Spectral Analysis Based on a Discrete Binary Model for Signal Stochastic Quantization and Calculating a Weighted Correlation Function Estimate

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Abstract. The paper considers the high-performance digital algorithm for estimating the power spectral density (PSD) by the correlogram method. This algorithm is developed on the binary analog-stochastic quantization basis of the investigated continuous signal. The mathematical model for the discrete representation of binary analog-stochastic quantization made it possible to analytically calculate the cosine Fourier transform of weighted window functions in the algorithm development. As a result of this, the developed algorithm does not require numerical integration operations. The main computational operations of the algorithm are the operations of summation and subtraction. The algorithm also does not require the calculation of correlation function estimates. All this increases the computational efficiency of the PSD estimation by the correlogram method. The experimental studies results of the algorithm are given. These results show that the proposed algorithm gives accurate PSD estimates in the presence of additive noise. The computational efficiency of the algorithm provides the ability to use it for estimating the PSD of complex signals.

Keywords: power spectral density, time-weighting function, binary analog stochastic quantization, sign signal, digital time readout

1 Introduction

Spectral analysis methods of signals are used in various fields of science and technology. In particular, this applies to acoustics, location, vibration diagnostics, radio frequency identification, etc. Under conditions of a priori statistical uncertainty, the regularity of changing signal parameters over time is determined by probabilistic laws. The spectral analysis of such signals involves estimating the power spectral density (PSD) over a finite time interval. PSD characterizes the distribution of the average signal power over frequencies within the analyzed frequency range.

One of the most common classical methods for assessing PSD is the correlogram method. This spectral analysis method can be used to study the frequency composition of complex multicomponent signals that meet the conditions of stationarity and ergodicity in time.

At present, the correlogram method for estimating PSD is carried out primarily in digital form. First, modern computing and software engineering allow you to create high-tech problem-oriented systems. Secondly, digital signal processing guarantees the accuracy of calculations. Moreover, traditionally, researchers use digital algorithms that are developed on the basis of an approximate implementation of the analog integration operation in discrete form by digital summation operations with a uniform sampling interval in time [1–4]. All digital algorithms developed in this way require a significant amount of calculated operations. Among these operations, a large number of digital multiplication operations have to be performed. As you know, digital multiplication operations are the most time-consuming computing operations. Using algorithms that require a large number of digital multiplication operations can lead to significant time costs. The consequence of this is a decrease in the efficiency of calculating PSD estimates.

A feature of classical algorithms for calculating the PSD estimate by the correlogram method is the need for preliminary calculation of the correlation function estimates sequence for the analyzed signal. It also reduces the computational efficiency of obtaining PSD estimates.

The correlogram method involves the use of weighted window functions (the so-called correlation windows) to attenuate the blurring effect of spectral components estimates. The use of weighted window functions involves the operation of weighing the correlation function estimates with the correlation window function samples. This procedure leads to the need for additional digital multiplication operations. As a result, the use of correlation windows reduces the computational efficiency of digital algorithms for estimating the PSD using the correlogram method.

Typically, the computational efficiency of digital signal processing is enhanced in three main ways [5–9]. Firstly, an increase in the speed of data processing programs is ensured by an increase in the overall performance of computing systems. Secondly, the developers of the algorithms carry out special preparation of the initial data in the form of ordered algebraic structures. Thirdly, the computational process is accelerated by refactoring and using various methods of optimizing program code. However, none of these methods can solve the problem of increasing the computational efficiency of digital signal processing in full. This is explained by the fact that the time characteristics of the execution of application programs are determined largely by the mathematical model of preparing data in digital form and by performing computational operations on this data. The use of binary quantization as the primary conversion of the analyzed continuous signal into a digital code can improve the computational efficiency of digital signal processing [10, 11]. The application of binary quantization has been investigated extensively in the past years [12–15]. In [16, 17], fast digital algorithms were developed for estimating the PSD based on the correlogram method using binary analog-stochastic quantization. The discrete-time mathematical model for this quantization type provided analytical calculation of analog integration operations in the development of these algorithms [18]. As a result, the need to perform a large number of digital multiplication operations is eliminated. The main operations of the algorithms are logical operations and arithmetic operations of summing and subtracting discrete values of the cosine function. Algorithms also do not require preliminary calculation of estimates of the correlation function. All this simplifies the digital procedures for calculating the PSD estimate by the correlogram method and reduces the overall complexity of the spectral analysis. However, the algorithms in [16, 17] were developed without taking into account the use of weight functions (correlation windows), which limits their application in practice.

Thus, the task of developing a computationally efficient algorithm for estimating the PSD based on the correlogram method is relevant. To solve this problem, it is fundamentally important to obtain simple computational procedures using the weighted window functions and reduce the number of digital multiplication operations. Moreover, such an algorithm should allow obtaining the PSD estimates with the necessary accuracy and frequency resolution.

2 Algorithm for estimating the PSD by the correlogram method based on binary analog-stochastic quantization of the analyzed signal

It was noted above that in [16, 17] fast algorithms for estimating the PSD based on the correlogram method were developed using a technique that allows analytical calculation of integration operations. Let us summarize this technique for developing a digital algorithm for calculating the PSD estimates by the correlogram method using weighted window functions.

The correlation function and weighted window functions are even functions. With this in mind, the PSD estimate using correlogram method is calculated as follows:

$$\hat{S}_{XX}(f) = 2\int_0^T w(\tau)\hat{R}_{XX}(\tau)\cos 2\pi f\tau d\tau \, , \tag{1}$$

where $\hat{R}_{XX}(\tau)$ is the correlation function estimate of the analyzed signal $X(t)$; $w(\tau)$ is the weighted window function; T is the length of time the spectral analysis.

Let within the time intervals $t \in [0;T]$ and $t \in [0;2T]$, the results of two independent operations of binary analog-stochastic quantization are sign signals:

$$z_1(t) = \operatorname{sgn}\{\overset{o}{x}(t) + \xi_1(t)\} \text{ and } z_2(t) = \operatorname{sgn}\{\overset{o}{x}(t) + \xi_2(t)\}, \tag{2}$$

where $\operatorname{sgn}\{...\}$ is the operator of the sign function; $\overset{o}{x}(t)$ is a centered implementation of the analyzed signal $X(t)$; $\xi_1(t)$ and $\xi_2(t)$ are auxiliary random signals.

Auxiliary signals $\xi_1(t)$ and $\xi_2(t)$ have a uniform distribution ranging from $-\xi_{\max}$ to $+\xi_{\max}$, where the value of $\xi_{\max}$ must exceed the highest possible value of the implementation $\overset{o}{x}(t)$ with a probability close to unity [10,11,18].

As an estimate of $\hat{R}_{XX}(\tau)$, we take an unbiased estimate [19–21]:

$$\hat{R}_{XX}(\tau) = \xi_{\max}^2 T^{-1} \int_0^T z_1(t) z_2(t+\tau) dt \,. \tag{3}$$

We introduce the notation:

$$g(\tau, f) = w(\tau) \cos 2\pi f \tau \,. \tag{4}$$

Then, taking into account (3) and (4), estimate (1) is:

$$\hat{S}_{XX}(f) = 2\xi_{\max}^2 T^{-1} \int_0^T g(\tau, f) \int_0^T z_1(t) z_2(t+\tau) dt d\tau \,. \tag{5}$$

In (5), we change the order of integration over the variables t and τ:

$$\hat{S}_{XX}(f) = 2\xi_{\max}^2 T^{-1} \int_0^T z_1(t) \int_t^{T+t} z_2(\tau) g(\tau - t, f) d\tau dt \,. \tag{6}$$

Further development of the digital algorithm for estimating the PSD was reduced to the practical implementation of analog integration operations in a discrete form. An effective calculation of these operations can be achieved using a discrete-time mathematical model for changing the values of the sign signals $z_1(t)$ and $z_2(t)$ in time. These signals are continuous functions and their values are limited in level by the values “–1” and “+1”. Therefore, we can unambiguously represent the signals $z_1(t)$ and $z_2(t)$ at the time intervals of their formation $t \in [0;T]$ and $t \in [0;2T]$ using the values of $z_1(t_0)$ and $z_2(t_0)$ at the initial moment of quantization time $t_0 = 0$ and the time samples $\{ t_i^{z_1} \}$ and $\{ t_j^{z_2} \}$, at which they change their value, where $1 \le i \le I$ and $1 \le j \le J$. $t_0^{z_1} = t_0^{z_2} = t_0 = 0$, $t_I^{z_1} = T$ and $t_J^{z_2} = 2T$ [14].

Taking into account the discrete-time representation of the sign signals $z_1(t)$ and $z_2(t)$, we write the estimate (6) as a sum:

$$\hat{S}_{XX}(f) = 2\xi_{\max}^2 T^{-1} z_1(t_0) \sum_{i=0}^{I-1} (-1)^i \int_{t_i^{z_1}}^{t_{i+1}^{z_1}} \int_t^{T+t} z_2(\tau) g(\tau - t, f) \, d\tau dt \,. \tag{7}$$

From (4) it follows that for a continuous and differentiable weighted window function $w(\tau)$ in the time interval $\tau \in [0;2T]$, the function $g(\tau, f)$ is also a continuous and differentiable function in this time interval. Then there exists a continuous function $G(\tau, f)$ for which the condition is satisfied [22]:

$$g(\tau, f)dt = dG(\tau, f)\,. \tag{8}$$

In (7), we change the order of integration. Then, taking into account (8), the integral over the variable t can be calculated analytically. After calculating this integral, we obtain:

$$\hat{S}_{XX}(f) = D_{XX}(T, f) + 2\xi_{\max}^2 T^{-1} z_1(t_0) \sum_{i=0}^{I} (-1)^i \lambda_i B(t_i^{z_1}, f)\,; \tag{9}$$

$$\lambda_i = 1, \text{ if } i = 0 \text{ and } i = I\,;\ \lambda_i = 2, \text{ if } 2 \le i \le I - 1\,;$$

$$D_{XX}(T, f) = 2(G(T, f)\hat{R}_{XX}(T) + G(0, f)\hat{R}_{XX}(0))\,; \tag{10}$$

$$B(t_i^{z_1}, f) = \int_{t_i^{z_1}}^{t_i^{z_1}+T} z_2(\tau) G(\tau - t_i^{z_1}, f) d\tau\,. \tag{11}$$

Relation (11) as well as (6) can be represented as a sum of integrals:

$$B(t_i^{z_1}, f) = z_2(t_i^{z_1}) \sum_{j=m(i)}^{m(i)+r(i)} (-1)^{j-m(l)} \int_{t_j^{z_2}}^{t_{j+1}^{z_2}} G(\tau - t_i^{z_1}, f) d\tau\,, \tag{12}$$

where $t_{m(i)}^{z_2} = t_i^{z_1}$ and $t_{m(i)+r(i)+1}^{z_2} = t_i^{z_1} + T$.

The integral in (12) is defined. It can be calculated numerically:

$$B(t_i^{z_1}, f) = z_2(t_i^{z_1}) \sum_{j=m(i)}^{m(i)+r(i)} (-1)^{j-m(l)} \Delta\tau_j G(\Delta t_{ji}, f)\,, \tag{13}$$

where $\Delta\tau_j = t_{j+1}^{z_2} - t_j^{z_2}$ and $\Delta t_{ji} = t_j^{z_2} - t_i^{z_1}$.

Relations (9) and (13) can be directly used to calculate the SDM estimates in discrete form. Moreover, we take into account that for a given duration of the spectral analysis time T, the maximum possible frequency resolution is $\Delta f = 1/T$. Then for $f_k = k\Delta f$ we finally get:

$$\hat{S}_{XX}(f_k) = D_{XX}(T, f_k) + A(\xi_{\max}, T) z_1(t_0) \sum_{i=0}^{I} (-1)^i \lambda_i B(t_i^{z_1}, f_k)\,, \tag{14}$$

$$B(t_i^{z_1}, f_k) = z_2(t_i^{z_1}) \sum_{j=m(i)}^{m(i)+r(i)} (-1)^{j-m(l)} \Delta\tau_j G(\Delta t_{ji}, f_k)\,, \tag{15}$$

$$A(\xi_{\max}, T) = 2\xi_{\max}^2 T^{-1} = 2\xi_{\max}^2 \Delta f = \text{Const}\,. \tag{16}$$

As follows from (14) and (15), the calculation of the PSD estimate was reduced to the discrete processing of the function $G(\tau, f)$. Relations (14) and (15) became the basis for the development of a digital algorithm for calculating PSD estimates. The main operations of this algorithm are the operations of summing and subtracting the samples of function $G(\tau, f)$, where $\tau = \Delta t_{ji}$ and $f = k\Delta f$.

The function $G(\tau, f)$ is primitive for the function $g(\tau, f) = w(\tau)\cos 2\pi f\tau$ in the time interval $\tau \in [0; 2T]$, where the weighted window function $w(\tau)$ is known. Therefore, the function $G(\tau, f)$ is also known, since its form is determined only by the type of the applied weighted window function $w(\tau)$. As an example, some of the main weighted window functions $w(\tau)$ used in calculating the PSD estimates and the corresponding functions $G(\tau, f)$ are presented in Table 1 [1, 23–25].

Table 1. Weighted window functions and functions $G(\tau, f)$

Window	$w(\tau)$	$G(\tau, f)$
Rectangular (Box Car) window	$\begin{cases} 1, & \lvert\tau\rvert \le T; \\ 0, & \lvert\tau\rvert > T. \end{cases}$	$\dfrac{\sin 2\pi f\tau}{2\pi f}$
Triangular (Bartlett) window	$\begin{cases} 1 - \dfrac{\lvert\tau\rvert}{T}, & \lvert\tau\rvert \le T; \\ 0, & \lvert\tau\rvert > T. \end{cases}$	$\left(1 - \dfrac{\tau}{T}\right)\dfrac{\sin 2\pi f\tau}{2\pi f} - \dfrac{1}{T}\dfrac{\cos 2\pi f\tau}{(2\pi f)^2}$
Cos(x) window	$\begin{cases} \cos\dfrac{\pi\tau}{2T}, & \lvert\tau\rvert \le T; \\ 0, & \lvert\tau\rvert > T. \end{cases}$	$\dfrac{1}{\pi}\left(\dfrac{\sin 0.5\pi\alpha\tau}{\alpha} + \dfrac{\sin 0.5\pi\beta\tau}{\beta}\right)$, $\alpha = (4f - \Delta f)$, $\beta = (4f + \Delta f)$
Hann (Raised-Cosine) window	$\begin{cases} 0.5 + 0.5\cos\dfrac{\pi\tau}{T}, & \lvert\tau\rvert \le T; \\ 0, & \lvert\tau\rvert > T. \end{cases}$	$\dfrac{0.5}{2\pi}\sum_{m=0}^{1}\left(\dfrac{\sin \pi\alpha\tau}{\alpha} + \dfrac{\sin \pi\beta\tau}{\beta}\right)$, $\alpha = (2f - m\Delta f)$, $\beta = (2f + m\Delta f)$

3 Experiments and results

Experimental studies of the developed algorithm were carried out using simulation methods. In accordance with this, a logical-mathematical model was developed for conducting computer experiments. The purpose of the experimental studies was to evaluate the metrological capabilities of the algorithm for calculating the PSD estimate. On the basis of discrete-event modeling, special software was developed to simulate the binary analog-stochastic quantization of a continuous signal. The discrete-event model of binary analog-stochastic quantization made it possible to abstract from the continuous quantization result and consider only the main events determined by time instants at which it changes its values. The model of centered signal implementation was the sum of the statistical-independent harmonic components in additive noise $e(t)$:

$$\overset{o}{x}(t) = \sum_{m=1}^{M} A_m \cos(2\pi f_m t + \varphi_m) + e(t)\,,$$

where f_m are the normalized frequencies, A_m are the normalized amplitudes, φ_m are the initial phases.

A model of this type reflects the dynamics of a real signal characterizing a wide class of physical processes. We can set a specific superposition for the arrangement of spectral components within a given frequency range and consider a complex signal as a combination of narrow-band signals. We also note that this model makes it possible to study the operability of the algorithm in the presence of noise interference and to examine its stability in calculating the PSD estimates in the presence of random background noise.

Normalized frequencies f_m were set in the range from zero to 0.5. This ensured the constancy of the frequency range of the presentation of the results of the PSD estimation for complex signals models that a priori occupy different frequency ranges in width. The amplitudes A_m were also interpreted as normalized. They were set in the range from zero to unity. Initial phases φ_k were set within the interval $-\pi \le \varphi_k \le \pi$ using a generator of randomly uniformly distributed quantities. Additive noise $e(t)$ was white noise with zero expectation and dispersion $\sigma_e^2 = 1$. In particular, the model contained five harmonic components. The parameters of these components are shown in Table 2.

Table 2. Harmonic components parameters

m	A_m	f_m	$A_m^2 / A_{\max}^2$, dB	Normalized PSD estimate, dB			
				Rectangular (Box Car) window	Triangular (Bartlett) window	Cos(x) window	Hann (Raised-Cosine) window
1	0.15	0.275	-16.48	-16.21	-16.32	-16.29	-19.52
2	0.25	0.3	-12.04	-12.07	-12.14	-12.17	-13.86
3	0.75	0.32	-2.5	-2.44	-2.47	-2.47	-3.04
4	1.0	0.33	0	0	0	0	0
5	0.35	0.375	-9.12	-9.73	-9.54	-9.55	-9.11

Using simulation, experiments were carried out in the course of which the possibility of frequency resolution and the accuracy of determining harmonic components were checked. Figures 1-4 show the normalized PSD estimates obtained for the Rectangular (Box Car) window, the Triangular (Bartlett) window, the Cos(x) window and the Hann (Raised-Cosine). PSD estimates were calculated with a resolution of 0.0001 units of normalized frequency. The results of determining the harmonic components are presented in Table 2.

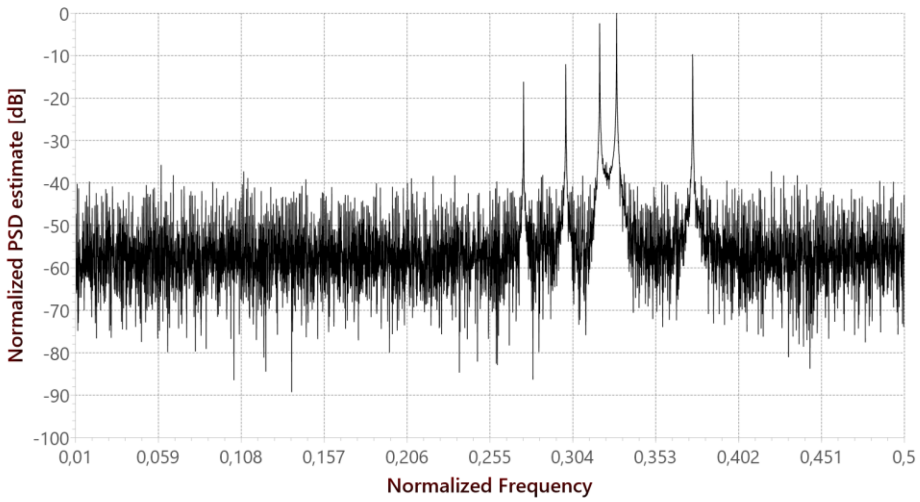

Fig. 1. Normalized PSD estimate, Rectangular (Box Car) window

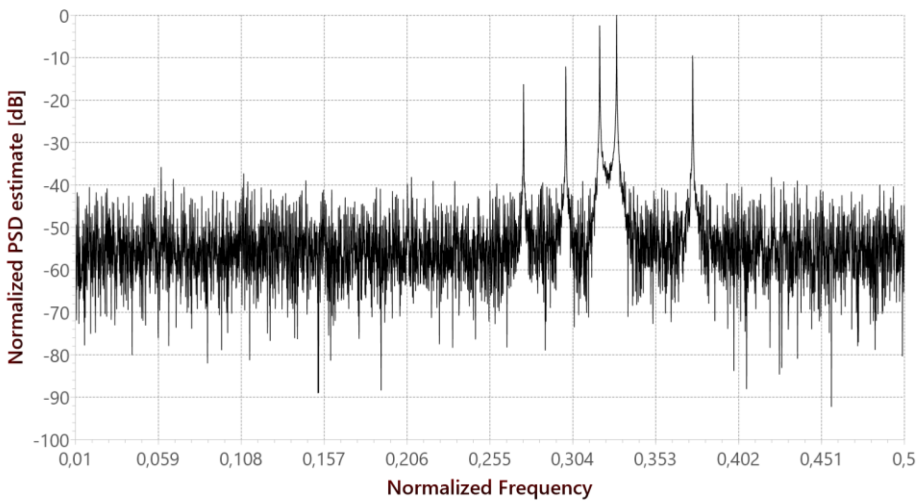

Fig. 2. Normalized PSD estimate, Triangular (Bartlett) window

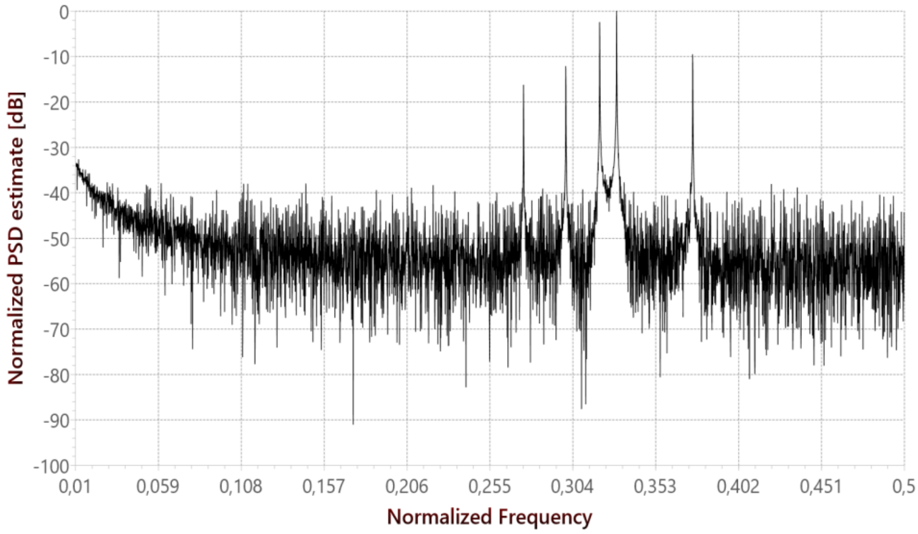

Fig. 3. Normalized PSD estimate, Cos(x) window

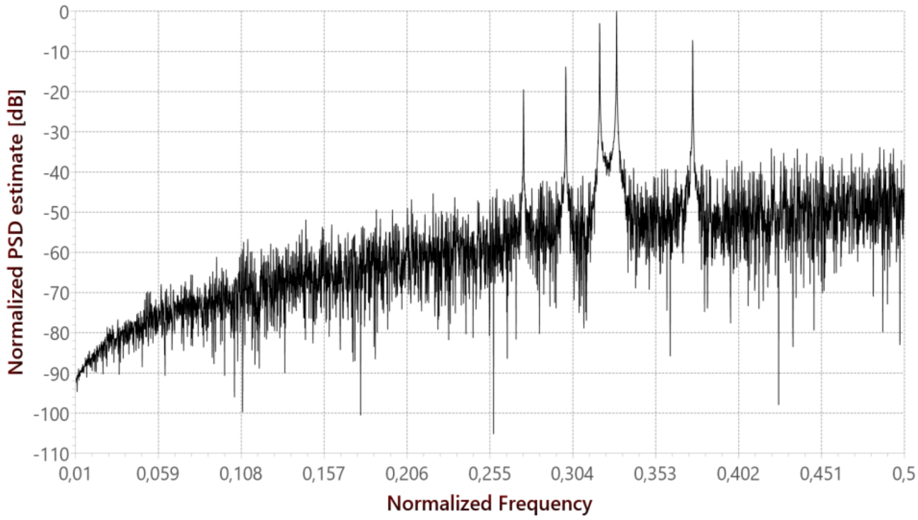

Fig. 4. Normalized PSD estimate, Hann (Raised-Cosine) window

On all graphs, we see a steady identification of all five harmonic components. The position of the harmonic components estimates in the spectrum corresponds to the initial values. False spectral lines are not present. Spectral line splitting is not observed. Spectral lines are clearly distinguishable. Strong harmonics do not mask weak ones. The spectral noise estimate remains at a fairly low level in relation to the harmonic component estimates. It practically does not exceed -40 dB. This indicates a good resolution and high stability of the algorithm to external additive noise.

4 Conclusion

Based on binary analog-stochastic quantization, we developed a digital algorithm for estimating the PSD by the correlogram method. During the development of the algorithm, the integration operations are calculated analytically. This eliminates the methodological error, which is the result of numerical integration operations. The main computational operations of the algorithm are the operations of summation and subtraction. Unlike the classical digital algorithms for estimating the PSD by the correlogram method, this algorithm requires calculating only two estimates of the correlation function $\hat{R}_{XX}(0)$ and $\hat{R}_{XX}(T)$. If $T > \tau_{kx}$, then $\hat{R}_{XX}(T) \to 0$, where τ_{kx} is the correlation interval of the analyzed signal. Then $G(T, f)\hat{R}_{XX}(T) \to 0$, and for (10) we will have $D_{XX}(T, f) \approx 2G(0, f)\hat{R}_{XX}(0)$. All this increases the computational efficiency of the estimation of the PSD by the correlogram method.

The considered algorithm can be implemented as a functionally complete software module. This module can find application in the composition of the metrologically significant software of multifunctional systems for operational frequency-time analysis of signals [26]. The practical use of such a module in integrated software should increase the efficiency of solving problems requiring the processing of complex multicomponent signals.

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