tag{8}$$

and the boundary conditions on the boundary of the inflow:

$$c(\bar{x}, t)|_{\Gamma_2} = c_s(\bar{x}, t) \tag{9}$$

In this approach, the displacement of the vessel walls and valve leaflets occurs under the influence of fluid flow in consideration of the walls rigidity, but the tissues tension and deformation forces give the contribution to the sum of the forces in the motion equations

$$\mathbf{f}(\mathbf{x}, t) = \int_{\Gamma} \mathbf{F}(\bar{q}, t) \cdot \delta(\mathbf{x} - X(\bar{q}, t)) \, d\bar{q} \tag{10}$$

$$\frac{\partial X}{\partial t}(\bar{q}, t) = \int_{\Omega} \mathbf{u}(\mathbf{x}, t) \cdot \delta(\mathbf{x} - X(\bar{q}, t)) \, d\mathbf{x} \tag{11}$$

is the vector of the body forces, is the deformation resistance force. The equations (10) and (11) use an integral transformation that uses three-dimensional Dirac function for the transition from Lagrangian to Eulerian coordinates. The equation (11) shows that the immersed boundary moves according to the local velocity of the ambient fluid, which is an analog of the adhesion condition:

$$\frac{\partial X(q, r, s, t)}{\partial t} = \mathbf{u}(X(q, r, s, t), t) \tag{12}$$

Adhesion condition is used to determine the movement of the immersed elastic boundary bounded by fluid flow. The formula from [7] is used to describe the deformation forces

$$F = \frac{\partial}{\partial s} k\tau + \frac{\partial^2}{\partial s^2} \left(k_s \left(\frac{\partial^2 X^0}{\partial s^2} - \frac{\partial^2 X}{\partial s^2} \right) \right), \tag{13}$$

where k is the elastic coefficient, $\tau = \frac{\partial X}{\partial s} / \left| \frac{\partial X}{\partial s} \right|$, k_s is the flexural rigidity coefficient. If we assume that the Lagrangian coordinates (q, r, s) are chosen in such a way that coordinate pair (q, r) defines the single fiber for the fixture s, and the formula $X(q_0, r_0, s)$ determines the parametric representation of the fiber $X^0(q, r, s) = X(q, r, s, t_0)$.

3 Solution method

In this research we will use immersed boundary method [12] to determine the movement of the valve leaflets and deformation of the vessel walls. In accordance with this method, we will calculate the fluid flow in the $\tilde{\Omega}$ parallelepiped, which includes Ω (see Fig. 1). The adhesion condition is required at the $\tilde{\Omega}$ boundaries. We will use a rectangular, actually non-uniform staggered grid $\tilde{\Omega}_h$ with h_{xi}, h_{yj}, h_{zk} pitches and staggered nodes, where the pressure, the velocity divergence and the concentration are determined at the center of the socket and the components of the velocity vector and external forces are determined at the socket boundaries to determine the fluid flow. The splitting scheme on physical factors is used to solve the equations (1) - (4):

$$\frac{u^* - u^n}{\triangle t} = -(u^n \cdot \nabla)u^* - \frac{1}{\rho}\nabla\sigma + f^n \tag{14}$$

$$\rho\triangle p^{n+1} - \nabla\rho \cdot p^{n+1} = \frac{\rho^2\nabla u^*}{\triangle t} \tag{15}$$

$$\frac{u^{n+1} - u^*}{\triangle t} = -\frac{1}{\rho}\triangle p^{n+1} \tag{16}$$

The numerical scheme consists of the following steps:

- The intermediate velocity field u^* is determined according to the known velocity values from the preceding time layer. For this purpose the equation (14) is solved by the stabilizing correction method.
- p^{n+1} is calculated from the formula (15) by the stabilizing correction method.
- The new velocity field u^{n+1} is determined according to the explicit formulas (16).
- The new concentration values are calculated from the transfer equation (7).
- The new values of the density and viscosity are defined according to the formulas (5) and (6).
- The velocity values at the points of the immersed boundary are approximated.
- The displacement of the free boundary is calculated.
- The deformation resistant forces are determined and the projections of the body forces vector are calculated in the grid nodes.

4 Results

The results of the numerical modelling of the artificial heart valve "UniLine" operation as well as the formation and growth of the blood aneurysm in large blood vessels are presented in this paragraph. The calculations were performed for the case of constant and variable density and viscosity in non-dimensional variables.

4.1 The calculation of the artificial heart valve "UniLine" operation

The artificial heart valve "UniLine" (see Fig. 2) is a stented tricuspid biological prosthetic device wherein inanimate, especially treated biological tissues are fixed on the support frame (stent) covered with a synthetic fabric. The high-precision leaflets cutting by the laser system, which enables to avoid the collagen fibers separation on the edge of the cut [2] is used in the production of this valve. We placed the valve inside a circular cylinder with length of 1 and radius r =0.11. The coefficient of the vessel walls elasticity is $k = 1 * 10^3$, for the valve leaflets is $k = 5 * 10^3$, the flexural rigidity coefficient is $k_s = 2 * 10^3$. The pressure drop $p_{in} - p_{out}$ varies continuously from 0 to 6 at times. The pitch on the spatial grid is 0.01 and in time it is the same (0.01).

Fig. 2. The artificial heart valve "UniLine".

The Figure 3 shows the movement of the valve leaflets and fluid flow through it at periodic increasing and decreasing of the pressure drop.

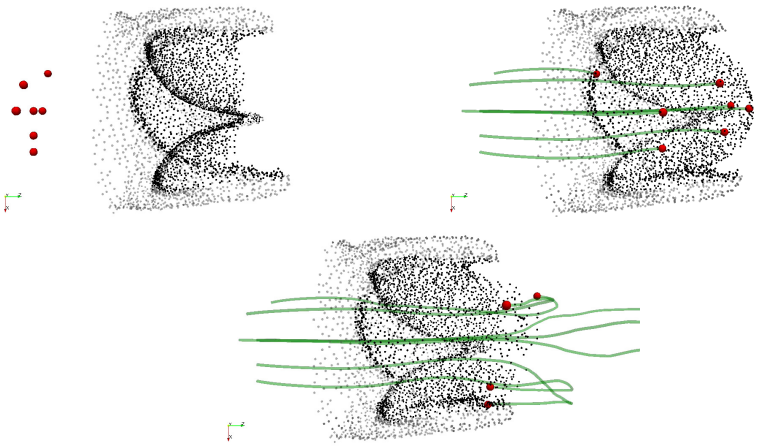

Fig. 3. The dynamics of the valve leaflets and the line of the certain particles. The direction of flow is indicated by the arrow. t =0, 0.7, 1.5.

As you can see in the Figure 3, the valve leaflets are opened when the pressure difference is changing, and then reset at pressure balancing.

Figure 4 by a dotted line shows a graph of fluid flow within the valve according to the time for the first three cycles. A leap fluid flow corresponds to each pulse, and the swings of the valve leaflets are reflected in the very slight oscillations of the graph when closing.

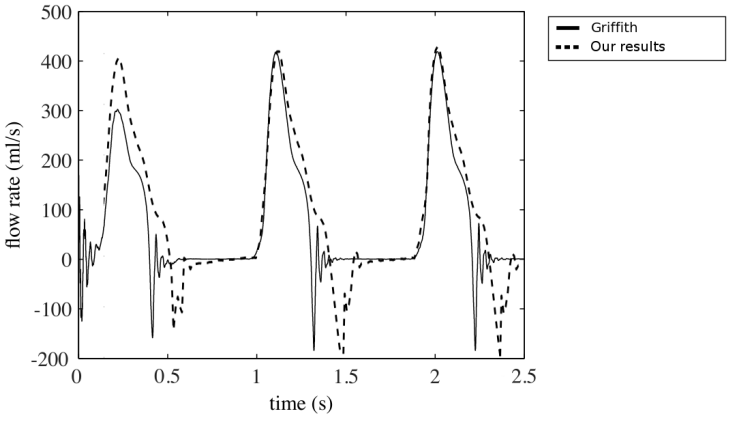

Fig. 4. Comparing flow rate versus time.

We use physiological conditions for the pressure presented at Figure 5. In the picture aorta pressure is input, ventricular pressure is the output one. Extension strength coefficient $k_s = 2 \cdot 10^3$, $k_b = 1 \cdot 10^2$ while valve opening and $k_b = 20 \cdot 10^3$ when valve closing. Viscosity is $\mu = 0.25 \cdot 10^{-2}$ Pa/s, density $\rho = 1 \cdot 10^3$ kg/m^3. Step on space is $h_x = h_y = h_z = 2 \cdot 10^{-3}$m, step on time $t = 1 \cdot 10^{-4}$s, vessel length $l = 0.1$ m, radius $r = 0.03$ m. Figure 4 also shows a comparison with the same calculation of the work [7]. The comparison shows that the rise and flow rate at the time of full valve opening match quite well. However, in our calculations, the valve closes a little more slowly, due to the differences in the parameters for rigidity. You can also note that in [7] the first flow peak is much smaller than the other. This is due to the fact that in this paper describe the initial conditions already strained leaflets, whereas in our calculation they rested without tension.

For the pressures analysis during "UniLine" valve operation we monitored their change at two points of this valve. The valve leaflets were moved under the influence of the fluid with constant, and then variable viscosity and density. These two points are marked in the Figure 6 and hereafter we will call them "active point" and the "mid-point". The "active point" is located on the one of the valve spindles at the point of the two adjacent leaflets fastening and the "mid-point" is located in the center of the leaflet.

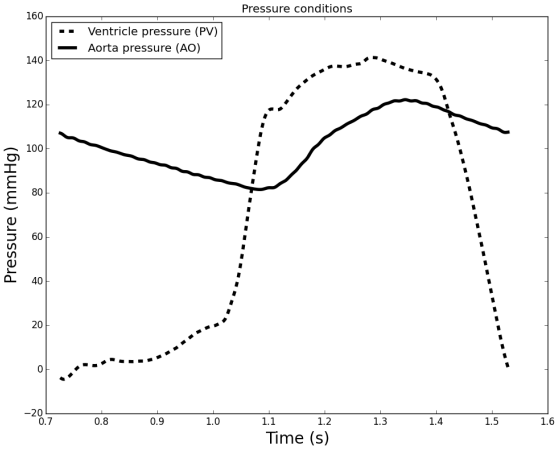

Fig. 5. The graph of physiological pressure.

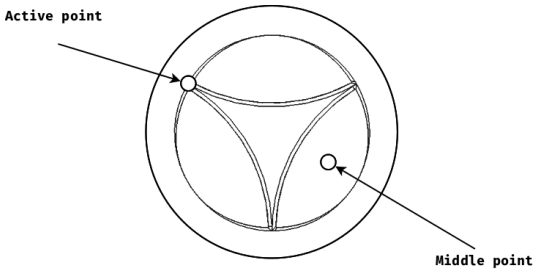

Fig. 6. The arrangement of the points spacing on the tissue annulus.

The Figure 7 shows graphs of the surface traction dependence on the time for these two points in the various parts of the valve for the cases of constant density and viscosity ($\rho_1 = \rho_2 = 1$, $\mu_1 = \mu_2 = 1 * 10^{-2}$, c=0%) and variable density and viscosity ($\rho_1 = 1$, $\rho_2 = 2$, $\mu_1 = 1 * 10^{-2}$, $\mu_2 = 2 * 10^{-2}$) for the two concentration values c = 20% and c = 40%.

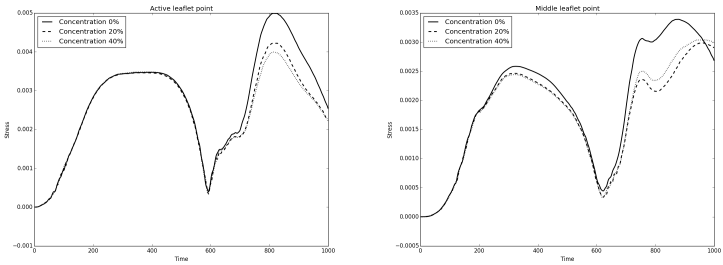

Fig. 7. The dependence of the traction on the time for two points on the valve for the cases without admixtures (continuous line) and with admixtures where =20% (dashed line) and with admixtures where c = 40% (dotted line).

4.2 The formation and growth of blood aneurysms in the large blood vessels

The presented in this research model can be successfully used to research the formation and growth of aneurysms in the large blood vessels. In this paragraph we will consider the growth process of the aneurysm in the large blood vessel under the periodically changing pressure, occurred during the blood circulation in the vessel. We considered the vessel of a constant cross section, the set of the cross-section centers is defined by the spline, the vessel radius is $r = 0.11$ and the walls rigidity is of $k = 2.5 * 10^3$. It is assumed that the aneurysm grows in areas with high pressure on the walls therefore the vessel rigidity is decreased in areas with high pressure. The Figure 8 shows the shape of the vessel during the formation of the aneurysm and also it shows the particle paths.

When the blood aneurysm is formed, the blood clots, which can be washed out by the flow and can get into the mainstream of the vessel, are formed in the area of aneurysm. Therefore we take into account the importance to investigate how a fairly dense and viscous area of the fluid will be spread in the vessel, which has the narrowing (angiostenosis). Two stenosis scenarios are under consideration: 1) insignificant vasoconstriction (15%), that happens gradually, 2) significant vasoconstriction (72,5%) at a small vessel area. The vessel wall stiffness is $4.5 * 10^3$, that is quite high, clot density is $\rho_2 = 2$, viscosity is $\mu_2 = 3 * 10^2$. In the latter case vessel wall stiffness decreases to $1 * 10^3$ due to linear law (before stenosis) and increses to the initial value at the stenosis area. Fig. 9, 10 show

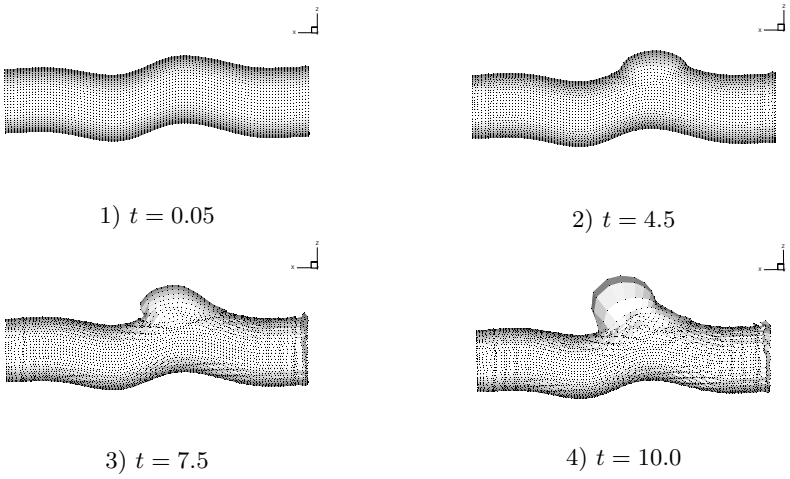

Fig. 8. The shape of the vessel during the formation of the aneurysm and some particle paths.

modeling results. In case the vessel has insignificant stenosis its part with strong concentration and, as a result, high density and viscosity is washed out by fluid flow (from vessel axis), the clot passes vessel constriction area by changing its form and leaves the computational domain. In case the stenosis is more significant the scenario takes more time. The clot reaches vessel constriction area, but only some part of it can passes through stenosis, the clot is washed out gradually and it never passes the stenosis. This situation leads to slight deformation of vessel walls before stenosis. The clot does not completely block the vessel and there is still flow inside, due to the insignificant density and viscosity of the clot, though its concentration changes more slowly compared with the first case.

5 Conclusion

We develop the model of the aneurysm and artificial heart valve operation, considering the blood circulation with variable density and viscosity. The immersed boundary method used for the realization of this model enables to get the movement patterns of the valve leaflets for various shapes, to analyze the stress rates, occurred during their movement and to predict the formation and growth of blood aneurysm in the blood vessels.

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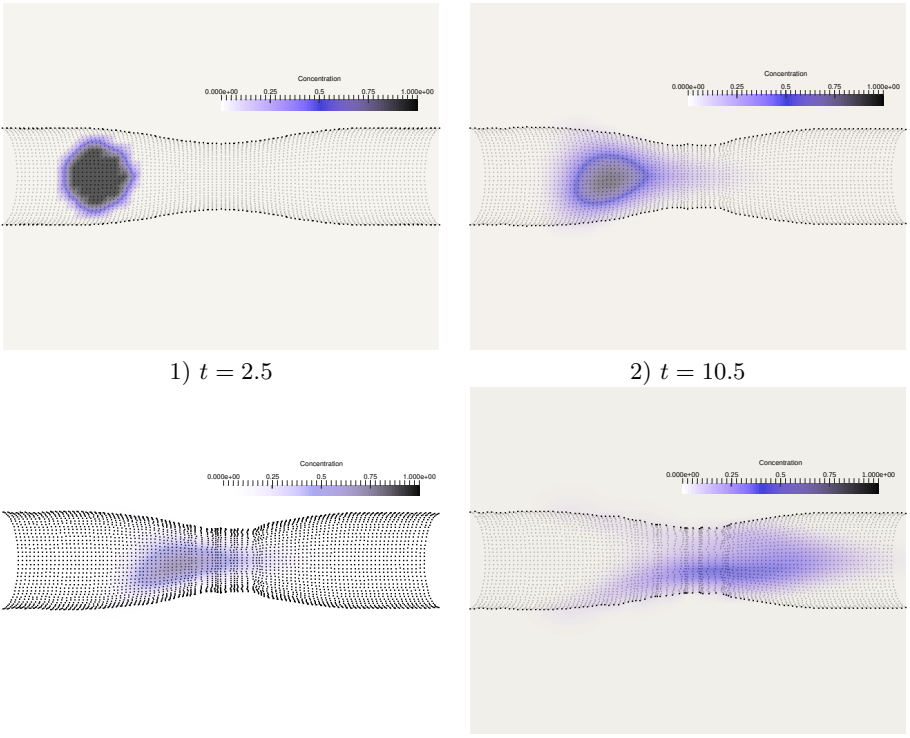

1) $t = 2.5$ 2) $t = 10.5$

3) $t = 19.5$ 4) $t = 34.0$

Fig. 9. The passage of the blood clot through the insignificant vessel angiostenosis, the coloring of the flow area is according to the concentration.

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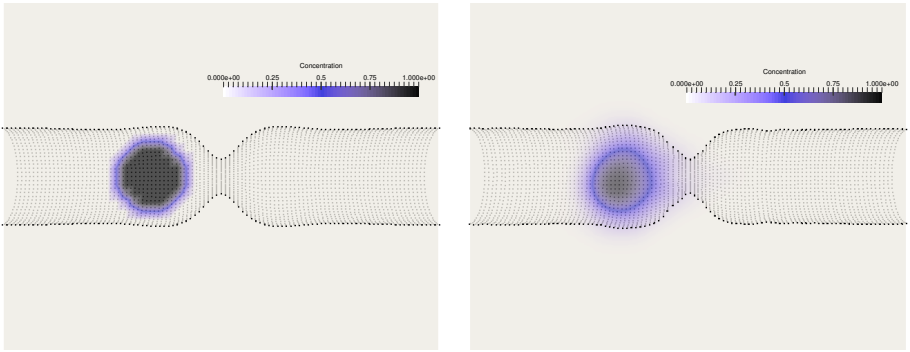
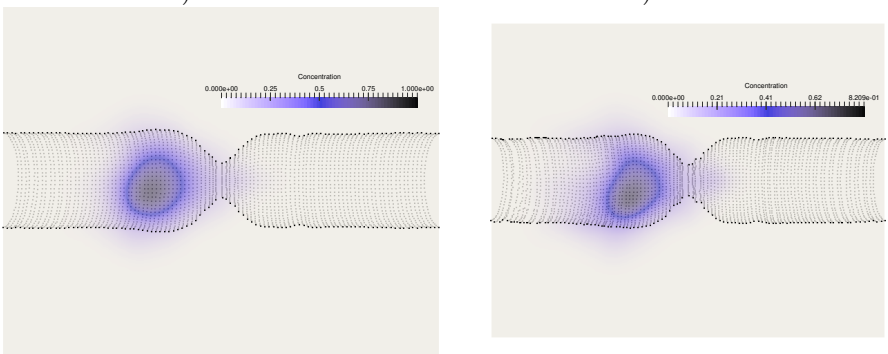


1) $t = 1.5$ 2) $t = 25.0$



3) $t = 52.2$ 4) $t = 72.5$

Fig. 10. The passage of the blood clot through the significant vessel angiostenosis, the coloring of the flow area is according to the concentration.

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