

# High-performance Calculation of the Relation between the Load of the Motor Vehicle Undercarriage, its Smoothness of the Ride and the Value of Unsprung Weights

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## Abstract

The article deals with the issue of loading the chassis of the car, in particular, its suspension elements, wheels, tires, as well as its smoothness of the ride and the value of unsprung weights. The calculations are performed in a non-linear formulation, taking into account the real characteristics of the vehicle. The obtained results of the calculations cast doubt on the generally accepted postulates, according to which the unsprung mass of the car has a significant negative impact on the load of its chassis (i.e., the smaller the value of unsprung masses, the better). The outcomes also clearly demonstrate the feasibility of using the full potential of high-performance computing to solve this class of scientific and technical problems related to statistical dynamics and comprising modeling and micro-profile of the road surface, without which it is impossible to numerically solve the problem in a nonlinear formulation, and the vehicle itself. A large amount of calculations and accurate modeling, adequate to the real objects of calculation, require the use of high-performance computing.

## Keywords 1

Sprung weight, unsprung weight, suspension, load, smoothness of the ride

## 1. Introduction

It is well known that the mass of the car is divided into sprung and unsprung, the latter being the mass of all the elements perceived by the elastic elements of the suspension. The remaining elements form an unsprung mass, which is several times smaller than the sprung mass. There is a trend to reduce the unsprung weight, because it is believed that this event will favorably affect the smoothness of the ride, stability and handling, traction-speed and fuel-economic characteristics of the car, the load of its suspension elements, wheels, and tires.

Naturally, the larger the mass, the more difficult it is to change the parameters of its movement, such as speed and direction of movement. However, this is true for both sprung and unsprung masses. Therefore, there is no doubt concerning the improvement of stability, handling, traction and braking dynamics with the unsprung mass decrease, but the question of improving the smoothness of the car, reducing the load on its suspension elements, wheels, tires remains rather controversial, because, on the one hand, the lower the value of the unsprung mass compared to the sprung one, the smaller the values of the kinematic characteristics of the latter, due to the fluctuations of the first from interaction with the irregularities of the support surface. At the same time, it is logical that the larger the unsprung mass, the less it will respond to external influences (with the same tire characteristics) and, correspondingly, its

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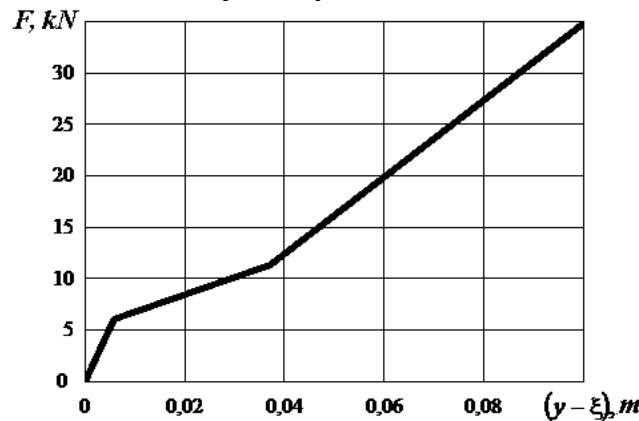
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effect on the sprung mass will be smaller. The same conclusions raise doubts about the increase in the load of elastic, dissipative suspension elements, wheels, and tires with an increase in the unsprung mass.

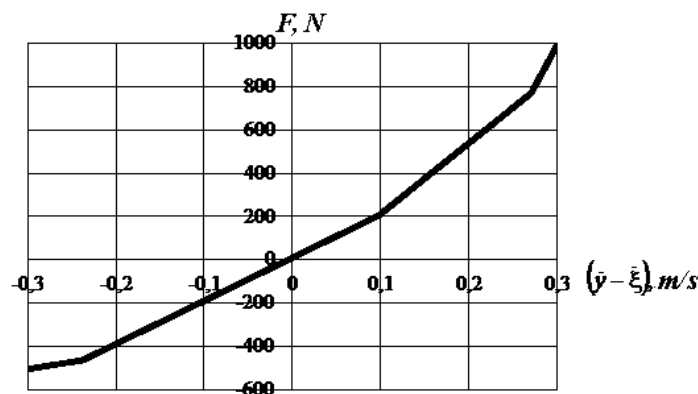
In [1], to study the effect of unsprung masses on the smoothness of the ride, the loading of elastic, dissipative suspension elements, tires, the amplitude-frequency characteristics of the movements, speeds, and accelerations of the sprung and unsprung masses were obtained. These calculation results allow to conclude that the reduction of unsprung masses, having a positive effect on the traction-dynamic, fuel-economic characteristics of the car, their stability and controllability, the load on the wheels, tires and road surface, still has little effect on the operation mode of the elastic and dissipative suspension elements, and on the smoothness of the ride, which contradicts the generally accepted opinion. However, the solution of the problem was carried out in a linear formulation, which could affect the results of calculations..

## 2. Calculation model

Let us complicate the problem and assume that the characteristics of the elastic and dissipative suspension elements are nonlinear, as shown in Figure 1 and 2. They show the characteristics of the springs and shock absorbers of the car, respectively.



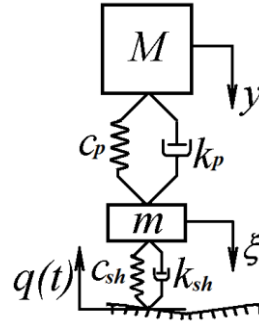
**Figure 1:** The characteristics of the springs



**Figure 2:** The characteristics of shock absorbers

Taking into account the assumption of the independence of the vibrations of the front and rear sprung parts, the design scheme will be as shown in Figure 1. Here  $y$ ,  $\xi$  is the vertical movements of sprung, unsprung masses. Accordingly,  $(\dot{y} - \dot{\xi})$  is the difference in the speeds of movement of sprung and unsprung masses, which is included in the characteristics of the shock absorber in Figure 2.

The calculation scheme is shown at the Figure 3.



**Figure 3:** Calculation scheme

The corresponding mathematical model is as follows:

$$\begin{cases} M\ddot{y} + c_p(y - \xi) + k_p(\dot{y} - \dot{\xi}) + tr_p \cdot \text{sign}(\dot{y} - \dot{\xi}) = 0; \\ m\ddot{\xi} - c_p(y - \xi) - k_p(\dot{y} - \dot{\xi}) + c_{sh}(\xi + q(t)) + k_{sh}(\dot{\xi} + \dot{q}(t)) = 0, \end{cases} \quad (1)$$

where  $M, m$  stand for the sprung and unsprung masses of a vehicle;  $c_p, k_p$  are the given coefficients of stiffness of the elastic element and the viscous resistance of the dissipative element;  $tr_p$  is the force of friction in the longitudinal direction in the suspension;  $c_{sh}$  is the coefficient of normal tire stiffness;  $q(t)$  is the kinematic effect from the roughness of the road surface.

The solution of the differential equations system will be obtained numerically, but this requires an array of ordinates of the micro profile of the road surface. Correlation functions for public roads are usually represented as:

$$R_q(x_s) = \sigma_q^2 \cdot e^{-\alpha_1|x_s|} \quad (2)$$

$$R_q(x_s) = \sigma_q^2 \cdot e^{-\alpha_2|x_s|} \cdot \cos \beta_1 x_s \quad (3)$$

$$R_q(x_s) = \sigma_q^2 \cdot \left( A_1 \cdot e^{-\alpha_3|x_s|} + A_2 \cdot e^{-\alpha_4|x_s|} \cdot \cos \beta_2 x_s \right) \quad (4)$$

where  $\sigma_q^2 = R_q(0)$  is dispersion, and  $A_1, A_2$  are weight coefficients;

$\alpha_i$  are the parameters that characterize the decay rate of the correlation relationship of the microprofile ordinates;

$\beta_i$  are the parameters that characterize the harmonic component of the microprofile;

$x_s$  is correlation interval.

For a number of roads, the parameters  $A_i, \alpha_i, \beta_i, \sigma_{qi}$  are given in the table. 1 [2].

**Table 1**

Parameter values for public roads

Road surface	$\sigma_q, M$	$A_1$	$A_2$	$\alpha_1 = \alpha_2 = \alpha_3, M^{-1}$	$\alpha_4, M^{-1}$	$\beta_1 = \beta_2, M^{-1}$
Cement concrete highway	0,005 – 0,012	1	0	0,15	0	0
Hard-topped highway	0,008 – 0,012	0,85	0,15	0,20	0,05	0,60
Smooth cobblestone highway	0,013 – 0,022	1	0	0,45	0	0
Broken cobblestone	0,025 – 0,033	0	1	0	0,10	0,238

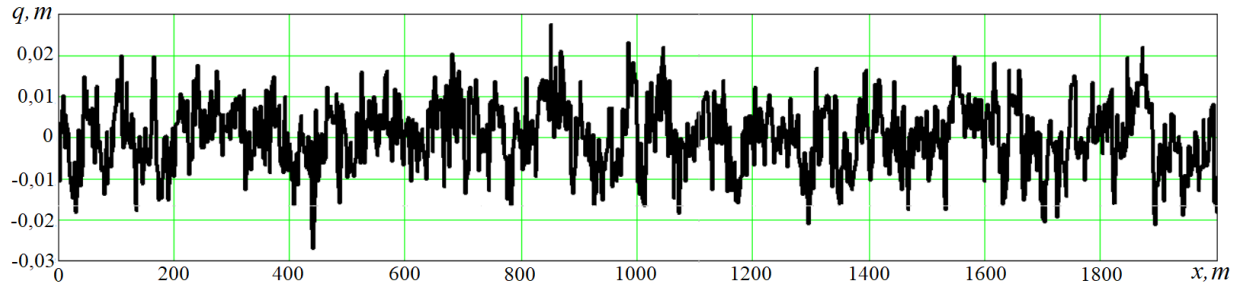
highway						
Worn concrete highway	0,013 – 0,025	0,85	0,15	0,50	0,20	2,0
Broken dirt road	0,100 – 0,140	0,55	0,45	0,085	0,08	0,235

Numerous research works have been devoted to the issue of modeling the micro profile of the road surface, in particular, [3–7].

By way of example, let us use the formula (4) of the correlation function of the hard-topped road. Here  $A_1 = 0,85$ ;  $A_2 = 0,15$ ;  $\sigma_q = 0,008 \text{ m}$ ;  $\alpha_1 = 0,2 \text{ c}^{-1}$ ;  $\alpha_2 = 0,05 \text{ c}^{-1}$ ;  $\beta_1 = 0,6 \text{ c}^{-1}$ . The equations of the forming filter featured as a system of first-order differential equations will be as follows [7]:

$$\begin{cases} \dot{q} = q_1 + \frac{b_0}{a_0} q_0; \\ \dot{q}_1 = q_2 + \frac{a_0 b_1 + a_1 b_0}{a_0^2} q_0; \\ \dot{q}_2 = -\frac{a_1}{a_0} q_2 - \frac{a_2}{a_0} q_1 - \frac{a_3}{a_0} q + \left( \frac{b_2}{a_0} - \frac{a_1}{a_0} \cdot \frac{a_0 b_1 - a_1 b_0}{a_0^2} - \frac{a_2 b_0}{a_0^2} \right) q_0. \end{cases} \quad (5)$$

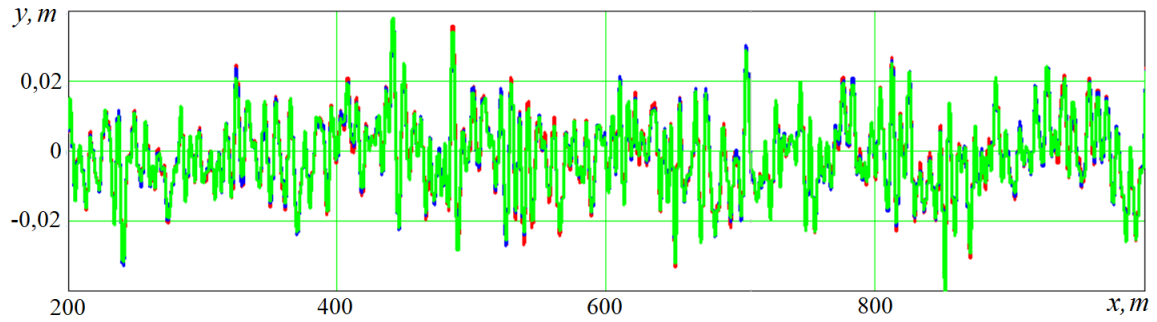
To solve the obtained differential equations, we will use the Runge-Kutta method of the first order. The corresponding simulated micro profile of the road surface is shown in Figure 4.



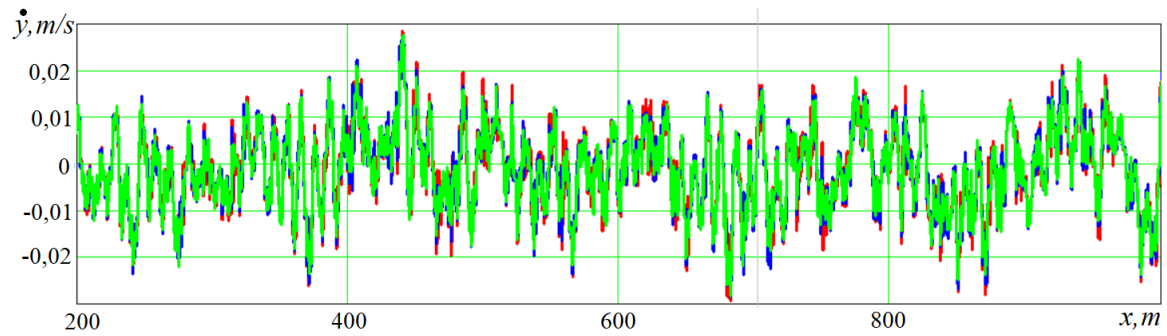
**Figure 4:** Simulated micro profile of a hard-topped highway

To conduct the numerical study, we will set the following tentative values:  $M = 2400 \text{ kg}$ ;  $c_{sh} = 1553,5 \text{ kN/m}$ ,  $k_p = 150 \text{ Ns/m}$ ,  $k_{sh} = 2544 \text{ Ns/m}$ , the reduced stiffness coefficient of the spring  $c_p = 168 \text{ kN/m}$ , the spring and the springer –  $c_p = 348,4 \text{ kN/m}$ . The given coefficients of the viscous resistance of the dissipative element are accepted, according to Figure 2, with the value of the friction force being  $1500 \text{ N}$ , the coefficient of friction between the spring sheets being  $0.18$ .

We will perform calculations for three values of the unsprung mass:  $500 \text{ kg}$ ,  $300 \text{ kg}$ ,  $100 \text{ kg}$  when the car is driving at the speed of  $60 \text{ km/h}$ . Figure 5 shows the vertical movements  $y$  of the sprung mass. In this drawing and in all the subsequent ones, the red color corresponds to the unsprung weight of  $500 \text{ kg}$ , the blue one to that of  $300 \text{ kg}$ , and the green one to that of  $100 \text{ kg}$ . As you can see, the movements at three different masses almost overlap. Moreover, the velocities of movement of the sprung masses do not significantly differ from each other for the three calculated cases presented in Figure 6.

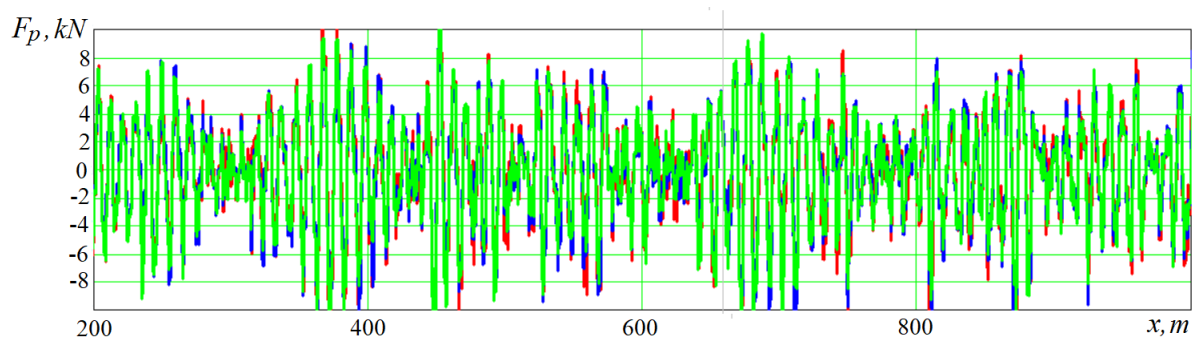


**Figure 5:** Vertical movements of sprung masses when driving a car on a hard-topped highway

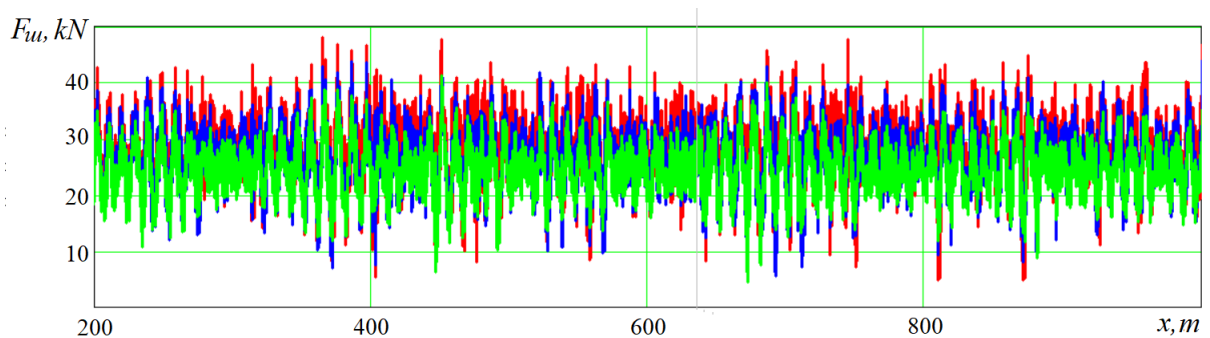


**Figure 6:** The speed of vertical movements of sprung masses when driving a car on a hard-topped highway

The loading of the elastic suspension elements can be assessed on the basis of Figure 7, which shows the impact of dynamic forces  $F_p$ . Figure 8 illustrates the loading of tires. Their more significant differences are due to different values of unsprung masses.

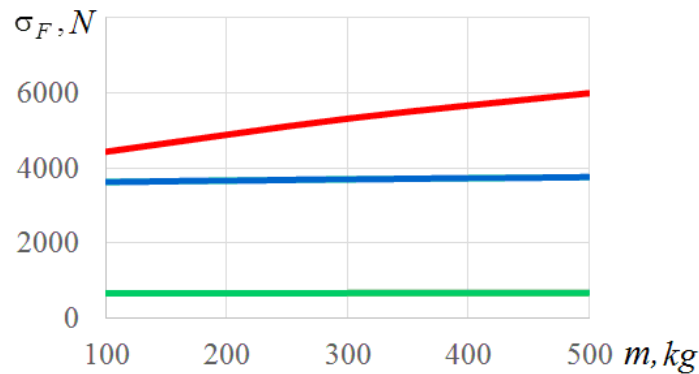


**Figure 7:** Dynamic forces' impact on elastic suspension elements when driving a car on a hard-topped highway

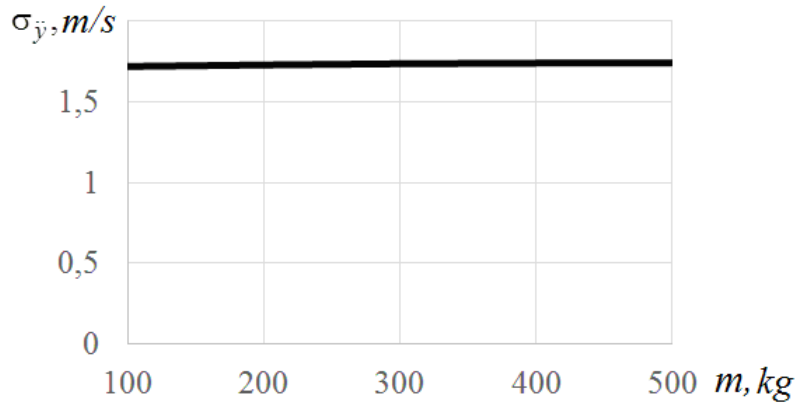


**Figure 8:** Dynamic forces' impact on tires when driving a car on a hard-topped highway

For greater clarity, Figure 9 shows the mean standard deviations (MSD) of dynamic loads on tires (red), on elastic suspension elements (blue), on shock absorbers (green) when driving a car at the speed of 60 km / h on a hard-topped highway. Figure 10 shows the MSD of the acceleration of the body. As can be seen from the results of the calculations presented in these figures, the change in the value of the unsprung mass does not significantly affect the loading of the elastic and dissipative suspension elements, the smoothness of the ride. It is the tires that will experience a noticeable increase in loads, both dynamic and static. Thus, an increase in the unsprung mass from 100 kg to 500 kg (i.e. by 400%) will lead to an increase in the static load by 16%, and the standard deviations of the dynamic forces – by about 35 %. The forces in the contact spot of the tire with the road will increase by approximately the same value. However, the increase in the MSD of dynamic forces on elastic suspension elements is about 3.5%, the one on the dissipative elements is 2.6%, and that of accelerations is less than 1.5%. The same situation is observed with other speeds of the vehicle.



**Figure 9:** MSD of dynamic loads



**Figure 10:** MSD of acceleration of sprung masses

Thus, the refined calculation in the nonlinear formulation confirms the conclusion made earlier in [1] that the reduction of unsprung masses has a negligible effect on the loading of elastic and dissipative suspension elements, and on the smoothness of the car. The load on the wheels, tires, and road surface is concurrently reduced.

In addition, it should be noted that the nonlinear formulation of the problem requires the use of high-performance computing, since the solution of complex systems of differential equations that take into account both the refined correlation functions of the microfiles of roads and the real inertial, elastic, dissipative characteristics of vehicle elements is difficult in most cases even if fairly modern PCs are employed. The current progress in the development of supercomputer methods allows us to solve the class of problems described in this article and related classes with a significant reduction in time and labor inputs [8, 9].

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