

Optimal Sensor Placement Problem for an Electro–pneumatic Actuator

Kornel Rostek¹

¹Warsaw University of Technology, Poland

e-mail: rostek@mchtr.pw.edu.pl

Abstract

In this paper, a method for formulating and solving the optimal sensor placement problem for an electro–pneumatic actuator is presented. The approach minimizes the number of additional sensors while maintaining maximum possible diagnosability and isolability. The proposed strategy is based on a Binary Diagnostic Matrix. Proposed isolability measure distinguish weak and strong isolability. It uses the branch-and-cut algorithm to find a solution.

1 Introduction

The quality of diagnosis is often characterized using the fault isolability. Different methods of Fault Detection and Isolation (FDI) can be compared with it.

Available measurements strongly affect the performance of an FDI system for a given industrial process. Additional sensors providing additional information about a process can improve the performance of an FDI system. From a practical point of view, it is vital to achieving the best possible FDI system performance with minimal additional costs. The problem of optimal sensor selection can be understood as a combinatorial problem of selecting the optimal set of measurements.

In recent years, numerous papers discussed different problems of the optimal sensor placement. The required minimum fault isolability of the diagnostic system is usually considered [1; 2]. Some of the proposed methods also maximize designed fault isolability using heuristic methods, e.g., genetic algorithms [3].

The model-based FDI considers faults as deviations from nominal values of process parameters or as unknown process inputs. If system model and measurements behave differently, then faults are detected. In [4], a method for searching for the optimal sensor set based on Analytical Redundancy Relations (ARRs) is proposed. First, all ARRs are found under the assumption that all sensor candidates are installed. Then, a sensor set is selected that minimizes the cost while satisfying detectability and isolability requirements. However, this solution is computationally expensive. A modified, incremental approach, using Minimal Structurally Overdetermined (MSO) sets, was proposed in [5]. In [6] the Binary Integer Programming is used to find the optimal sensor set using the set of all possible MSO sets. FDI requirements were ensured using non-linear constraints. The resulting problem is computationally difficult to solve. This

method was further improved in [7] and [8]. There, FDI requirements were specified as linear constraints. As the cost function is linear, the problem falls into Binary Integer Linear Programming (BILP). It can be efficiently solved with a branch-and-bound algorithm with standard Linear Programming (LP) solver. Those methods were thoroughly compared in [9]. Budgetary constraints were analyzed in [10]. The branch-and-bound algorithm is used to obtain the optimal solution. Regardless of chosen method, simple, qualitative methods of analysis of fault isolability are insufficient. Generalized, quantitative method of fault isolability analysis is required.

This paper presents the method of an optimal sensor placement for diagnostic purposes using linear constraints and a linear objective function. The method uses a new measure of isolability proposed by the author. The main contribution of this metric is that both weak and unidirectionally strong isolability properties are considered and distinguished. Various additional optimization constraints are analyzed. The model of an electro–pneumatic actuator was used as an example illustrating the procedure.

The paper is organized as follows. In Section 2, the preliminary definitions used in this work are given. Section 3 defines the measure of fault isolability. Section 4 presents the proposed optimization procedure. Section 5 describes the example of an electro–pneumatic actuator. Conclusions and final remarks section finalizes this paper.

2 Preliminaries

A signal sensitive to faults is considered as a diagnostic signal in FDI. A symptom is a value of a diagnostic signal which indicates fault or faults. In the case of multi-valued diagnostic signals, one fault type may generate different values of each diagnostic signal. A fault signature is a vector of diagnostic signal values associated with a particular fault [11]. In case of multi-valued diagnostic signals, multiple values of a single diagnostic signal can be associated with a fault. The specific vector of values of diagnostic signals is called an alternative signature [11]. In the case of binary diagnostic signals, each fault has exclusively one alternative signature, while for multi-valued diagnostic signals there might be multiple alternative signatures.

There are different definitions of fault isolability. Generally, faults are considered isolable when at least some of their signatures are different [12].

Binary Diagnostic Matrix (BDM) or Incidence Matrix is a form of notation of a relationship specified by the Cartesian product of diagnostic signals sets $S = \{s_j : j = 1, 2, \dots, J\}$

Table 1: The Binary Diagnostic Matrix example. 1 in j^{th} row and i^{th} column means that f_i is detectable with signal s_j .

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
s_1	1	1	1		1			
s_2		1	1	1	1			
s_3				1	1			
s_4						1	1	1
s_5							1	1

and faults $F = \{f_i : i = 1, 2, \dots, n\}$. Each row displays sensitivity of a given diagnostic signal to each fault. Each column $V_i = [v_{1,i}, v_{2,i}, \dots, v_{J,i}]^T$ of binary diagnostic matrix V can be associated with a fault f_i . Often column V_i is called signature of fault f_i . An example of binary diagnostic matrix is shown in Table 1.

The basic definition of isolability can be formulated in the context of BDM in the following way [13]:

Definition 1.

Faults $f_k, f_m \in F$ are weakly isolable if their signatures are different.

In the example from Table 1 all faults with exception of a pair (f_2, f_3) are weakly isolable. A weak isolability in some applications is not sufficient. It is possible that due to a different sensitivity of diagnostic tests or process dynamics some signals appear earlier and match a signature of a different, weakly isolated fault. In the above example, the appearance of only the signal s_1 may be insufficient to indicate the fault f_1 reliably. Later signals s_2 or s_3 may appear indicating faults f_2, f_3 or f_5 . Therefore a stronger isolability property is required [13].

Definition 2.

A structure is unidirectionally strongly isolating if it is weakly isolating and if no column in the structure matrix can be obtained from any other column by turning an arbitrary number of "1"s into "0"s or by turning an arbitrary number of "0"s into "1"s.

In a unidirectionally strongly isolating structure, each pair of faults differs in at least two entries. Firstly, where "1" is in the first column and "0" in the other one and secondly, where "0" is in the first column and "1" in the other one. A weak isolability is a necessary condition for a strong isolability.

In Table 1 faults f_5 and f_6 are unidirectionally strongly isolable.

From Definition 2 following statement can be extrapolated:

Definition 3.

The signature V_i is excluding a fault f_k if V_i is different than V_k and V_i cannot be obtained from V_k by turning "1"s into "0"s.

Opposite does not have to be true.

Definition 4. *Faults $f_k, f_m \in F$ are weakly isolated iff each alternative fault signature $\phi(f_k)$ excludes the fault f_m , or each alternative fault signature $\phi(f_m)$ excludes the fault f_k .*

If faults are mutually excluding each other, then they are strongly isolable.

Definition 5. *Faults $f_k, f_m \in F$ are unidirectional strongly isolable iff each alternative fault signature $\phi(f_k)$ excludes*

the fault f_m , and each alternative fault signature $\phi(f_m)$ excludes the fault f_k .

In Table 1 signature V_2 is excluding f_1 . Opposite is not true so they are not strongly isolable.

Commonly, in FDI an exoneration assumption is accepted. It states that a lack of symptoms exonerates a fault. It means that all symptoms must appear for a fault isolation. This assumption is not always valid. Due to dynamics of symptoms and different sensitivity to faults, they may not appear simultaneously or may even not appear at all.

3 Measure of isolability

An implementation of the measure of isolability for a Binary Diagnostic Matrix was proposed in [14]. The value of this measure is calculated in two steps:

1. Calculate the value of the following discrete function for all possible ordered pairs of faults:

$$D: F \times F \rightarrow \{0, 1\}, \quad (1)$$

where: F is the set of faults and $f_k \in F, k = 1 \dots K$ are particular faults. It is assumed that the value $D(f_k, f_m) = 1$ when the appearance of all symptoms of the fault f_k excludes the fault f_m . If this is not true, then $D(f_k, f_m) = 0$.

2. Calculate the value of the measure as:

$$\psi = \frac{1}{(K-1)K} \sum_{k=1}^K \sum_{\substack{m=1 \\ m \neq k}}^K D(f_k, f_m). \quad (2)$$

In the case of multi-valued diagnostic signals, the conditional isolability metric was proposed in [15]. The first step of calculation of the value of the proposed metric (1) needs to be slightly modified in order to take into account conditional isolability. Instead of assigning exclusively values 0 or 1 to each ordered pair of faults, the $D(f_k, f_m)$ can take any value from the range [0, 1]. Let $D(f_k, f_m)$:

$$D(f_k, f_m) = \frac{\text{card}(\{\phi : \phi \in \Phi(f_k) \wedge \phi \text{ excludes } f_m\})}{\text{card}(\Phi(f_k))}, \quad (3)$$

where: $\Phi(f_k)$ is the set of all alternative signatures of the fault f_k .

The formula (3) generalizes the first step of calculation of the proposed measure. It can be understood as a fraction of all alternative signatures of f_k that excludes f_m . In the case of binary diagnostic signals, there is always only one alternative signature $\phi(f_k)$. The value of $D(f_k, f_m)$ is then equal to 0 or 1. Consequently, in the case of binary diagnostic signals, the formula (3) is equivalent to formulation below the formula (1).

The proposed metric of isolability makes it possible to distinguish unidirectional strong isolability from weak isolability. If $D(f_k, f_m) = 1 \vee D(f_m, f_k) = 1$, then the signature of the fault f_k excludes the fault f_m or the signature of the fault f_m excludes the fault f_k . Therefore, according to Definition 4, the faults are weakly isolable. Moreover, if $D(f_k, f_m) = 1 \wedge D(f_m, f_k) = 1$, then the signature of the fault f_k excludes the fault f_m and the signature of the fault f_m excludes the fault f_k . Thus, faults f_k and f_m are unidirectionally strongly isolable as defined in Definition 5.

The value of the presented measure of isolability can be interpreted as a mean fraction of all diagnoses that can be

excluded, after the occurrence of a single fault. The measure of isolability takes the maximal value when all pairs of faults are unidirectionally strongly isolable. In such a case, each single fault signature excludes $(K-1)$ other faults (the fault does not exclude itself). Then $\sum_{k=1}^K \sum_{\substack{m=1 \\ m \neq k}}^K D(f_k, f_m) = (K-1)K$ and the value of the measure of isolability is equal to $\frac{(K-1)K}{(K-1)K} = 1$.

4 Problem formulation for BDM

In this section, only binary diagnostic signals are analyzed. If a fault f_k is unisolable from f_m , then $D(f_k, f_m)$ is equal to 0. Otherwise, it is equal to 1. The value of $D(f_k, f_m)$ can be calculated in the following way:

$$x_{D_{k,m}} = D(f_k, f_m) = \max_{x_{s_j}} \{x_{s_j} : v_{j,k} \neq 0 \wedge v_{j,m} = 0\}, \quad (4)$$

where: x_{s_j} is the decision variable, which indicates that j^{th} diagnostic signal is available. This formula states that $D(f_k, f_m)$ is equal to 1 if at least one diagnostic signal s_j is sensitive to the fault f_k and not sensitive to the fault f_m . The shorthand notation $x_{D_{k,m}}$ will be used instead of $D(f_k, f_m)$ as a variable in the description of an optimal sensor placement problem.

Similarly, the variable x_{s_j} can be expressed as:

$$0 \leq x_{s_j} \leq \min_{x_i} \{x_i : x_i \text{ is necessary to calculate } s_j\}, \quad (5)$$

where: x_i is the decision variable, which indicates that i^{th} sensor is available. If even one of the sensors necessary for the diagnostic signal s_j is unavailable, then this signal cannot be used. The inequality relation \leq is used, because, even if all required sensors are available, the diagnostic signal may be not of interest, e.g., due to a too high cost of development of necessary models.

Example 1.

In Tab. 2 an example of a simple BDM is presented. There are three faults and three diagnostic signals. Each diagnostic signal requires two sensors to be available.

Table 2: Simple BDM and sensor requirements for diagnostic signals.

	f_1	f_2	f_3
$s_1(x_1, x_2)$	1	1	1
$s_2(x_1, x_3)$		1	1
$s_3(x_2, x_3)$			1

The following equations can be constructed:

$$\begin{aligned} x_{s_1} &\leq \min\{x_1, x_2\}, \\ x_{s_2} &\leq \min\{x_1, x_3\}, \\ x_{s_3} &\leq \min\{x_2, x_3\}, \\ x_{D_{2,1}} &= D(f_2, f_1) = \max\{x_{s_2}\} = x_{s_2}, \\ x_{D_{3,1}} &= D(f_3, f_1) = \max\{x_{s_2}, x_{s_3}\}, \\ x_{D_{3,2}} &= D(f_3, f_2) = \max\{x_{s_3}\} = x_{s_3}, \\ x_{s_1}, x_{s_2}, x_{s_3} &\geq 0, \\ x_i, s_{s_j} &\in \{0, 1\}, \quad i, j = 1 \dots 3. \end{aligned} \quad (6)$$

The pairs of faults for which $D(f_k, f_m) = 0$ were omitted.

The objective function maximize $\frac{1}{6} \sum_{k=1}^K \sum_{\substack{m=1 \\ m \neq k}}^K x_{D_{k,m}}$ with constraints (6) is a difficult, constrained, non-linear optimisation problem.

4.1 Additional constraints

Fault detectability

Generally, it is not possible to determine which faults will be detectable before solving the basic optimal sensor placement problem. In practice, detectability of the most important faults is often required. In a special case, this requirement can refer to all faults.

The detectability of a given fault can be interpreted as the possibility to distinguish this fault from the state without faults. To satisfy detectability requirements, an additional constraint can be added in the following way:

$$D(f_k, \text{faultless state}) = \max_{x_{s_j}} \{x_{s_j} : v_{j,k} \neq 0\} = 1. \quad (7)$$

This ensures that there is at least one signal sensitive to fault f_k .

If the problem with this additional constraint becomes infeasible, then it is impossible to meet the detectability requirements.

Example 2.

The detectability requirements for the problem introduced in Example 1 can be formulated in the following way:

$$\begin{aligned} f_1 : \quad &\max\{x_{s_1}\} = x_{s_1} = 1, \\ f_2 : \quad &\max\{x_{s_1}, x_{s_2}\} = 1, \\ f_3 : \quad &\max\{x_{s_1}, x_{s_2}, x_{s_3}\} = 1. \end{aligned} \quad (8)$$

Isolability constraints

For some critical subset of faults, it may be beneficial to require the solution of the optimal sensor placement problem to isolate these faults. These requirements can be fulfilled by introducing additional equality constraints. For example, if it is important that a fault f_k is isolable from a fault f_m , then the following constraint should be added:

$$x_{D_{k,m}} = 1. \quad (9)$$

If unidirectional strong isolability is desired, then two constraints should be added:

$$\begin{aligned} x_{D_{k,m}} &= 1, \\ x_{D_{m,k}} &= 1. \end{aligned} \quad (10)$$

If the isolability requirements cannot be satisfied, then the constrained problem will be infeasible.

Example 3.

For the diagnostic system introduced in Example 1, if it is required that the fault f_3 is isolable from both f_1 and f_2 , then the following constraints should be added:

$$\begin{aligned} x_{D_{3,1}} &= 1, \\ x_{D_{3,2}} &= 1. \end{aligned} \quad (11)$$

5 Optimal sensor placement problem for an electro-pneumatic actuator

To demonstrate an example of the optimal sensor placement formulation, an electro-pneumatic valve actuator will be discussed. Fig. 1 illustrates the actuator [16]. It consists

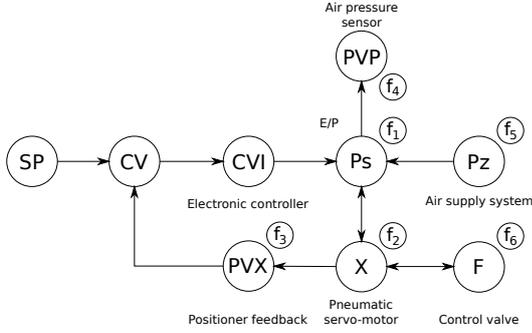


Figure 1: Causal graph of the electro-pneumatic actuator [16]. SP – set point, CV – control value, CVI – control value of the electro-pneumatic transducer, PVP – pressure measurement in servo-motor chamber, PVX – stem displacement, F – flow rate.

of an electronic controller, an electro-pneumatic converter, a servo-motor, a control valve, and an electro-mechanical stem position feedback. The list of available measurements includes the control value CV, the control value of the electro-pneumatic transducer CVI, the stem displacement measurement X, and the pressure in the chamber of the servo-motor. Tab. 3 lists the considered faults. Tab. 4

Fault	Faulty component
f_1	E/P transducer
f_2	Pneumatic servomotor
f_3	Position feedback
f_4	Pressure sensor fault
f_5	Supply air pressure
f_6	Control valve

specifies the considered binary diagnostic matrix.

Table 4: List of considered diagnostic signals for the electro-pneumatic actuator.

Signal	Residual	f_1	f_2	f_3	f_4	f_5	f_6
s_1	$X - f(CV)$	1	1	1		1	1
s_2	$X - f(CVI)$	1	1	1		1	1
s_3	$X - f(P_s)$		1	1		1	1
s_4	$P_s - f(CV)$	1	1	1	1	1	1
s_5	$P_s - f(CVI)$	1	1	1	1	1	1

Using Tab. 4, the maximum value of the metric of isolability ψ can be calculated as:

$$\psi = \frac{1}{(K-1)K} \sum_{k=1}^K \sum_{\substack{m=1 \\ m \neq k}}^K D(f_k, f_m) = \frac{9}{30} = 0.3. \quad (12)$$

To find the diagnostic structure with $\psi = 0.3$ and the minimum number of required sensors the optimal sensor placement problem should be formulated as (13).

$$\begin{aligned} & \underset{x}{\text{minimize}} && x_{CV} + x_{CVI} + x_{P_s} + x_X \\ & \text{s.t.} && \frac{1}{30} \sum_{k=1}^K \sum_{\substack{m=1 \\ m \neq k}}^K x_{D_{k,m}} = 0.3, \\ & && x_{s_1} \leq x_X, \\ & && x_{s_1} \leq x_{CV}, \\ & && x_{s_2} \leq x_X, \\ & && x_{s_2} \leq x_{CVI}, \\ & && x_{s_3} \leq x_X, \\ & && x_{s_3} \leq x_{P_s}, \\ & && x_{s_4} \leq x_{P_s}, \\ & && x_{s_4} \leq x_{CV}, \\ & && x_{s_5} \leq x_{P_s}, \\ & && x_{s_5} \leq x_{CVI}, \\ & && x_{D_{2,1}} \leq x_{s_3}, \\ & && x_{D_{3,1}} \leq x_{s_3}, \\ & && x_{D_{5,1}} \leq x_{s_3}, \\ & && x_{D_{6,1}} \leq x_{s_3}, \\ & && x_{D_{1,4}} \leq x_{s_1} + x_{s_2}, \\ & && x_{D_{2,4}} \leq x_{s_1} + x_{s_2} + x_{s_3}, \\ & && x_{D_{3,4}} \leq x_{s_1} + x_{s_2} + x_{s_3}, \\ & && x_{D_{5,4}} \leq x_{s_1} + x_{s_2} + x_{s_3}, \\ & && x_{D_{6,4}} \leq x_{s_1} + x_{s_2} + x_{s_3}, \\ & && x_{CV}, x_{CVI}, x_X, x_{P_s}, x_{s_j}, x_{D_{k,m}} \in \{0, 1\}, \\ & && k, m = 1 \dots 6, j = 1 \dots 5. \end{aligned} \quad (13)$$

To ensure that all faults are detectable the following constraints should be added:

$$\begin{aligned} f_1 : & \quad x_{s_1} + x_{s_2} + x_{s_4} + x_{s_5} \geq 1, \\ f_2 : & \quad x_{s_1} + x_{s_2} + x_{s_3} + x_{s_4} + x_{s_5} \geq 1, \\ f_3 : & \quad x_{s_1} + x_{s_2} + x_{s_3} + x_{s_4} + x_{s_5} \geq 1, \\ f_4 : & \quad x_{s_4} + x_{s_5} \geq 1, \\ f_5 : & \quad x_{s_1} + x_{s_2} + x_{s_3} + x_{s_4} + x_{s_5} \geq 1, \\ f_6 : & \quad x_{s_1} + x_{s_2} + x_{s_3} + x_{s_4} + x_{s_5} \geq 1. \end{aligned} \quad (14)$$

The problem (13) with (14) was solved using a Coin-or-branch-and-cut (Cbc) solver and a PuLP modeler. The following solution was returned by the solver: $x_{CV} = 1.0, x_{CVI} = 0.0, x_{P_s} = 1.0, x_X = 1.0$. Consequently, the optimal sensor set for given constraints is $\{CV, P_s, X\}$ and the resulting BDM is presented in Tab. 5. All of the con-

Table 5: Optimal BDM for the electro-pneumatic actuator.

	f_1	f_2	f_3	f_4	f_5	f_6
s_1	1	1	1		1	1
s_3		1	1		1	1
s_4	1	1	1	1	1	1

sidered faults are detectable and the value of the isolability measure is $\psi = 0.3$.

6 Conclusion

In this paper, the sensor placement problem was addressed for an electro–pneumatic actuator. A key contribution of this work is the introduction of a new measure of fault isolability as an objective function or constraint to Linear Programming problem. It distinguishes weak and unidirectionally strong isolability. A strategy of introducing new variables which allow obtaining BILP problem was presented. This strategy makes it possible to use efficient tools to find optimal sensors sets.

In this paper, the method was applied to a Binary Diagnostic Matrix, but the proposed measure of fault isolability can describe multi-valued systems such as Fault Information Systems (FIS).

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