Covariance Error Analysis for Pedestrian Dead Reckoning with Foot Mounted IMU *

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Abstract. We investigate dead reckoning with foot-mounted inertial measurement unit. To improve accuracy of navigation the ZUPT (zero velocity update) technic is commonly used, aiding the IMU with information on zero foot velocity during the stance phase of a step. This information is fed to a extended Kalman-type filter. The state vector of the filter usually contains position, velocity and orientation of the IMU. We show that the ZUPT condition can be written in two ways of which the most commonly used yields inconsistent results. For that purpose we employ covariance analysis. We suggest a decomposition of the error equations into the so-called dynamic and kinematic errors and decompose these equations into four simple subsystems. For each subsystem, error covariance can be written in explicit formulas.

Keywords: foot mounted IMU · pedestrian dead reckoning · ZUPT

1 Introduction

Pedestrian navigation is a fast developing branch of navigation where different systems are used to determine the user's position and velocity. Some systems are based on smartphones and use internal sensors (accelerometers, gyroscopes and magnetometers) and GNSS receivers, others use the WiFi or lidar signals. Following [1], [2], we investigate dead reckoning with foot-mounted inertial measurement units (IMU). To improve accuracy of foot-mounted systems the ZUPT technic is used [3]. The idea of ZUPT is to aid the IMU with information on zero foot velocity during stance phase of the step. This information is fed to an extended Kalman filter (EKF) or a UKF filter [4]. The state vector of a filter usually contains position, velocity and orientation parameters of the IMU. Some authors tried to add sensors errors to the state vector, but this usually doesn't give significant improvement of accuracy.

In our opinion, analytical formulas for accuracy bounds are of interest. In the EKF framework the navigation accuracy is determined by the errors covariance.

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In a recent paper [5] covariance analysis of pedestrian navigation was done under certain simplifying assumptions. One of the assumptions was that the IMU is accurate enough so that the computed and the true velocity of the system during the ZUPT phase are very close. In our experience this assumption is not always valid, especially for low grade IMU with unstable gyro drift, and taking it can lead to some paradoxes. For example, when looking at the covariances, the azimuth angle seems observable. By the definition given in [6], the corresponding EKF is *inconsistent*. Note that inconsistency of EKF is a well-known feature in the SLAM community, where a modification of EKF preserving observability properties was suggested in [8] [4].

In this paper, we investigate analytically covariances of EKF for pedestrian dead reckoning. We make simplifications other than in [5], assuming that the ZUPT phase is instantaneous. We provide analytical formulas for covariances of errors in velocity, position and orientation. We write down two forms of ZUPT equation and by looking at covariances claim that only one of them yields a consistent EKF. One feature of our approach is that we transform the error equations into the so-called dynamic and kinematic errors. This transform is well-known in high precision inertial navigation [7], but, to our knowledge, is not used in the pedestrian dead-reckoning settings.

2 Notation

Definitions Consider an IMU mounted on a pedestrian's foot. The IMU contains three component micro electro mechanical accelerometer and gyroscope. Let us introduce some definitions and coordinate frames.

- M is the sensitive mass of the accelerometer triad. When we talk about IMU position, we mean position of M.
- O is some reference point fixed on the ground. We shall set O as the start point of the pedestrian trajectory.
- $On = On_1n_2n_3$ is the navigation frame fixed on the ground. The On_3 axis points up; direction of other axes will be specified later.
- $\boldsymbol{p}_n, \boldsymbol{v}_n$ are the position and velocity of M in the On frame. Here and below the subscripts denotes the coordinate frame the vectors are referred to.
- $Ms = Ms_1s_2s_3$ is the sensor frame, fixed on the IMU body. The axes Ms_1, Ms_2, Ms_3 coincide with accelerometer axes of sensitivity. Orientation of Ms relative to On is given by the quaternion $\boldsymbol{q}_{ns}$. The angular velocity vector of Ms relative to Mn projected onto the Ms axes is denoted $\boldsymbol{\omega}_s$.
- $M's' = M's'_1s'_2s'_3$ is the computed sensor frame. The position $\boldsymbol{p}'_n$ of the point M' is the computed position of M. Orientation of $M's'$ is close to orientation of Ms and is given by the vector of small rotation $\boldsymbol{\beta}_s$. The angular velocity vector of $M's'$ relative to Mn projected onto $M's'$ is denoted by $\boldsymbol{\omega}'_s$.

[4] The authors would like to thank Mikhail Pikhletsky (pikhletsky.mikhail @huawei.com), who called our attention to these results.

– $Mz = Mz_1z_2z_3$ is the so-called virtual platform frame. This reference frame is close to the On frame. Orientation of Mz relative to Mn is given by the vector of small rotation $\boldsymbol{\beta}_n$. Orientation of Ms relative to Oz is given by the quaternion $\boldsymbol{q}_{ns'}$.

It is convenient to set the origin of all the frames at the same point O, introducing the frames Os, Os', On, Oz. The frames form the following diagram:

$$\begin{array}{ccc} On & \xrightarrow{\boldsymbol{\beta}_n} & Oz \\ \uparrow{\scriptstyle \boldsymbol{q}_{ns'}} & & \uparrow{\scriptstyle \boldsymbol{q}_{ns'}} \\ Os' & \xrightarrow{\boldsymbol{\beta}_s} & Os \end{array}$$

Denote by $R_{ns} = R(\boldsymbol{q}_{ns})$ the rotation matrix corresponding to the quaternion $\boldsymbol{q}_{ns}$. Coordinates transformation can be written as $\boldsymbol{p}_n = R_{ns}\boldsymbol{p}_s$. Denote by $\Delta\boldsymbol{q}(\boldsymbol{\beta}_s)$ quaternion corresponding to the vector of small rotation $\boldsymbol{\beta}_s$.

Let $\boldsymbol{f}_s$, $\boldsymbol{\omega}_s$ be the specific force and the absolute angular rate of IMU. Accelerometer and gyro measurements are written as ($\boldsymbol{r}_s$, $\boldsymbol{\varepsilon}_s$ are the errors):

$$\boldsymbol{f}'_s = \boldsymbol{f}_s + \boldsymbol{r}_s \qquad \boldsymbol{\omega}'_s = \boldsymbol{\omega}_s + \boldsymbol{\varepsilon}_s$$

Mechanization and error equations Let X be the state vectors and X' be the computed state vector:

$$X = \begin{bmatrix} \boldsymbol{p}_n \\ \boldsymbol{v}_n \\ \boldsymbol{q}_{ns} \end{bmatrix}, \qquad X' = \begin{bmatrix} \boldsymbol{p}'_n \\ \boldsymbol{v}'_n \\ \boldsymbol{q}_{ns'} \end{bmatrix}.$$

The mechanization equations and the computed mechanization equations can be written as [1], [2]:

$$\begin{aligned} \dot{\boldsymbol{p}}_n &= \boldsymbol{v}_n, \\ \dot{\boldsymbol{v}}_n &= R(\boldsymbol{q}_{ns})\boldsymbol{f}_s + \boldsymbol{g}_n, \\ \dot{\boldsymbol{q}}_{ns} &= -\frac{1}{2}\breve{\boldsymbol{\omega}}_s \cdot \boldsymbol{q}_{ns}. \end{aligned} \qquad (1) \qquad \begin{aligned} \dot{\boldsymbol{p}}'_n &= \boldsymbol{v}'_n, \\ \dot{\boldsymbol{v}}'_n &= R(\boldsymbol{q}_{ns'})\boldsymbol{f}'_s + \boldsymbol{g}_n, \\ \dot{\boldsymbol{q}}_{ns'} &= -\frac{1}{2}\breve{\boldsymbol{\omega}}'_s \cdot \boldsymbol{q}_{ns'}. \end{aligned} \qquad (2)$$

Here $\breve{\boldsymbol{\omega}}$ is the quaternion $(0, \omega_1, \omega_2, \omega_3)$, the dot is quaternion multiplication.

For aided navigation and accuracy analysis we need the errors vectors and the errors equations [2]. The errors vector is $x = (\Delta\boldsymbol{p}_n, \Delta\boldsymbol{v}_n, \boldsymbol{\beta}_n)^T$, where

$$\Delta\boldsymbol{p}_n = \boldsymbol{p}'_n - \boldsymbol{p}_n, \quad \Delta\boldsymbol{v}_n = \boldsymbol{v}'_n - \boldsymbol{v}_n,$$

and $\boldsymbol{\beta}_n$ is the small rotation angle vector between the virtual platform and the reference frame: $\boldsymbol{q}_{ns} = \Delta\boldsymbol{q}(\boldsymbol{\beta}_n) \cdot \boldsymbol{q}_{ns'}$. Note that the dimensions of X, x are different, thus state update is a bit different from the standard EKF formulas.

The errors equations can be written as:

$$\begin{aligned} \Delta\dot{\boldsymbol{p}}_n &= \Delta\boldsymbol{v}_n, \\ \Delta\dot{\boldsymbol{v}}_n &= \boldsymbol{f}'_n \times \boldsymbol{\beta}_n + \boldsymbol{r}_n, \\ \dot{\boldsymbol{\beta}}_n &= \boldsymbol{\varepsilon}_n. \end{aligned} \qquad (3)$$

The ZUPT observations in errors are usually written as (see, e.g. [2])

$$\boldsymbol{z} = Z' - Z = \boldsymbol{v}'_n = \Delta \boldsymbol{v}_n + \Delta \boldsymbol{z}. \tag{4}$$

3 Covariance analysis

Dynamic errors Observability analysis of (3), (4) is rather difficult. We suggest to make a change of variables, introducing the so-called *dynamic errors* $\delta \boldsymbol{p}_n$, $\delta \boldsymbol{v}_n$ as in [7]:

$$\Delta \boldsymbol{p}_n = \delta \boldsymbol{p}_n + \boldsymbol{p}'_n \times \boldsymbol{\beta}_n, \qquad \Delta \boldsymbol{v}_n = \delta \boldsymbol{v}_n + \boldsymbol{v}'_n \times \boldsymbol{\beta}_n. \tag{5}$$

This change of variables is best understood with the following formulas:

$$\Delta \boldsymbol{v}_n = \boldsymbol{v}'_n - \boldsymbol{v}_z + \boldsymbol{v}_z - \boldsymbol{v}_n = \underbrace{\boldsymbol{v}'_n - \boldsymbol{v}_z}_{\delta \boldsymbol{v}_n} + (I - \boldsymbol{\beta}_n \times)\boldsymbol{v}_n - \boldsymbol{v}_n = \delta \boldsymbol{v}_n + \boldsymbol{v}_n \times \boldsymbol{\beta}_n$$

Below we call $\Delta \boldsymbol{p}_n, \Delta \boldsymbol{v}_n$ *the full errors.* In dynamic errors (3) can be written as

$$\begin{aligned} \dot{\delta \boldsymbol{p}}_n &= \delta \boldsymbol{v}_n - \boldsymbol{p}_n \times \boldsymbol{\varepsilon}_n, \\ \dot{\delta \boldsymbol{v}}_n &= -\boldsymbol{g}_n \times \boldsymbol{\beta}_n - \boldsymbol{v}_n \times \boldsymbol{\varepsilon}_n + \boldsymbol{r}_n, \\ \dot{\boldsymbol{\beta}}_n &= \boldsymbol{\varepsilon}_n. \end{aligned} \tag{6}$$

ZUPT measurements in dynamic errors In dynamic errors (4) becomes

$$\boldsymbol{z} = \delta \boldsymbol{v}_n + \boldsymbol{v}'_n \times \boldsymbol{\beta}_n + \Delta \boldsymbol{z}. \tag{7}$$

The last formula looks strange: when $v'_{n1} \neq 0$ or $v'_{n2} \neq 0$, β_3 enters the measurement, though the azimuth is obviously unobservable. Thus when we construct a EKF aided with (7) the filter is inconsistent [6], [8]. This means that ZUPT in the form (4), which is the most commonly used, is also wrong. Let us look at the problem from the mechanical point of view: the measured ZUPT velocity must be attributed to the body frame Ms, or, equivalently, to the virtual platform frame Oz (rotation $q_{ns'}$ between these frames is known), while the computed velocity $\boldsymbol{v}'_n$ is in the navigation frame On. The difference of two quantities is $\boldsymbol{v}'_n - \boldsymbol{v}_z$. We get the following:

Corollary 1. *The correct formula for ZUPT observations is* $\boldsymbol{z} = \boldsymbol{v}'_n - \boldsymbol{v}_z \approx \delta \boldsymbol{v}_n$.

To summarize, we got two ZUPT observations forms, referred to as forms D, F, where form F is unsound, but commonly used.

$$\text{Form D – in dynamic errors:} \quad \boldsymbol{z} = \delta \boldsymbol{v}_n + \Delta \boldsymbol{z}, \tag{8}$$

$$\text{Form F – in full errors:} \quad \boldsymbol{z} = \delta \boldsymbol{v}_n + \boldsymbol{v}'_n \times \boldsymbol{\beta}_n + \Delta \boldsymbol{z}. \tag{9}$$

Note that equations (6), (9) are equivalent to (3), (4). Note also some additional advantages of using dynamic errors. First, $\boldsymbol{f}'_n$, which is a time varying function, is absent in (6). Thus the state matrix is time invariant. Second, the equations can be decomposed into that of horizontal and vertical motion, as shown below.

Covariance analysis: case D For analytics, we assume that the step duration T is constant, the zero velocity interval can be considered very short, and the noises $\boldsymbol{r}_n$, $\boldsymbol{\varepsilon}_n$ in (3) are white noises with PSD σ_f^2, σ_ω^2. The ZUPT error $\boldsymbol{\Delta z}$ is assumed a random vector with zero mean and mean square σ_z^2. For brewity we drop the primes in computed values, writing p_1 instead of p_1', etc.

First we consider the system (6), (8). We can split it into three weakly coupled and one independent systems of equations in horizontal motion $(\delta p_2, \delta v_2, \beta_1)$, $(\delta p_1, \delta v_1, \beta_2)$, azimuth rotation β_3, and vertical motion $(\delta p_3, \delta v_3)$. Since we are interested in horizontal motion, we write down the first three:

$$\begin{aligned} \delta\dot{p}_1 &= \delta v_1 - p_2\varepsilon_3 + p_3\varepsilon_2, \\ \delta\dot{v}_1 &= -g\beta_2 + r_1 + v_3\varepsilon_2 - v_2\varepsilon_3, \\ \dot{\beta}_2 &= \varepsilon_2, \\ z_1 &= \delta v_1 + \Delta z_1. \end{aligned} \tag{10}$$

$$\begin{aligned} \delta\dot{p}_2 &= \delta v_2 + p_1\varepsilon_3 - p_3\varepsilon_1, \\ \delta\dot{v}_2 &= g\beta_1 + r_2 - v_3\varepsilon_1 + v_1\varepsilon_3, \\ \dot{\beta}_1 &= \varepsilon_1, \\ z_2 &= \delta v_2 + \Delta z_2. \end{aligned} \tag{11}$$

$$\dot{\beta}_3 = \varepsilon_3. \tag{12}$$

The systems are time dependent but periodic; observations take place at every $t_k = kT, k = 0, 1, 2, \ldots$ The sequence of covariance matrices of x in standard notation is written as

$$\begin{aligned} P_k^- &= F P_{k-1}^+ F^T + GQG, \\ P_k &= P_k^- - P_k^- H^T (H P_k^- H^T + R)^{-1} H P_k^-. \end{aligned} \tag{13}$$

Here P_k denotes posteriori covariance at t_k (after ZUPT); P_k^- denotes a priori covariance at t_k (before ZUPT). Whenever possible, we drop the index k. A system is observable, if the covariance matrix tends to a stationary solution. This solution can be found as the stationary point of (13).

Let us start the analysis with (10), (11). We assume that the pedestrian walks over horizontal ground, thus $v_3 \ll v_1, v_2$, and neglect the terms $v_3\varepsilon_2$, $v_3\varepsilon_1$ in the velocity equations. Dropping the position equations (we look at that later), and neglecting the ZUPT error in the measurement equations [5], we obtain

$$\begin{aligned} \delta\dot{v}_1 &= -g\beta_2 + r_1 - v_2\varepsilon_3, \\ \dot{\beta}_2 &= \varepsilon_2, \\ z_1 &= \delta v_1. \end{aligned} \tag{14}$$

$$\begin{aligned} \delta\dot{v}_2 &= g\beta_1 + r_2 + v_1\varepsilon_3, \\ \dot{\beta}_1 &= \varepsilon_1, \\ z_2 &= \delta v_2. \end{aligned} \tag{15}$$

The solution of (13) for (14), (15) can be found as

$$P_{\delta v_i \beta_j} = 0, \quad P_{\delta v_i} = 0, \quad P_{\beta_i} = \frac{\sigma_\omega^2 T}{2}\left(\sqrt{1 + 4\varkappa_i^2} + 1\right), \tag{16}$$

$$\begin{aligned} P_{\delta v_i}^- &= \frac{\sigma_\omega^2 g T^3}{2}\left(\sqrt{4\varkappa_i^2 + 1} + 1\right) + \sigma_{fi}^2 T, \\ P_{\beta_i}^- &= \frac{\sigma_\omega^2 T}{2}\left(\sqrt{4\varkappa_i^2 + 1} + 3\right), \qquad P_{\delta v_i \beta_j}^- = -\frac{\sigma_\omega^2 g T^2}{2}\left(\sqrt{4\varkappa_i^2 + 1} + 1\right). \end{aligned} \tag{17}$$

[5] The last assumption can be argued since in real world the person's foot is moving during the stance stage, but we can't proceed further analytically without it.

Here $i \neq j$ are 1 or 2 and we denote

$$\sigma_{fi}^2 = \sigma_f^2 + \overline{v_j^2}\sigma_\omega^2, \qquad \varkappa_i = \frac{\sigma_{fi}^2}{\sigma_\omega^2 g^2 T^2}, \tag{18}$$

where the bar denotes the mean value of a function during a step. Now we turn to (12). The system is completely unobservable, and $\beta_{3|k}$ is just random walk:

$$P_{\beta_{3|k+1}} = P_{\beta_{3|k}} + \sigma_\omega^2 T. \tag{19}$$

Covariance analysis: case F Here we prove inconsistency of form F. We can split (6), (9) into three weakly coupled and one independent systems of equations in (β_3), $(\delta p_2, \delta v_2, \beta_1)$, $(\delta p_1, \delta v_1, \beta_2)$, $(\delta p_3, \delta v_3)$, of which we consider the first three. Under the same assumptions as above, and dropping again the equations for $\delta p_1, \delta p_2$, we obtain

$$\begin{aligned} \delta\dot{v}_1 &= -g\beta_2 + r_1 - v_2\varepsilon_3, \\ \dot{\beta}_2 &= \varepsilon_2, \\ z_1 &= \delta v_1 + v_2\beta_3. \end{aligned} \tag{20}$$

$$\begin{aligned} \delta\dot{v}_2 &= g\beta_1 + r_2 + v_1\varepsilon_3, \\ \dot{\beta}_1 &= \varepsilon_1, \\ z_2 &= \delta v_2 - v_1\beta_3. \end{aligned} \tag{21}$$

$$\dot{\beta}_3 = \varepsilon_3. \tag{22}$$

The systems (20) — (22) are interconnected due to the terms $v_2\beta_3$, $v_1\beta_3$ in the measurement equations and can be shown to be observable in the non-stationary sense. To proceed with covariance analysis, we again assume that v_1, v_2 are small at the ZUPT phase. We conclude the following.

Proposition 1. *When ZUPT computed velocity is close to zero, stationary covariances of* $\delta v_1, \beta_2, \delta v_2, \beta_1$ *for* (20), (21) *exist and are given by the asymptotic formulas* (16).

With this result we can rewrite the observations equations in β_3 as

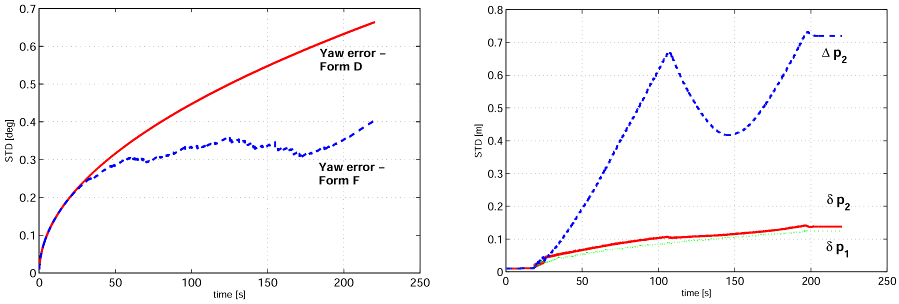
$$\begin{aligned} \dot{\beta}_3 &= \varepsilon_3, \\ z_1 &= \delta v_1 + v_2\beta_3, \\ z_2 &= \delta v_2 - v_1\beta_3, \end{aligned} \qquad P_{\delta v_i} = \frac{\sigma_\omega^2 g T^3}{2}\left(\sqrt{4\varkappa_i^2 + 1} + 1\right) + \sigma_{fi}^2 T. \tag{23}$$

Here δv_i, $i = 1, 2$ can be treated as observation noises with known intensity $P_{\delta v_i}$. Seeking again for the stationary solution of (13) for (23), we obtain, dropping terms of higher order in v_1, v_2

$$P_{\beta_3} = \sqrt{\sigma_\omega T}\left(\frac{v_2^2}{P_{\delta v_1}} + \frac{v_1^2}{P_{\delta v_2}}\right)^{-1/2}. \tag{24}$$

Corollary 2. *For ZUPT observation in form F the heading angle is false observable with error covariance given by* (24). *The EKF is inconsistent. For ZUPT observation in form D the heading angle is unobservable. The EKF is consistent.*

Thus writing error equations in dynamic errors with ZUPT observation in form D is preferable. This result is illustrated in Fig. 1.A, where we compare STD of yaw errors with ZUPT observations in forms D and F. Further on, we assume that ZUPT equations in form D only are used.



A: B:

Fig. 1. A: Yaw angle STD with D and F ZUPT observation models. We see that in case F the yaw STD doesn't grow steadily, as it should. B: Dynamic and full position errors STD $P^{1/2}_{\delta p_1}$, $P^{1/2}_{\delta p_2}$, $P^{1/2}_{\Delta p_2}$. The full position error STD changes with the distance travelled due to the presence of $p_1\beta_3$ term in the formula for that error. The walk was for 3 minutes forward-backward.

Covariances of dynamic position errors Position errors δp_1 δp_2 are unobservable, and behave like random walk. To find parameters of this random walk, we turn again to covariance iterations, now for (10), setting there covariances for $\delta v_1, \delta v_2, \beta_1, \beta_2$ to their stationary values (16). We obtain iterations

$$\begin{aligned} P_{\delta p_i|k+1} &= P_{\delta p_i|k} + Q_i + \sigma_\omega^2 T \overline{p_j^2}, & P_{\delta p_1\beta_3|k+1} &= P_{\delta p_1\beta_3|k} - \sigma_\omega^2 T\,\overline{p_2}, \\ P_{\delta p_1\delta p_2|k+1} &= P_{\delta p_1\delta p_2|k} - \sigma_\omega^2 T\,\overline{p_1 p_2}, & P_{\delta p_2\beta_3|k+1} &= P_{\delta p_2\beta_3|k} + \sigma_\omega^2 T\,\overline{p_1}. \end{aligned}$$

where

$$Q_i = \frac{\sigma_{fi}^2 T^3}{4} \frac{\sqrt{1+4\varkappa_i^2}+1}{2\varkappa_i^2 + \sqrt{1+4\varkappa_i^2}+1}, \qquad i = 1, 2. \tag{25}$$

Covariances of full position errors For applications one is more interested not in dynamic, but in full errors. Since

$$\Delta p_1 = \delta p_1 + p_2\beta_3, \qquad \Delta p_2 = \delta p_2 - p_1\beta_3,$$

the full position errors are modulated by the distance travelled. To find out how, suppose that the pedestrian walks strictly in the On_1 direction: $p_2(t) \equiv 0$. Then we can write for a series of k steps (assuming initial errors to be zeros):

$$\begin{aligned} P_{\Delta p_1 \Delta p_2} &= 0, & P_{\Delta p_1 \beta_3} &= 0, \\ P_{\Delta p_2 \beta_3} &= \sigma_\omega^2 kT(\overline{p_1} - p_1), & P_{\beta_3} &= \sigma_\omega^2 kT, \\ P_{\Delta p_2} &= Q_2 k + \sigma_\omega^2 kT(\overline{p_1^2} + p_1^2 - 2\overline{p_1}p_1), & P_{\Delta p_1} &= Q_1 k. \end{aligned}$$

Contrary to (18), here the bar denotes the average during the whole walk.

Corollary 3. *Position error in the direction of travel is uncorrelated with the azimuth error. Position error perpendicular to the direction of travel is correlated* *to the azimuth error with correlation coefficient given by the formulas*

$$K_{\Delta p_2 \beta_3} = \left[1 + \frac{\overline{p_1^2} - \overline{p_1}^2}{(\overline{p_1} - p_1)^2} + \frac{\frac{Q_2}{\sigma_\omega^2 T}}{(\overline{p_1} - p_1)^2} \right]^{-\frac{1}{2}}$$

When the pedestrian travels far ($p_1 = \max$), the coefficient reaches its maximum; when the pedestrian returns to the center position ($p_1 = \overline{p_1}$), the coefficient becomes zero: position and azimuth become uncorrelated. The difference between covariances of dynamic and full position errors is illustrated in Fig. 1.B.

4 Conclusions

Covariance analysis of pedestrian dead reckoning with foot mounted IMU was done analytically under simplifying assumptions. The main result of the paper is that care must be taken when writing the ZUPT measurement equations for EKF in the case of low grade sensors. If written in one commonly used form, these equations yield inconsistent EKF. A consistent form of ZUPT equations was suggested. Another result is that when writing the error equations of IMU in the so called dynamic errors, these equations can be decomposed into four nearly uncoupled subsystems to simplify the covariances analysis. Experiments results are not discussed here for lack of space.

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