

# Comparison of Two Distributed Fault Diagnosis Approaches based on Binary Integer Linear Programming (BILP) Optimization

Erdal Taskent<sup>1</sup> and Vicenç Puig<sup>2</sup>

<sup>1</sup>Reliability, Risk, and Safety Engineering Specialist, Izmir, Turkey

e-mail: [etaskent@hotmail.com](mailto:etaskent@hotmail.com)

<sup>2</sup>Advanced Control System Research Group, Universitat Politècnica de Catalunya (UPC), Barcelona, Spain

e-mail: [vicenc.puig@upc.edu](mailto:vicenc.puig@upc.edu)

## Abstract

Two distributed fault diagnosis approaches were compared, by analogy, to determine which is more efficient regarding computational complexity. The first approach considered all “locally computed” global and compound sets with minimal cardinality using a heuristic optimization method while minimizing subsystems interactions (communication). The second approach aimed at obtaining minimal coupled MSOs for minimizing the number of common links between MSOs by adding constraints in already existing optimal sensor placement algorithm, which uses BILP, but not in a distributed context. As a result of comparison, complexity of both approaches is characterized.

## 1. Introduction

For complex systems with large-scale distribution and communication constraints, it is appropriate to use distributed approaches [1]. Distributed approaches are more reliable than centralized approaches in case of the failure of the centralized diagnoser (also in decentralized schemes). Moreover, distributed approaches are preferred because of lack of scalability and efficiency of centralized solutions during online analysis for large-scale systems since that can be dealt with complexity by partitioning the system into subsystems [11]. Failures in communication links and nodes, and degraded diffusion through the affected nodes by propagation of overloads can lead to cascading failures. Moreover, transmission delays increasing the detection time can affect diagnostic accuracy [2]. Reducing communication costs in distributed contexts requires minimizing data transfer between local subsystems [1]. Therefore, distributed algorithms should consider the requirements of computational and communication efficiency. To deal with computational complexity in distributed algorithms, efficient approaches for the sensor placement analysis and to compute feasible MSO sets need to be developed.

The Minimal Structurally Over-determined (MSO) set approach offers an alternate way to find all ARR's.

According to [9], a minimal structurally over-determined subsystem (MSO subsystem) is a part of the over-constrained part of a system graph from which removal of one constraint will make the subsystem to become just

constrained, i.e., structural redundancy 1. Therefore, each MSO set will consist of in any case one constraint that can be used as an ARR.

The global FMSO sets are obtained from the set of local FMSO sets, and the union of locally computed shared sets which forms a compound FMSO set that includes at least one shared FMSO set whose fault support is not empty, contains equations from at least two subsystems.

In this paper, two distributed fault diagnosis approaches were compared, by analogy, to determine which is more efficient regarding computational complexity. The first approach [1] considered generates all “locally computed” global and compound sets with minimal cardinality using a heuristic optimization method while minimizing subsystems interactions (communication). The second approach [4] aimed at obtaining minimal coupled MSOs, for minimizing the number of common links between MSOs by adding constraints in already existing optimal sensor placement algorithm [3] which uses BILP, but not in a distributed context.

The two considered approaches deal with the problem of distributed fault diagnosis (local diagnosis with minimum global diagnosis) that aims to obtain a set of optimal local diagnosers that guarantee the same properties as a global diagnoser. Both approaches target to provide the maximum possible detectability and isolability that can be achieved for a system given a set of measurements.

In the first approach [1], the Fault-Driven Minimal Structurally Overdetermined (FMSO) Set concept is introduced, which can be directly used to construct an ARR (or residual generator). A heuristic optimization method to obtain the minimal cardinality set of compound FMSO sets is used in [1]. This optimization procedure can be improved by using BILP optimization as proposed in [2] utilizing MSOs (each is sensitive to a set of faults) and the structural equivalence to the compound FMSO sets formation shown.

In the second approach [4], after applying sensor placement algorithm proposed in [3], a set of minimum coupled set of MSOs are obtained. The adaptation presented in [4] aims at placing the sensors not only to guarantee detectability and isolability properties but also to facilitate the partition of a system into various subsystems by reducing number of links (communication) within a

system. This algorithm also minimizes the number of sensors to be installed thus reducing overall cost.

For solving the BILP optimization without the need of previous computation of the complete MSOs set, which is a computationally complex task, some methods have been developed [7]. In [12], an efficient method to finding all the minimal sensors set for maximum fault detectability and isolability from a structural model is proposed.

The structure of the paper is as follows: Section 2 presents a four tank system used as case study along the paper. Sections 3 and 4 present the two distributed fault diagnosis approaches. Section 5 presents the comparison of the two approaches using the case study presented in Section 2. Finally, Section 6 draws the main conclusions.

## 2. Case Study

The case study used to compare both approaches is based on four tank system, proposed in [2], and is shown in Fig. 1.  $V_1, V_2, V_3, V_4$  are the volumes of water in each tank,  $q_{12}, q_{23}, q_{34}, q_4$  represents the flows of water through each pipe ( $P_1, P_2, P_3, P_4$ ),  $u_1$  and  $u_2$  represents the water sources.

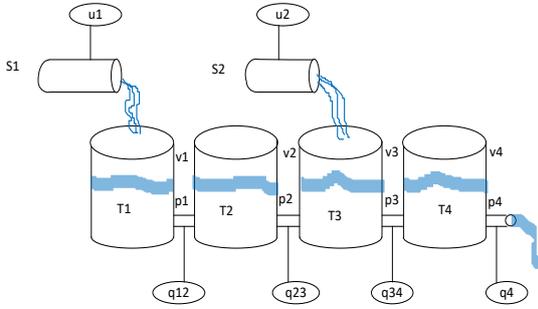


Figure 1: Four Tank System [2], [4].

The four tank system is modeled through the following equations [4]:

$$e_1 : \dot{v}_1 = \frac{1}{C_{T1} + f_1} (q_{in1} - q_{12}) \quad e_4 : q_{in1} = u_1$$

$$e_2 : q_{12} = \frac{v_1 - v_2}{R_{v_{12}} + f_2} \quad e_5 : v_1 = y_1$$

$$e_3 : v_1 = \int \dot{v}_1 dt \quad e_6 : q_{12} = y_2$$

$$e_7 : \dot{v}_2 = \frac{1}{C_{T2} + f_3} (q_{12} - q_{23}) \quad e_{10} : v_2 = y_3$$

$$e_8 : q_{23} = \frac{v_2 - v_3}{R_{v_{23}} + f_4} \quad e_{11} : q_{23} = y_4$$

$$e_9 : v_2 = \int \dot{v}_2 dt$$

$$e_{12} : \dot{v}_3 = \frac{1}{C_{T3}} (q_{in2} + q_{23} - q_{34}) \quad e_{15} : q_{in2} = u_2$$

$$e_{13} : q_{34} = \frac{v_3 - v_4}{R_{v_{34}} + f_5} \quad e_{16} : q_{34} = y_5$$

$$e_{14} : v_3 = \int \dot{v}_3 dt$$

$$e_{17} : \dot{v}_4 = \frac{1}{C_{T4} + f_6} (q_{34} - q_4) \quad e_{19} : v_4 = \int \dot{v}_4 dt$$

$$e_{18} : q_4 = \frac{v_4}{R_{v_4}} \quad e_{20} : v_4 = y_6$$

## 3. First Approach: Fault-Driven Minimal Structurally Overdetermined Sets

### 3.1 Background concepts

This approach [1], [6] establishes a structure connecting FMSOs with minimum number of shared measurements (communication) from neighboring subsystems by an iterative matching procedure.

The global FMSO sets,  $\Phi$ , are obtained from the set of local FMSO sets  $\Phi^l$ , and locally computed shared FMSO sets  $\Phi^s$  and shared CMSO sets  $\Psi^s$  (different subsystems) which forms a compound FMSO set [1], [6].

The shared CMSO (Clear Minimal Structurally Overdetermined) set, whose fault support is empty, corresponds to the measurements (internal (subsystem  $i$ ) and from neighboring subsystems (shared variables)).

The FMSO sets including equations with shared variables are called shared FMSO sets [1], [6].

The shared variables  $X_i^s = \{q_{12}, v_2, q_{23}, v_3, q_{34}, v_4\}$ , which  $X_i^l$  don't include, are considered as known variables [1], [6].

Compound FMSO set ( $\varphi'$ ): A global FMSO set that includes at least one shared FMSO set whose fault support is not empty, contains equations from at least two subsystems [1], [6].

The optimal compound FMSO set selection is performed by a heuristic method.

A local FMSO set for any subsystem  $\Sigma_i$  is also an FMSO set of  $\Sigma$ , hence a global FMSO set [1], [6].

A local FMSO set ( $\varphi \in \Phi_i^l$ )'s equations include local and shared variables of  $\Sigma_i$  and only involve the fault  $f_i$ . To achieve detectability of fault  $f_i$ , only the equations included in  $\varphi$  required [1], [6].

The concept of compound FMSO set allow us to establish the relation between FMSO sets for the subsystems and FMSO sets for the global system [1], [6].

To illustrate the previous concepts the example used in [1], [2], [6] is used that considers the first tank of the proposed case study

$$\begin{aligned} e_1 : \dot{v}_1 &= \frac{1}{C_{T1} + f_1} (q_{in1} - q_{12}) \\ e_2 : q_{12} &= \frac{v_1 - v_2}{R_{v_{12}} + f_2} \\ e_3 : v_1 &= \int \dot{v}_1 dt \\ e_4 : q_{in1} &= u_1 \\ e_5 : v_1 &= y_1 \\ e_6 : q_{12} &= y_2 \end{aligned} \quad (1)$$

Then, the set of shared FMSO sets  $\Phi_i^s$  is  $\{\varphi_1, \varphi_2, \varphi_3\}$ :

$$\begin{aligned}
\varphi_1 &= \{e_2, e_5\}, \text{ where :} \\
X_{\varphi_1} &= \{v_1\}, Z_{\varphi_1} = \{q_1, v_2, y_1, y_2\}, F_{\varphi_1} = \{f_2\} \\
\varphi_2 &= \{e_1, e_2, e_3, e_4\}, \text{ where :} \\
X_{\varphi_1} &= \{v_1, v_1, q_{in1}\}, Z_{\varphi_2} = \{q_1, v_2, u_1\}, F_{\varphi_2} = \{f_1, f_2\} \\
\varphi_3 &= \{e_1, e_3, e_4, e_5\}, \text{ where :} \\
X_{\varphi_1} &= \{v_1, v_1, q_{in1}\}, Z_{\varphi_2} = \{q_1, u_1, y_1\}, F_{\varphi_2} = \{f_1\} \quad (2)
\end{aligned}$$

$$\varphi \subseteq \Sigma_i, X_\varphi \subseteq X_i^l, Z_\varphi \cap X_i^s \neq \emptyset, \text{ and } Z_\varphi \subseteq (Z_i \cup X_i^s) \quad (3)$$

The procedure to compute a global FMSO set  $\varphi^c$ , starts by searching in the bipartite graph  $G(X, \Gamma)$  for a matching that covers each shared variable of  $X_{\varphi_r}^s$  ( $\varphi_r$  is the root FMSO set) given by Figure 4 in [1], [6].

According to the operational procedure of Algorithm 1 in [6], [1], it is possible to get the set of all global FMSO sets  $\Phi$  from the set of local FMSO sets  $\Phi^l$  shared FMSO sets  $\Phi^s$  and shared CMSO sets  $\Psi^s$ .

Considering all the possible root FMSO sets, 164 compound FMSO sets are computed for this system. Added to  $\varphi_4 = \{e_1, e_3, e_4, e_5, e_6\} \in \Phi_1^l$ , which is a local FMSO set for subsystem  $\Sigma_1$ , the 165 global FMSO sets are found for  $\Phi$  [1], [5].

### 3.2 Distributed diagnosis

Given a set of faults, measurements and local models for every subsystem, we now construct local diagnosers that together make the entire system completely diagnosable. Using the Algorithm 2 and definitions of Chapter 2 in [1], [6], we can develop a local full diagnosis for every subsystem.

These results demonstrate that all considered faults can be detected and isolated, e.g. in the considered example, detectability is achieved for  $f_1$  using  $\varphi_4 \in \Phi_i^l$  (local FMSO) of Table 4 in [1], [6] (no additional measurement is needed). For  $f_2$ , detectability is achieved obtaining a compound FMSO set  $\varphi_9 \in \Phi_1^c$  lumping  $\varphi_1 \in \Phi_1^s$  (as root FMSO set) with  $\psi_1 \in \Psi_1^s$  and  $\psi_2 \in \Psi_2^s$ . Optimal compound FMSO sets from 164 compound FMSO sets are obtained by heuristic method as presented in Table 1.

|   |                 |  |
|---|-----------------|--|
| $\Phi_1^c = \{\varphi_9\}$<br>$\varphi_9 = \{e_2, e_5, e_6, e_{10}\}$   | $F_{\varphi_1}$ | $F_{\varphi_9} = \{f_2\}$                                    |
| $\Phi_2^c = \{\varphi_{10}, \varphi_{11}\}$<br>$\varphi_{10} = \{e_6, e_7, e_9, e_{10}, e_{11}\}$<br>$\varphi_{11} = \{e_8, e_{10}, e_{11}, e_{13}, e_{16}, e_{20}\}$ | $F_{\varphi_2}$ | $F_{\varphi_{10}} = \{f_3\}$<br>$F_{\varphi_{11}} = \{f_4\}$ |
| $\Phi_3^c = \{\varphi_{12}\}$<br>$\varphi_{12} = \{e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{20}\}$  | $F_{\varphi_3}$ | $F_{\varphi_{12}} = \{f_5\}$                                 |
| $\Phi_4^c = \{\varphi_{13}\}$<br>$\varphi_{13} = \{e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$  | $F_{\varphi_4}$ | $F_{\varphi_{13}} = \{f_6\}$                                 |

Table 1: Optimal compound FMSO sets  $\Phi_i^c$  ( $i = 1..4$ ) obtained by heuristic method for distributed diagnosis [1], [6].

Algorithm 2, using a heuristic optimization method, produces a minimal cardinality set of compound (global) FMSO sets while minimizing subsystems interactions.

## 4. Second Approach: Minimal Coupled MSOs

### 4.1 Background concepts

In the second approach [4], the graph  $G(V, E)$  representing the set of MSOs is obtained considering that

- the MSOs are the graph vertices collected in a set  $V$ ,
- the measured input/output variables are the graph edges collected in a set  $E$ .

Each *M*SO set will consists of in any case one constraint that can be used as an ARR. MSOs represent the redundancies in the system and can form the basis for fault detection and isolation. As given in [2], for the running example, there were 165 MSOs generated using the algorithm proposed in [12]. For example for the first tank, the only *M*SO =  $\{e_1, e_3, e_4, e_5, e_6\}$  as  $e_5$  is the redundant equation. The number of *ARR*s generated in this way will be larger than the set of *ARR*s found from a single complete matchings (ranking algorithm), and get a set of *ARR*s for each of these matchings (the number of *ARR*s was 16 ( $C_1, \dots, C_{16}$ ) as obtained in [4]).

The second approach by *ARR*s in original [4] performs better concerning computational complexity, without the need of previous computation of the complete MSOs set. For the comparison on a common basis, in this work we used MSOs instead of *ARR*s in the second approach. The analysis is to be shown with MSOs as the same carried out by *ARR*s, judging that the inference will be equivalent. After obtaining the model from the sets of equations or set of all MSOs, sensor placement algorithm [3] is applied.

A binary matrix  $W = [wij]$  of size  $n \times k$  containing the set of MSOs (the row set) and sensors (the column set) is formed. Matrix  $W$  refers to the set of sensor faults an *M*SO is sensitive to. In the same way, the binary matrix  $V = [vij]$  of size  $n \times l$  relates the set of MSOs (the row set) and process faults (the column set). These relations are known as fault signature matrix (FSM) [9]. After obtaining  $W$  and  $V$  (the process faults not shown here) according to [4], the values of  $W$  and  $V$  are used to find various constraints in (6) [4], [13].

### 4.2 Minimizing the coupling between MSOs

In order to facilitate the distributed implementation of the fault diagnosis systems, the sensors should be placed such that the coupling between MSOs is minimized. This is achieved by adding additional constraints that minimizes the number of common links between MSOs [4], [13]. First, a constraint that reduces the number of row links coupling is written in compact form as

$$\begin{pmatrix} W_{ixj} & (0)_{ixp} & -I_{ixi} & (0)_{ixu} \end{pmatrix} (r)_{ix1} \leq 0_{ix1} \quad (4)$$

An analog constraint could be added to minimize the row links coupling as follows

$$\begin{pmatrix} (W_{ixj})^T & (0)_{j \times y} & -I_{j \times j} \end{pmatrix} (c)_{j \times 1} \leq 0_{j \times 1} \quad (5)$$

The MSOs (the row set (vertices)) are added.

Additional constraints were added in the existing optimal sensor placement algorithm using Binary Integer



for that to be comparable with the second approach [4] in terms of matrix sizes.

Then, the matrix size increase for the second approach to obtain minimal coupled MSOs, by adding constraints in already existing optimal sensor placement algorithm (BILP) [3], was demonstrated to see which approach is more efficient.

### 5.1 First approach: Fault-Driven Minimal Structurally Overdetermined Sets [1]

This approach [1] used a heuristic method (Algorithm 2) in optimizing the compound FMSO sets, after generating all “locally computed” global and compound sets using Algorithm 1 [1], a structure connecting FMSOs with minimum number of shared measurements (communication) from neighboring subsystems:

- 164 compound FMSO sets
- 165 global FMSO sets (1 local FMSO set)

Using the definitions given in Section 3.1, we can demonstrate a sketch for the optimal compound FMSO set selection, the shared FMSO /CMSO sets, e.g., for subsystem 1 ( $S_1$ ):

|  |   |
|--|---|
|  | <u>subsystem 1 (<math>S_1</math>)</u>   |
| for <u>subsystem 1 (<math>S_1</math>)</u>                | <u>shared FMSO sets</u><br>(as root FMSO set)   |
| <i>optimal</i>   |   |
| a <u>compound FMSO set</u> $\varphi_9 \in \Phi_1^c$      | <u>lumping</u> $\varphi_1 \in \Phi_1^s$   |
| with $\psi_1 \in \Psi_1^s$ and $\psi_2 \in \Psi_2^s$ .   |   |
| <u>subsystem 1 (<math>S_1</math>)</u><br>shared CMSO set | <u>subsystem 2 (<math>S_2</math>)</u><br>(neighboring subsystems)<br>shared (variable) CMSO set |
|  | (also <u>shared FMSO sets</u> )<br><u>from neighboring subsystems</u>                           |

(7)

The shared CMSO set, whose fault support is empty, corresponds to the internal measurements ( $S_i$ ) and measurements from neighboring subsystems (shared variables).

From the Table 1,  $\Phi_i^c$  ( $i = 1..4$ ) by a heuristic method, 5 optimal compound FMSO sets for the 4 subsystems are obtained from 164 compound FMSO sets as given below:

$$\Phi_1^c = \{\varphi_9\}, \Phi_2^c = \{\varphi_{10}, \varphi_{11}\}, \Phi_3^c = \{\varphi_{12}\}, \Phi_4^c = \{\varphi_{13}\}.$$

$$\begin{aligned} \varphi_9 &= \{e_2, e_5, e_6, e_{10}\} \\ \varphi_{10} &= \{e_6, e_7, e_9, e_{10}, e_{11}\} \\ \varphi_{11} &= \{e_8, e_{10}, e_{11}, e_{13}, e_{16}, e_{20}\} \\ \varphi_{12} &= \{e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{20}\} \\ \varphi_{13} &= \{e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\} \end{aligned} \quad (8)$$

The red colored equation relates to the shared CMSO set (variable) from neighboring subsystems, corresponding minimal subsystems interactions (one or possibly more

from each nearest neighbor), the blue colored equation relates to the shared CMSO set from subsystem  $i$  ( $S_i$ ), the black colored equation to the shared FMSO set from  $S_i$  (the root), and the yellow colored equation to the shared FMSO set from a neighboring subsystem.

### 5.2 Applying Khorasani’s BILP Approach [2] to the First Approach in Optimizing the Compound FMSO Sets by Analogy

Khorasani [2] used in this approach the Binary Integer Linear Programming (BILP) for optimization.

The optimization problem takes into account the relationship between measurements and MSOs (each MSO is sensitive to a set of faults).

A distributed MSO selection is to design an algorithm that selects  $MSO_i$  (locally) in a way that we add a minimum number of measurements to develop a local diagnoser (agent) for each subsystem.

The optimal shared variables selection is performed by the Binary Integer Linear Programming (BILP).

$$MSO_{k-l} = \underbrace{MSO_i(\text{measurements})}_{M_i} + \underbrace{\text{measurements}}_{M_o} \quad (9)$$

(the part of CMSO set from neighboring subsystems)

where  $M_o$  represents the set of measurements (not belonging to subsystem  $i$ ) we need to communicate to the subsystem  $S_i$  along with the set of measurements,  $M_i$  associated with the subsystem  $S_i$ .

$MSO_i$  corresponds, in the optimal compound FMSO set, to the shared FMSO set and the part of the shared CMSO set as the internal measurements in subsystem  $i$  ( $S_i$ ).

$M_o$  corresponds, in the optimal compound FMSO set, to the part of the shared CMSO set as the measurements from the neighboring subsystems of  $S_i$ .

According to [2], using a MSO is equivalent to using the measurements ( $M_i$ ) that are included in the MSO, and we need to include this in the optimization problem. For example, consider  $MSO_{11}$ , it has three measurements  $M_{11} = \{u_1, y_1, y_2\}$ . Using  $MSO_{11}$  in a local diagnosis subsystem means we need to communicate these measurement streams to that subsystem to achieve global diagnosability for the faults that belong to that subsystem.

We need to demonstrate that this model can be considered as structurally equivalent to the compound FMSO set model of the first approach, and hence, we can use this approach, which uses BILP, in the first approach for that to be comparable with the second approach: As given in [2], for the running example there were 165 MSOs generated, 3 measurements in the subsystem 1, and 8 measurements for the entire system.

Subsystem 1 has two faults of interest, and the goal is to be able to isolate them from any of the 6 faults in the complete system.

Therefore, to solve the optimization problem in [2] for subsystem 1 ( $S_1$ ), matrix A has

177 rows (equal to the number of constraints):

- 2 constraints to guarantee the local detectability of  $f_1$  and  $f_2$ ,
- 10 constraints to guarantee the local isolability of  $f_1$  and  $f_2$  from the other faults, and
- 165 constraints to capture the relationship between the MSOs and the measurements and

173 columns (equal to the number of binary variables):

- 8 constraints for the measurements
- 165 constraints for the MSOs and

b is a vector with 177 elements (equal to the number of constraints).

Table 5 shows, in the  $MSO_{k-l}$  form, the minimum number of shared measurements (from neighboring subsystems and possibly one from each neighbor) obtained with the BILP Optimization.

Table 5: Set of augmented measurements to each subsystem model [2].

| Subsystem | $MSO_{k-l}$ | Set of augmented measurements |                   |
|-----------|-------------|-------------------------------|-------------------|
| $S_1$     | $MSO_{k-l}$ | $y_3$                         | $(S_2)$           |
| $S_2$     | $MSO_{k-l}$ | $y_2, u_2, y_6$               | $(S_3, S_1, S_4)$ |
| $S_3$     | $MSO_{k-l}$ | $y_4, y_6$                    | $(S_2, S_4)$      |
| $S_4$     | $MSO_{k-l}$ | $y_5$                         | $(S_3)$           |

From the Table 6, the solution obtained in the first approach by heuristic method the measurements from neighboring subsystems as the CMSO sets ( $\psi_i \in \Psi_i^S$ ) of  $\varphi_{9,10,11,12,13}$  (5 optimal compound FMSO sets  $\Phi_i^c$ , ( $i = 1..4$ ) for the 4 subsystems) is shown.

Table 6: Set of augmented measurements to each subsystem model – first approach solution by heuristic method [1].

|       |                |            |            |              |
|-------|----------------|------------|------------|--------------|
| $S_1$ | $\varphi_9$    | $y_1, y_2$ | $y_3$      | $(S_2)$      |
| $S_2$ | $\varphi_{10}$ | $y_3, y_4$ | $y_2$      | $(S_1)$      |
|       | $\varphi_{11}$ | $y_3, y_4$ | $y_5, y_6$ | $(S_3, S_4)$ |
| $S_3$ | $\varphi_{12}$ | $u_2, y_5$ | $y_4, y_6$ | $(S_2, S_4)$ |
| $S_4$ | $\varphi_{13}$ | $y_6$      | $y_5$      | $(S_3)$      |

As seen from the Table 5 and Table 6, all the measurements (one from each neighbor) from the neighboring subsystems are the same except for  $S_2$ ,  $u_2$  in [2] is selected instead of  $y_5$  in [1] from the same subsystem ( $S_3$ ). This difference comes from the use of shared FMSO set from a neighboring subsystem ( $e_{13}$  used in  $\varphi_{11}$ ) in this approach [1] (see equation (8) and Table 5). For both application ([2], Table 5 and [1], Table 6), subsystem 2 is the only subsystem that shares a variable with a second order connected subsystem, all the other subsystems only need to communicate with their first order (nearest) connected subsystems. To minimize the number of

measurements from the other subsystems, as given in the cost function in Eq. 9 [2], the cost will incur only the external measurements from the neighboring subsystems and **not** the measurements **as** internal in  $S_i$  and **as** the use of shared FMSO set from a neighboring subsystem with the root shared set ( $S_i$ ).

Therefore, we could decide that both application lays on the same basis and Khorasgani's approach using BILP (i.e., the optimization stage) obtains the **optimal** equivalent results so as to be used in the first approach for comparison.

The above case is that the shared FMSO set's fault support is not empty and then there are two faults ( $f_4, f_5$ ) included in  $\varphi_{11}$  corresponding two shared FMSO sets, but, since the local diagnoser  $\varphi_{12}$  will respond to its subsystem's ( $S_3$ ) fault ( $f_5$ ), in achieving global diagnosability,  $\varphi_{11}$  can act for only the fault ( $f_4$ ) of the root shared set ( $S_2$ ) not the one ( $f_5$ ) of the shared set in the neighbor ( $S_3$ ).

Alternatively by another algorithm, a different compound FMSO set for  $S_2$  can also *optimally* select the same measurements from the neighbors in Table 5 (possibly more than one from each first order (nearest) neighbor) which provides a practical advantage by not needing to transfer data over long distances, which can be costly and error-prone [2].

This approach [2] now establishes a structure connecting FMSOs, in compound, with minimum number of shared measurements from neighboring subsystems.

If we apply Binary Integer Linear Programming (BILP) instead of a heuristic method in the first approach, for the four tanks ( $S_1, S_2, S_3, S_4$ ), similar to the analysis in [2]:

subsystem 1 ( $S_1$ ), matrix A has

176 rows (equal to the number of constraints):

- **164** constraints to capture the relationship between the FMSOs and the measurements

In this case, **164** constraints are used for the relationship corresponding to the number of all compound FMSO sets computed in the first approach except for the one local FMSO set.

Since we have **165 global** FMSOs (165 constraints) in this approach as given in [1], the total number of columns (c) for each subsystem is 173.

for all the subsystems, the matrix A:

|                                       |   |    |     | total (r) | total (c) |
|---------------------------------------|---|----|-----|-----------|-----------|
| <b>subsystem 1 (<math>S_1</math>)</b> |   |    |     |           |           |
| 2 faults                              | 2 | 10 | 164 | 176 (r)   | 173 (c)   |
| 3 measurements                        |   |    |     |           |           |
| <b>subsystem 2 (<math>S_2</math>)</b> |   |    |     |           |           |
| 2 faults                              | 2 | 10 | 164 | 176       | 173       |
| 2 measurements                        |   |    |     |           |           |
| <b>subsystem 3 (<math>S_3</math>)</b> |   |    |     |           |           |
| 1 fault                               | 1 | 5  | 164 | 170       | 173       |

2 measurements

$$\begin{array}{l} \text{subsystem 4 (S}_4\text{)} \\ \text{1 fault} \\ \text{1 measurement} \end{array} \quad \begin{array}{cccc} 1 & 5 & 164 & 170 & 173 \end{array} \quad (10)$$

for the system ( $S_1, S_2, S_3, S_4$ ):

$$176 + 176 + 170 + 170 = 692 \text{ (r)}$$

The analysis results in (692, 173) element matrix. This matrix size is to be processed if we apply BILP to the first approach for optimizing the compound FMSO sets, which is to be compared with the one to be obtained in Section 5.3.

### 5.3 Second Approach: Minimal Coupled MSOs

The analysis is performed for the four tank example:

As obtained in the second approach in Section 4.1, using the methodology in [3] as the first step:

$$\begin{array}{l} \text{n number of MSOs} \\ \text{k number of candidate sensors} \\ \text{q = 8, } \lambda = 165, \text{ and} \\ \text{l number of process faults} \end{array} \quad \begin{array}{l} = 165 \\ = 8 \\ = 2 \end{array} \quad (11)$$

$$\begin{pmatrix} -W & kI_n \\ 0_{l \times k} & -V^T \\ I_k & -W^T \\ 0_{C_2^l \times k} & -V_{ff}^T \\ G_2 & -V_{fs}^T \\ G_3 & -V_{ss}^T \end{pmatrix} \begin{pmatrix} q \\ \lambda \end{pmatrix} \leq \begin{pmatrix} \beta \\ -1_{l \times 1} \\ 0_{k \times 1} \\ -1_{C_2^l \times 1} \\ 0_{l,k \times 1} \\ 1_{C_2^k \times 1} \end{pmatrix} \quad (12)$$

The number of rows (*i.e.*, constraints)

- The *MSO selector* constraints (11) in [3] involve
$$\text{n}(\lambda) = 165 \text{ rows}$$
- The detectability constraints (14) and (17) in [3] involve
$$l + k(q) = 10 \text{ rows}$$
- The isolability constraints (20), (24), and (30) in [3] involve

$$C_2^l + l.k + C_2^k = 1 + 16 + 28 = 45 \text{ rows}$$

$$(165 + 10 + 45) = 220 \text{ rows}$$

$$\text{n}(\lambda) + k(q) = 165 + 8 = 173 \text{ columns}$$

The analysis results in as (220, 173) element matrix.

This is the matrix size for the four tank example, before adding constraints (the rows) in the following, as obtained with the existing optimal sensor placement algorithm.

The matrix size by adding additional constraints, the row set, to choose MSOs that form a system with minimum coupling (communication) was shown below:

From Section 4.2, using equations (4) and (5) [4], we checked how these additional constraints act.

$$\begin{array}{l} (W_{165 \times 8} \quad 0_{165 \times 165} - I_{165 \times 165} \quad 0_{165 \times 2}) \quad (r)_{165 \times 1} \leq 0_{165 \times 1} \\ (W_{8 \times 165} \quad 0_{8 \times 6} \quad -I_{8 \times 8}) \quad (c)_{8 \times 1} \leq 0_{8 \times 1} \end{array} \quad (13)$$

For each constraint, with the values as the numbers of MSOs ( $\text{n}(\lambda) = 165$ ) and (candidate) sensors ( $q = 8$ ) in columns, a simple validation performed first to see that the corresponding number of columns does not change by this approach as 173 in total as a joint effect of the constraints, thus maintaining the number of columns obtained with using [3]:

The rows added for each constraint as 165 and 8 are given in equation (13).

$$165 \times 8$$

$$8 \times 165$$

If we add the number of rows obtained with two additional constraints:

$$165 + 8 = 173 \text{ rows}$$

And totally, 173 rows are added.

As many as the column number before adding constraints the rows are added.

If we add the additional number (173) of rows (constraints) obtained in this section after adding constraints (the rows) in eq. (13), to the number of rows in (220, 173) matrix.

$$(220 + 173) = 393 \text{ rows}$$

We obtain the resulting (393, 173) element matrix.

In comparison, it is shown that for optimization the second work [4] performs better as (393, 173) element matrix than the first approach [1] processing a (692, 173) matrix in terms of computational complexity.

## 6. Discussion and Conclusions

Two distributed fault diagnosis approaches were compared, by analogy (i.e., the matrix sizes) to determine their efficiency in the case of using, for both, a Binary Integer Linear Programming (BILP) for optimization using a four-tank system example.

Though, as demonstrated in the first approach [1], the Fault-Driven Minimal Structurally Overdetermined (FMSO) Sets can be directly used to construct an ARR or residual generator, the matrix sizes to be processed for computing the optimal sets (apart from the computation of all global (compound) sets) were assessed.

Since the first approach used a heuristic optimization method to obtain the minimal cardinality set of compound FMSO sets, we applied Khorasgani's BILP optimization method, utilizing a structurally equivalent model to the compound FMSO sets formation, to the first approach to decide that Khorasgani's approach using BILP obtains the equivalent results so as to be used in the first approach for the purpose of comparison.

Then, we applied Binary Integer Linear Programming (BILP) instead of a heuristic method for the four tanks to find the matrix size to be processed in this case.

In the second approach [4], minimum coupled MSOs for minimizing the number of common links (communication) between MSOs are obtained by adding constraints (the row set/MSOs) in already existing optimal sensor placement algorithm [3], which uses BILP, but not in a distributed context and thus uses the complete set of MSOs.

For the comparison on a common basis, in this work we used MSOs instead of ARRs in performing the analysis of the second approach.

In the original second approach [4], in applying sensor placement algorithm, for solving the BILP optimization without the need of previous computation of the complete MSOs which requires a high computation time, the ARRs (the model) were generated using ranking algorithm and all the possible ARRs in addition to the primary ARRs obtained from the set of system model equations.

After solving the sensor placement problem, the algorithm ensures a set of minimum coupled (minimal sensors) set of MSOs for maximum fault detectability and isolability.

In comparison, it is shown that the second work performs better in terms of the matrix sizes to handle. Then again, it is preferential to use the second approach with ARRs in original [4] concerning computational complexity, in that case resulting as (95, 24) matrix.

In addition to this work, adding redundant sensors (the column set) to obtain the ideal solution (best matching) in the fault signature matrix can be shown.

As the most efficient approach from the required MSO sets point of view *causal computation* approach can be studied which needs no MSO sets.

The uncertainty in the system could be studied by using statistical and stochastic methods for robust distributed fault detection and isolation.

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