Iterative Algorithm of Optical Triangulation Sensors Signals Superposition for Measuring Solid Deformation

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***Abstract*—The article was described as the algorithms for contour superposition. The first contour is the etalon signal. The first contour output signal of the optical triangulation sensor (measured object contour). The contour superposition method was developed on the hypothesis that etalon contour consists of line equations and measured contour consists of points sequence. The superposition includes two processing stages. The first stage is dividing the source sequence point into point subsequences. The second stage is making an equation that links line equations with point subsequences. The superposition parameters are calculated from this equation. The article was shown several iterative algorithms which differ source data (the line types for etalon signal). The error measuring of solid deformation was shown in the example side wear railway rail. The developing superposition algorithms were compared with superposition by control points.**

Keywords—iterative, triangulation, superposition, deformation, measurement.

I. Introduction

Measurement solid deforming is the actual issue of some organizations, which specializes in the production control of parts and geometry measuring of extended objects. Mechanical methods of measurement include specific gadgets: calipers, feelers, gauges, and complex devices on their basis. Mechanical methods are widely spread in cases if automation is not possible and if it is not economically feasible. The automation methods of measurement link with optical triangulation sensors. These sensors allow measuring the contour of aim objects. The main issue of developers for measurement solid deforming is the superposition of measuring the contour of aim objects and contour of etalon (no wear) objects. If the superposition issue is done then measurement solid deforming is an obvious task. It consists of defining the distance between reference points or square of wear object part.

II. Overview of Methods Contours Superposition

The methods of contours superposition include several areas of research. The first of the early ones is superposition by reference points. This method is widely spread at the current time due to simplicity. As rule, the reference points are:

– the points of lines intersection (for example, the point of lines intersection which makes right angle);

– the corner connection of line and known radius fillet (*note*: fillet is a concave architectural fragment, representing the outline of a quarter of a circle or a segment of a curve close to this shape [1]).

The point's coordinate of measuring aim object contour and no wear objects are defined the superposition parameters (rotate angle and plain offsets) due to equation system:

$$\begin{cases} v_i = x_i \cdot \cos(\alpha) - y_i \cdot \sin(\alpha) + x0; \\ w_i = x_i \cdot \sin(\alpha) + y_i \cdot \cos(\alpha) + y0, \end{cases} \qquad (1)$$

where (x_i, y_i) is reference points coordinate of no wear object; (v_i, w_i) is reference points coordinate of aim object; $i = 1, \dots n$, n is the number of reference points; (α, $x0$, $y0$) are the superposition parameters.

The least-square method is used for estimation superposition parameters in some cases [2,3]. As the system (1) contains a nonlinear trigonometric function, so this equation cannot be solved by the standard algorithm (for example, by Cramer's rule or matrix's rule). But if function sine and cosine expression in the Taylor series and limit the first two terms, then the system (1) transforms into a linear equation system. This way was showed at research [4].

As rule, the contour of measuring aim object contains not as many reference points (from 2 to 10 points). So high measurement error of coordinate some reference points or fails of detection point leads to increase superposition error and as consequence to high measurement error of solid deforming. Also if the amount of reference points is less some threshold then superposition parameters are not measured and solid deforming is not measured too. This way is very popular at commercial applying because measurement with high error is worse than failure. The fail measurement can be restored by approximation and interpolation of nearby points.

Also the other method of superposition is widely spread in scientific research and commercial applying. It is useful for objects in which contour is described as a second-order curve (circle, ellipse, parabola, etc.).In this case the superposition is defined by approximation with parametric equation [4-11] which described known contour type. Two main ways of estimating superposition parameters are used in this research area:

– applying least square method;

– applying invariant not depending on rotation.

The disadvantage of this method is limit applying. It can be used for only specific objects. The shape is described as a second-order curve. Also the article [7] was shown an additional serious disadvantage. If the aim object contains only part of the measurement point then differences approximation method give differences result which does not correspond with a real object.

The novel method for estimating superposition parameters for the close contour of digital image (the coordinates of the points are discrete value) was presented at research [12]. The contour encodes the sequence of the complex number. The superposition parameter is defined by calculating the dot product (scalar product) of two vectors that describe the object's contour.

But this method can be used if two conditions are true:

– contour should be close;

– contour should be transformed so that all coordinate points can be described as a discrete value.

As rule, these both conditions are not providing for contour which is measured by optical triangulation sensors.

The novel method for signals superposition of triangulation optical sensors is described in this article. The method is the generalization of the first method by reference points. It was developed for one aim. It is decreasing measurement error by making use of more points than the number of reference points in measurement contour.

III. Describing Developed Superposition Method

The developed superposition method was based on the modification (1). The modification allows using (1) for processing subsequence of points:

$$\begin{cases} v_{ij} = x_{ij} \cdot \cos(\alpha) - y_{ij} \cdot \sin(\alpha) + x0; \\ w_{ij} = x_{ij} \cdot \sin(\alpha) + y_{ij} \cdot \cos(\alpha) + y0, \end{cases} \tag{2}$$

where i is subsequence order number, i = 1, .. n, n is the amount of subsequence; j is point's number at subsequence, j = 1, .. mi, mi is the amount of point at i-th subsequence.

If in (1) (x_i, y_i) is coordinates reference points of no wear objects, then in (2) (x_{ij}, y_{ij}) is defined line equation:

$$y_{ij} = f_i(x_{ij}), \tag{3}$$

where $f_i(x_{ij})$ is an analytical equation of i-th line which corresponds i-th subsequence.

After the transform (2):

$$\begin{cases} x_{ij} = (v_{ij} - x0) \cdot \cos(\alpha) + (w_{ij} - y0) \cdot \sin(\alpha); \\ y_{ij} = -(v_{ij} - x0) \cdot \sin(\alpha) + (w_{ij} - y0) \cdot \cos(\alpha). \end{cases} \tag{4}$$

Using the variables $A = \cos(\alpha)$, $B = \sin(\alpha)$:

$$\begin{cases} x_{ij} = (v_{ij} - x0) \cdot A + (w_{ij} - y0) \cdot B; \\ y_{ij} = -(v_{ij} - x0) \cdot B + (w_{ij} - y0) \cdot A; \\ A = (1 - B^2)^{1/2}. \end{cases} \tag{5}$$

The estimating unknown parameters with using (3) and (5) by a least-square method is given:

$$F(A, B, x0, y0) = \sum_{i=1}^{n} \sum_{j=1}^{mi} \left(y_{ij} - f_i(x_{ij}) \right)^2 \to \min \tag{6}$$

$$\begin{aligned} F(A, B, x0, y0) = \sum_{i=1}^{n} \sum_{j=1}^{mi} \{ &-(v_{ij} - x0) \cdot B + (w_{ij} - y0) \cdot A - \\ &- f_i((v_{ij} - x0) \cdot A + (w_{ij} - y0) \cdot B) \}^2 \to \min \end{aligned} \tag{7}$$

The result equation is given by a system of partial derivatives:

$$\left\{ \frac{\partial F}{\partial B} = 0; \quad \frac{\partial F}{\partial x0} = 0; \quad \frac{\partial F}{\partial y0} = 0; \quad A = (1 - B^2)^{1/2} \right. \tag{8}$$

The system (5) and (8) contains the expression A=(1–B^2)$^{1/2}$. This expression does not equal the Pythagorean trigonometric identity A^2+B^2=1. Using expression A=(1–B^2)$^{1/2}$ leads to the limit of the task that cosine cannot take a negative value. Let be this limit has been held at a task which is presented below.

If the rotate angle can take all possible values then both expressions A = (1–B^2)$^{1/2}$ and A = –(1–B^2)$^{1/2}$ can be included in (8). So the solving about a sign of rotate angle is defined by addition metrics. The metric is the distance between superposition contours.

IV. Examples

Examples of commercial applying are shown below:

1) superposition with a contour which is described two straight lines (see. Fig. 1a – the example superposition with target contour which is represented a rectangular profile, see. Fig. 1b – the example contour of measurement railway rails and contour of no wear rails which are needed matching to each other);

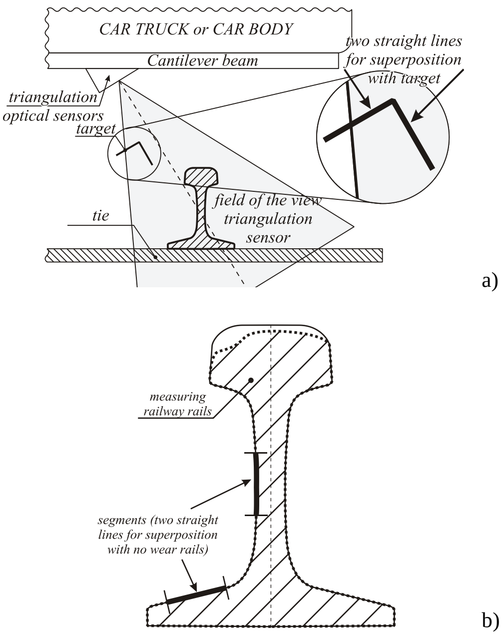

Fig. 1. Examples superposition of contour which is described two straight lines.

2) superposition with a contour which is described several straight lines (see. Fig. 2 – the example superposition with the contour of a drill-pipe joint);

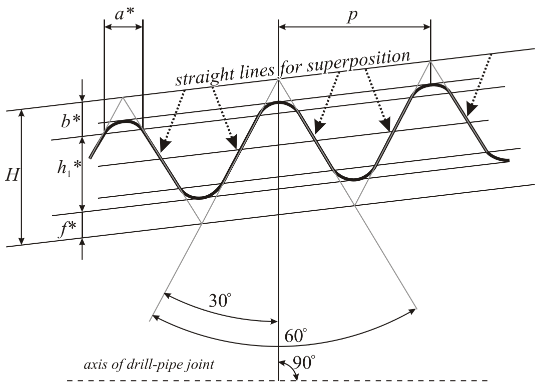

Fig. 2. Examples superposition of contour of a drill-pipe joint which is described several straight lines.

3) superposition with a contour which is described straight line and circle part (see. Fig. 3 – the example superposition measured rail contour and no wear rail).

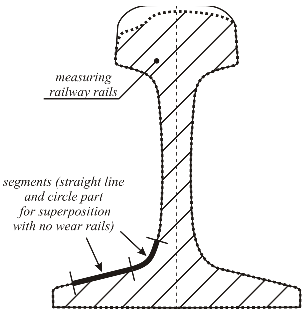

Fig. 3. Example of superposition with contour describing straight line and circle part.

The developing algorithms are possible if the point sequence of optical triangulation sensors can be divided into subsequence. Preprocessing algorithms of contour were shown at articles [13-15]. Preprocessing allows divide source data of point into subsequence by derivative and curvature which calculate by every point of the contour. Let be the measured sequence points on the plane defines the contour of the aim object. Points define two subsequences with coordinates (v_{1j}, w_{1j}) and (v_{2j}, w_{2j}). Subsequences belong to different straight lines. The subsequence of points (v_{1j}, w_{1j}) belong to line $y = k_1x+b_1$ etalon contour and points (v_{2j}, w_{2j}) belong to line $y = k_2x+b_2$. The task of superposition is to find transform. After transforming the points (v_{1j}, w_{1j}) should belong to the line $y = k_1x+b_1$ and points (v_{2j}, w_{2j}) should belong to the line $y = k_2x+b_2$ (see. Fig. 4).

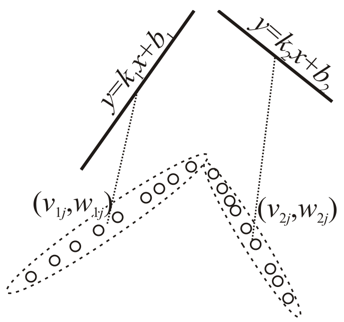

Fig. 4. The task of superposition points with two straight lines.

According to the above:

$$\begin{pmatrix} v_{1j} \\ w_{1j} \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & x0 \\ \sin(\alpha) & \cos(\alpha) & y0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{1j} \\ y_{1j} \\ 1 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} v_{2j} \\ w_{2j} \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & x0 \\ \sin(\alpha) & \cos(\alpha) & y0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{2j} \\ y_{2j} \\ 1 \end{pmatrix} \tag{10}$$

where $y_{1j} = k_1x_{1j} + b_1$, $y_{2j} = k_2x_{2j} + b_2$.

After transformation (9):

$$x_{1j} = \cos(\alpha)(v_{1j} - x0) + \sin(\alpha)(w_{1j} - y0) \tag{11}$$

$$y_{1j} = -\sin(\alpha)(v_{1j} - x0) + \cos(\alpha)(w_{1j} - y0) \tag{12}$$

After replacements $A = \cos(\alpha)$, $B = \sin(\alpha)$ in (11) and (12):

$$x_{1j} = A(v_{1j} - x0) + B(w_{1j} - y0) \tag{13}$$

$$y_{1j} = -B(v_{1j} - x0) + A(w_{1j} - y0) \tag{14}$$

After substitutions (13) and (14) at an equation $y_{1j} - k_1x_{1j} - b_1 = 0$, the unknown parameters (α, $x0$, $y0$) can be defined by the least-square method:

$$\begin{gathered} F(A, B, x0, y0) = \sum_{j=1}^{m1} \{[-B(v_{1j} - x0) + A(w_{1j} - y0)] - \\ - k_1[A(v_{1j} - x0) + B(w_{1j} - y0)] + b_1\}^2 \to \min \end{gathered} \tag{15}$$

where $m1$ is the amount of point at subsequence.

According to (8) the partial derivatives define the following equation:

$$\frac{\partial F(A, B, x0, y0)}{\partial B} = N1 \cdot B + M1 = 0,$$

$$\frac{\partial F(A, B, x0, y0)}{\partial x0} = N2 \cdot x0 + M2 = 0,$$

$$\frac{\partial F(A, B, x0, y0)}{\partial y0} = N3 \cdot y0 + M3 = 0,$$

$$A = (1 - B^2)^{1/2},$$

where $N1 = 2\sum_{j=1}^{m1} (v_{1j} - x0 + k_1(w_{1j} - y0)) \times$

$\times (b_1 - B(w_{1j} - y0) + A \cdot k_1(v_{1j} - x0))$,

$M1 = 2\sum_{i=1}^{n} (v_{1j} - x0 + k_1(w_{1j} - y0))^2$,

$N2 = -2\sum_{i=1}^{n} (B + A \cdot k_1) \times$

$\times (b_1 + B \cdot v_{1j} - A(w_{1j} - y0) + k(A \cdot v_{1j} + B(w_{1j} - y0)))$,

$M2 = 2\sum_{i=1}^{n} (B + A \cdot k_1)^2$,

$N3 = 2\sum_{i=1}^{n} (A - B \cdot k_1) \times$

$\times (b_1 - A \cdot r_i + B(p_i - x0) + k(B \cdot r_i + A(p_i - x0)))$,

$M3 = 2\sum_{i=1}^{n} (A - B \cdot k_1)^2$.

So, the equation system should be

$$\begin{cases} N1\cdot B + M1 = 0; \\ N2\cdot x0 + M2 = 0; \\ N3\cdot y0 + M3 = 0; \\ A = \left(1 - B^2\right)^{1/2}. \end{cases}, \qquad (16)$$

Only points (v_{1j}, w_{1j}) were used to get (16). If points (v_{2j}, w_{2j}) add to (16) then it will rewrite as:

$$\begin{aligned} F(A,B,x0,y0) = \sum_{j=1}^{m1} \{ & [-B(v_{1j}-x0) + A(w_{1j}-y0)] - \\ & - k_1[A(v_{1j}-x0) + B(w_{1j}-y0)] + b_1\}^2 + \\ + \sum_{j=1}^{m2} \{ & [-B(v_{2j}-x0) + A(w_{2j}-y0)] - \\ & - k_2[A(v_{2j}-x0) + B(w_{2j}-y0)] + b_2\}^2 \to \min \end{aligned} \qquad (17)$$

So the coefficient $N1$ is rewritten as:

$$\begin{aligned} N1 = {} & 2\sum_{j=1}^{m1} \left(v_{1j} - x0 + k_1(w_{1j} - y0)\right)\times \\ & \times\left(b_1 - B(w_{1j} - y0) + A\cdot k_1(v_{1j} - x0)\right) + \\ & + 2\sum_{j=1}^{m2} \left(v_{2j} - x0 + k_2(w_{2j} - y0)\right)\times \\ & \times\left(b_2 - B(w_{2j} - y0) + A\cdot k_2(v_{2j} - x0)\right) \end{aligned}$$

The coefficient $M1$, $N2$, etc. are rewritten similarly way.

The system (16) is solved by the iteration method. The finish algorithm for estimation parameters includes the next steps:

1. The first approximation $x0$, $y0$, B, $A=(1–B^2)^{1/2}$ is defined.

2. The coefficients $N1$, $N2$, $N3$, $M1$, $M2$, $M3$ are calculated by variables (k_1, b_1), (k_2, b_2) and (v_{1j}, w_{1j}), (v_{2j}, w_{2j}), (A, B, $x0$, $y0$).

3. The values $B' = -\dfrac{M1}{N1}$, $x0' = -\dfrac{M2}{N2}$, $y0' = -\dfrac{M3}{N3}$, $A' = \left(1 - (B')^2\right)^{1/2}$ are calculated.

4. Assignment $A=A'$, $B=B'$, $x0=x0'$, $y0=y0'$, $\alpha=\arcsin(B)$.

5. Go to the step №2.

An amount step from 5 to 2 is the iteration count.

The above algorithm allows us to process two subsequences of point which belong at two straight lines. The algorithm can be extended at three and more lines similarly way.

Let be the measured sequence points on the plane defines the contour of aim object which contains a straight line and circle part. Measured sequence points define two subsequences with coordinates (v_{1j}, w_{1j}) and (v_{2j}, w_{2j}). Subsequences (v_{1j}, w_{1j}) belong to a straight line $y=kx+b$. Subsequences (v_{2j}, w_{2j}) belong to the circle part, where R is circle radius (see. Fig. 5).

If the center of the circle has been set at point (0,0) then the algorithm can be simplified. The circle equation is $x^2+y^2=R^2$ at that case.

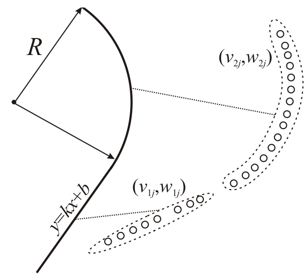

Fig. 5. The task of superposition points with straight lines and circle part.

The following expressions have been taken from (8):

$$x_{2j} = \cos(\alpha)(v_{2j} - x0) + \sin(\alpha)(w_{2j} - y0), \qquad (18)$$

$$y_{2j} = -\sin(\alpha)(v_{2j} - x0) + \cos(\alpha)(w_{2j} - y0), \qquad (19)$$

After transformation and simplifies, the parameter α disappears at the equation. The equation does not depend from a rotate angle:

$$(v_{2j} - x0)^2 + (w_{2j} - y0)^2 = R^2, \qquad (20)$$

Also, this task is characterized by some features in compares with the previously developed algorithm. The unknown parameters (α, $x0$, $y0$) at the first algorithm link to each other. So, it cannot be estimated separately.

The separate estimation of the unknown parameters at the current algorithm is possible. The circle (20) does not depend on the rotate angle. And straight-line equation does not depend on $x0$, $y0$ because solving for the line is infinity which joins (α, $x0$, $y0$) each other.

So the common way includes two stages. The first stage is estimation parameters $x0$, $y0$. The second stage is the estimation rotate angle by known $x0$, $y0$. The next example shows the equation for estimating $x0$, $y0$. The center of the circle is estimated by three points (v_1, w_1), (v_2, w_2), (v_3, w_3). The following system of equations has been defined by substitution points at (12):

$$\begin{cases} (v_1 - x0)^2 + (w_1 - y0)^2 = R^2; \\ (v_2 - x0)^2 + (w_2 - y0)^2 = R^2;, \\ (v_3 - x0)^2 + (w_3 - y0)^2 = R^2. \end{cases} \qquad (21)$$

The systems (22) and (23) are equal (21):

$$\begin{cases} (v_1 - x0)^2 + (w_1 - y0)^2 = (v_2 - x0)^2 + (w_2 - y0)^2; \\ (v_1 - x0)^2 + (w_1 - y0)^2 = (v_3 - x0)^2 + (w_3 - y0)^2. \end{cases} \qquad (22)$$

$$\begin{cases} 2\cdot x0(v_1 - v_2) + 2\cdot y0(w_1 - w_2) = (v_1^2 - v_2^2) + (w_1^2 - w_2^2); \\ 2\cdot x0(v_1 - v_3) + 2\cdot y0(w_1 - w_3) = (v_1^2 - v_3^2) + (w_1^2 - w_3^2). \end{cases} \qquad (23)$$

Using the least-square method allows defining the expression for estimating $x0$, $y0$:

$$g(x0,y0) = \sum_{i=1}^{N}\sum_{j=i+1}^{N} \left(x0\cdot dv_{ij} + y0\cdot dw_{ij} + dvw_{ij}\right)^2, \qquad (24)$$

where $dv_{ij}=v_i-v_j$, $dw_{ij}=w_i-w_j$, $dvw_{ij}=0.5\cdot((v_i^2-v_j^2)+(w_i^2-w_j^2))^2$, N is the amount of point (*note*: points (v_i, w_j), (v_j, w_j) belong the subsequence (v_{2j}, w_{2j}), $N = m2$).

The solving estimation (24) is

$$\begin{cases} \dfrac{\partial g(x0,y0)}{\partial x0} = 0, \\ \dfrac{\partial g(x0,y0)}{\partial y0} = 0; \end{cases}, \quad \begin{cases} x0\cdot A11 + y0\cdot A12 = B1, \\ x0\cdot A21 + y0\cdot A22 = B2; \end{cases},$$

$$x0 = \frac{B1 \cdot A22 - B2 \cdot A21}{A11 \cdot A22 + \cdot A12 \cdot A21}, \quad (25)$$

$$y0 = \frac{B2 \cdot A11 - B1 \cdot A12}{A11 \cdot A22 + \cdot A12 \cdot A21}, \quad (26)$$

where $A11 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} dv_{ij}^{2}$, $A12 = A21 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} dv_{ij} dw_{ij}$,

$A22 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} dw_{ij}^{2}$, $B1 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} dv_{ij} \cdot dvw_{ij}$,

$B2 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} dw_{ij} \cdot dvw_{ij}$.

The rotate angle α is estimated by known $x0$, $y0$. The equation joins points (v_{1j}, w_{1j}) and straight-line $y=kx+b$ by the following expression:

$$k\left[\cos(\alpha)(v_{1j} - x0) + \sin(\alpha)(w_{1j} - y0)\right] + b -$$
$$- \left[-\sin(\alpha)(p_i - x0) + \cos(\alpha)(r_i - y0)\right] = 0$$

Denote

$$cv_{1j} = v_{1j} - x0, \; cw_{1j} = w_{1j} - y0$$

A trigonometric function is expanded at the series:

$$\cos(\alpha) \approx 1 - 0.5\alpha^2, \; \sin(\alpha) \approx \alpha$$

After substitution:

$$k\left[\left(1 - 0.5\alpha^2\right) \cdot cv_{1j} + \alpha \cdot cw_{1j}\right] + b -$$
$$- \left[-\alpha \cdot cv_{1j} + \left(1 - 0.5\alpha^2\right) \cdot cw_{1j}\right] = 0$$

Using the least-square method allows defining the expression for estimating α:

$$g(\alpha) = \sum_{i=1}^{m1} \left\{k\left[\left(1 - 0.5\alpha^2\right) \cdot cp_i + \alpha \cdot cr_i\right] + \right.$$
$$\left. + b - \left[-\alpha \cdot cp_i + \left(1 - 0.5\alpha^2\right) \cdot cr_i\right]\right\}^2 \to \min \quad (27)$$

where $m1$ is the amount point.

The solving estimation (27) is

$$\frac{\partial g(\alpha)}{\partial \alpha} = 0, \; L0 + L1 \cdot \alpha + L2 \cdot \alpha^2 + L3 \cdot \alpha^3 = 0 \quad (28)$$

where $L0 = 2\sum_{i=1}^{m1} \left(cv_{1j} + cw_{1j} \cdot k\right) \cdot \left(b - cw_{1j} + cv_{1j} \cdot k\right)$,

$$L1 = 2\sum_{i=1}^{m1} \left(cr_i + cp_i \cdot k\right) \cdot \left(b - cr_i + cp_i \cdot k\right) + \left(cp_i + cr_i \cdot k\right)^2,$$

$$L2 = 2\sum_{i=1}^{m1} \left\{\left(cv_{1j} + cw_{1j} \cdot k\right) \cdot \left(cw_{1j} - cv_{1j} \cdot k\right) + \right.$$
$$\left. + \left(\frac{cw_{1j}}{2} + \frac{cv_{1j}}{2} \cdot k\right) \cdot \left(cv_{1j} + cw_{1j} \cdot k\right)\right\},$$

$$L3 = 2\sum_{i=1}^{m1} \left(\frac{cw_{1j}}{2} - \frac{cv_{1j} \cdot k}{2}\right) \cdot \left(cw_{1j} - cv_{1j} \cdot k\right).$$

The cubic equation has three roots. One of these is a real number and the other two roots are complex numbers.

If the rotate angle is more than 10° then the iteration procedure can increase the accuracy of estimation.

The finish algorithm for estimation parameter α includes the next steps:

1. The first approximation α_k is defined, where $k = 0$.

2. The transform matrix is calculated as

$$M = \begin{bmatrix} \cos(\alpha_k) & \sin(\alpha_k) \\ -\sin(\alpha_k) & \cos(\alpha_k) \end{bmatrix}.$$

3. The point's coordinate is calculated by angle value:

$$\begin{bmatrix} cv_{1j} \\ cw_{1j} \end{bmatrix} = M \begin{bmatrix} v_{1j} - x0 \\ w_{1j} - x0 \end{bmatrix} = \begin{bmatrix} \cos(\alpha_k) & \sin(\alpha_k) \\ -\sin(\alpha_k) & \cos(\alpha_k) \end{bmatrix} \cdot \begin{bmatrix} v_{1j} - x0 \\ w_{1j} - x0 \end{bmatrix}.$$

4. The variables $L0$, $L1$, $L2$, and $L3$ are calculated by equations which are shown above.

5. Three roots are defined by (27).

6. The estimation of parameter α is the root which is a real number.

7. Accuracy estimation of the rotate angle is $k = k+1$, $\alpha_k = \alpha_{k-1} + \alpha$.

8. Go to the step №2.

The exit from the iteration procedure is by criteria $|\alpha_k - \alpha_{k-1}| < thr$, where thr is a threshold. The threshold is 10^{-9} radians at the experiment.

V. Numerical Simulation

The main goal of task superposition is defining solid deforming. The procedure of measurement solid deforming of railway rails by wear side is presented below [16]. The wear side is defined as the distance between the point at no wear rail and measuring rail at the deep 13 mm below the running surface (see. Fig. 6).

The rail contour with known wear side was used for defining the accuracy of the measurement wear side at the experiment. The white noise with known dispersion σ_n^2 was added to abscissa and ordinate of each point's contour.

Superposition parameters are calculated according to algorithms that are described above. And the side wear is rated after superposition measured rail and no wear rail. The side wears compare with real value. The result of a comparison is the root mean square (RMS). The result of numerical simulation was showed for three measurement methods:

– by two reference point which is defined as points where headrail goes into the neck rail;

– by two straight lines (see. Fig. 1b);

– by straight lines and circle part (see. Fig. 3);

The result of a measurement error is showed in Fig. 7.

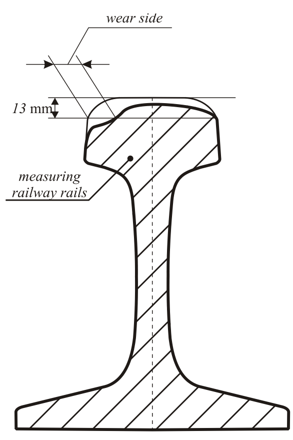

Fig. 6. Wear side.

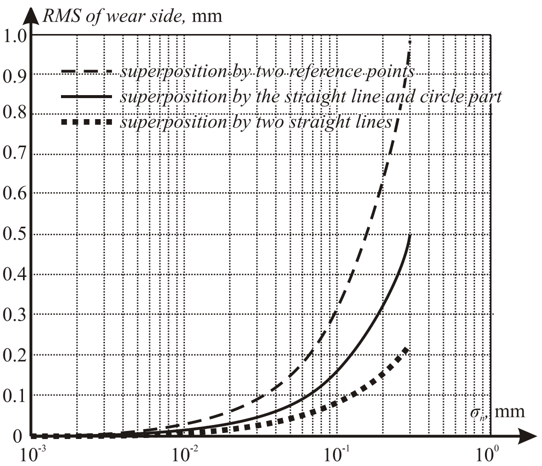

Fig. 7. RMS of wear side.

VI. Natural Experiment

The developed algorithm has been realized at software for a track measuring car. Calculation side wears with developed superposition algorithm by straight lines shows that error estimation is much more than side wears with superposition algorithm by two reference points.

The results of measurements of both algorithms are shown in Fig. 8 and 9. The superposition by two reference points (*note*: reference point is the point of the end headrail and begin web) is better than superposition by straight lines because points of side headrail are matched better with contour no wear rail.

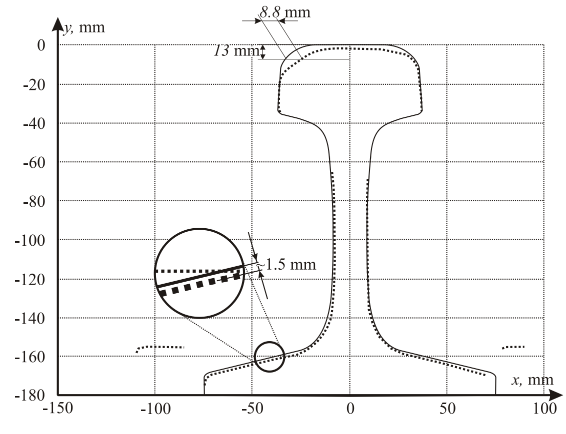

Fig. 8. Measurement side wears with the algorithm by two reference points.

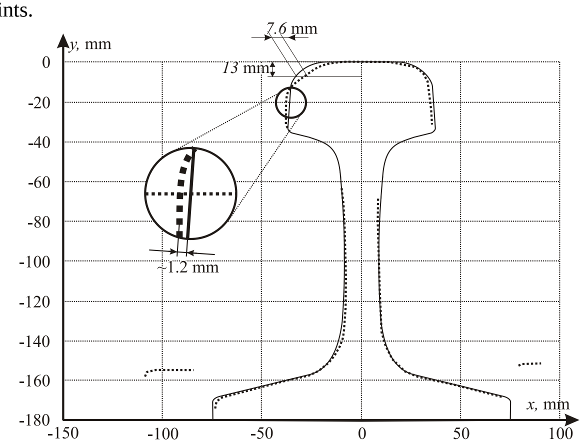

Fig. 9. Measurement side wears with the algorithm by two straight lines.

The points of side headrail are matched with contour no wear rail. This superposition leads to correct measurements of side wear.

But the points of foot rail are not matched with the contour of no wear rail in Fig. 8 by comparison with the case in Fig. 9.

Analyzing this situation shows that the problem links with measurements object (rail). Making tolerances and deformation under mechanical stress are the reasons that measurements object is not matched the model of no wear rail. This disadvantage leads to error estimation of wear which is not satisfied requirements for the correct rating of railway condition.

The additional procedure has been developed for fixing this disadvantage. The idea is rotation measured rail contour. After rotation, the point of side headrail should match with the contour of no wear rail.

The contour of measured rail after superposition by two straight lines is shown in Fig. 10. The line $y=k_1x+b_1$ describes side headrail. Also the subsequence of points is shown in Fig. 10 which belongs to the line $y=k_1x+b_1$.

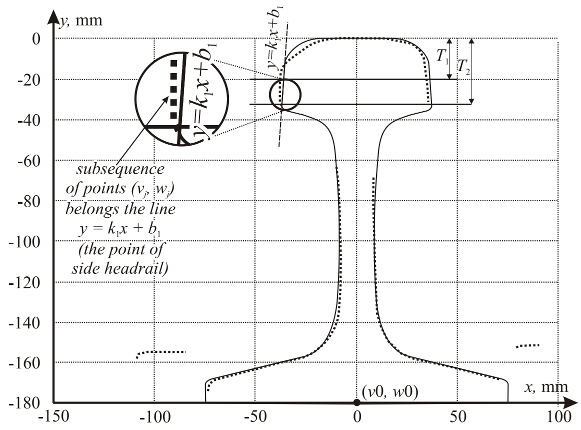

Fig. 10. Additional superposition.

Subsequence of points is defined as contour points which perform the requirements that $T_1<y<T_2$. Denote subsequence of point as (v_j, w_j), $j = 1, .. m1$. Also Fig. 10 shows point $(v0, w0)$. The contour rotates around this point that points (v_j, w_j) belongs the line $y = k_1x+b_1$ (*note*: the parameters T_1, T_2, $(v0, w0)$ is defined by experiment; for rail $R65$ [16]: $T_1 = 20$ mm, $T_2 = 30$ mm, $(v0, w0) = (0, -180)$).

The math model for linking points (vj, wj) and equation $y = k_1x+b_1$ is described as:

$$x_j = (v_j - v0)\cdot\cos(\alpha) - (w_j - w0)\cdot\sin(\alpha) + v0, \quad (29)$$

$$y_j = (v_j - v0)\cdot\sin(\alpha) + (w_j - w0)\cdot\cos(\alpha) + w0, \quad (30)$$

The estimating unknown parameter α with using (29), (30) and equation $y-k_1x-b_1 = 0$ by the least-square method is given:

$$F(\alpha) = \sum_{j=1}^{m1}\left[(v_j - v0)\sin(\alpha) + (w_j - w0)\cos(\alpha) + w0 - k_1\left((v_j - v0)\cos(\alpha) - (w_j - w0)\sin(\alpha) + v0\right) - b_1\right]^2 \to \min \quad (31)$$

Denote:

$$vr_j = v_j - v0, \; wr_j = w_j - w0$$

Also, the small-angle α can be a replacement as:

$$\cos(\alpha) \approx 1, \ \sin(\alpha) \approx \alpha$$

The expression $F(\alpha)$ after the replacement is

$$F(\alpha) = \sum_{j=1}^{m1} \left[vr_j \cdot \alpha + wr_j + w0 - - k_1 \left(vr_j - wr_j \cdot \alpha + v0 \right) - b_1 \right]^2 \to \min \quad (32)$$

Also, denote:

$$p_1 = -\left(w0 + k_1 \cdot v0 - b_1 \right), \quad (33)$$

The final expression of $F(\alpha)$ is

$$F(\alpha) = \sum_{j=1}^{m1} \left(vr_j \cdot \alpha + wr_j - k_1 \left(vr_j - wr_j \cdot \alpha \right) - p_1 \right)^2 \to \min \quad (34)$$

The least-square method $\frac{\partial F(\alpha)}{\partial \alpha} = 0$ allows the expression for estimation rotate angle:

$$\alpha = \frac{N}{M}, \quad (35)$$

where

$$N = k_1^2 \sum_{j=1}^{m1} vr_j \cdot wr_j + k_1 \sum_{j=1}^{m1} vr_j^2 + k_1 p_1 \sum_{j=1}^{m1} wr_j + + p_1 \sum_{j=1}^{m1} vr_j - k_1 \sum_{j=1}^{m1} wr_j^2 - \sum_{j=1}^{m1} vr_j \cdot wr_j \quad (36)$$

$$M = k_1^2 \sum_{j=1}^{m1} wr_j^2 + 2k_1 \sum_{j=1}^{m1} vr_j \cdot wr_j + \sum_{j=1}^{m1} vr_j^2 \quad (37)$$

So (35) has been deduced with assumptions about the value of α then iteration procedure can increase the accuracy of estimation.

The finish algorithm for estimation parameter includes the next steps:

1. The first approximation α_k is defined, where $k = 0$.

2. The transform matrix is calculated as:

$$M = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix},$$

3. The point's coordinate is calculated by angle value:

$$\begin{bmatrix} vr_j \\ wr_j \end{bmatrix} = M \cdot \begin{bmatrix} v_j - v0 \\ w_j - w0 \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \cdot \begin{bmatrix} v_j - v0 \\ w_j - w0 \end{bmatrix}.$$

4. The variables N and M are calculated by (35) and (36).

5. Define $\alpha = \frac{N}{M}$.

6. Accuracy estimation of rotate angle is $\alpha_k = \alpha_{k-1} + \alpha$, where $k = k+1$.

7. Go to the step № 2.

The exit from the iteration procedure is by criteria $|\alpha_k - \alpha_{k-1}| < thr$, where thr is a threshold. The threshold is 10^{-9} radians at the experiment.

VII. Conclusion

The iteration algorithms of signals superposition of triangulation optical sensors for measurement solid deforming were shown in the current article. The basis of method superposition consists in join points of measurement contour and pattern no wear object which is defined as equation lines. The superposition issue leads to iteration solving of the equation. The unknown variables of the equation are superposition parameters. They are the angle of rotation and offset along abscissa and ordinate. The numerical simulation shows that the developed algorithms characterize less measurement error than the algorithm of superposition by reference points. The developed method allows reducing the part of fail measurements. The fail happens in case no measure fragment of contour which contains reference point. The developed method can be used as a redundant solving of superposition issue if the main procedure of superposition leads to failing measurement. This approach allows an increase in one of the important technical and operational parameters. It is the survivance of the measuring system.

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