

# From Weighted Conditionals with Typicality to a Gradual Argumentation Semantics and back (Extended Abstract)

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## Abstract

A fuzzy multi-preferential semantics has been recently proposed for weighted conditional knowledge bases with typicality, and used to develop a logical semantics for Multilayer Perceptrons, by regarding a deep neural network (after training) as a weighted conditional knowledge base. Based on different variants of this semantics, we propose some new gradual argumentation semantics, and relate them to the family of the gradual semantics. This also suggests an approach for defeasible reasoning over a weighted argumentation graph, building on the proposed semantics.

This extended abstract reports about some work [24] investigating the relationships between the weighted conditional knowledge bases with typicality, under a fuzzy semantics, and gradual argumentation semantics [17, 36, 21, 22, 2, 5, 3, 46], and discusses some extension of this work in the direction of allowing defeasible reasoning over weighted argumentation graphs [26].

Argumentation is a reasoning approach which, in its different formulations and semantics, has been used in different contexts in the multi-agent setting, from social networks [42] to classification [4], and it is very relevant for decision making and for explanation [47]. The argumentation semantics are strongly related to other non-monotonic reasoning formalisms and semantics [20, 1].

Our starting point in this work is a preferential semantics for commonsense reasoning which has been proposed for a description logic with typicality. Preferential description logics have been studied in the last fifteen years to deal with inheritance with exceptions in ontologies, based on the idea of extending the language of Description Logics (DLs) by allowing for non-strict forms of inclusions, called *typicality or defeasible inclusions*, of the form  $\mathbf{T}(C) \sqsubseteq D$  (meaning “the typical  $C$ -elements are  $D$ -elements” or “normally  $C$ 's are  $D$ 's”), with different preferential semantics [28, 13] and closure constructions [15, 14, 29, 8, 44, 16, 27]. Such defeasible inclusions correspond to Kraus, Lehmann and Magidor (KLM) conditionals  $C \sim D$  [40, 41], and defeasible DLs inherit and extend some of the preferential semantics and closure constructions developed within preferential and conditional approaches to commonsense reasoning by Kraus, Lehmann and Magidor [40], Pearl [43], Lehmann [41], Geffner and Pearl [23], Benferhat et al. [7].

In previous work [33], a concept-wise multi-preferential semantics for weighted conditional knowledge bases (KBs) has been proposed to account for preferences with respect to different concepts, by allowing a set of typicality inclusions of the form  $\mathbf{T}(C) \sqsubseteq D$  with positive or

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negative weights, for some distinguished concepts  $C$ . The concept-wise multi-preferential semantics has been first introduced as a semantics for ranked DL knowledge bases [32], where conditionals are given a positive integer rank, and later extended to weighted conditional KBs, in the two-valued and in the fuzzy case, based on a different semantic closure construction in the spirit of Lehmann’s lexicographic closure [41] and Kern-Isberner’s  $c$ -representations [37, 38], but exploiting multiple preferences with respect to concepts.

The concept-wise multi-preferential semantics has been proven to have some desired properties from the knowledge representation point of view. In the two-valued case [32], it satisfies the KLM properties of a preferential consequence relation [40, 41], it allows to deal with specificity and irrelevance and avoids inheritance blocking or the “drowning problem” [43, 7], and deals with “ambiguity preservation” [23]. The plausibility of the concept-wise multi-preferential semantics has also been supported [30, 31] by showing that it is able to provide a logical interpretation to Kohonen’s Self-Organising Maps [39], which are psychologically and biologically plausible neural network models. In the fuzzy case, the KLM properties of non-monotonic entailment have been studied in [25], showing that most KLM postulates are satisfied, depending on their reformulation and on the choice of fuzzy combination functions. It has been shown [33] that (both in the two-valued and in the fuzzy case) the multi-preferential semantics allows to describe the behavior of Multilayer Perceptrons (MLPs), after training, in terms of a preferential interpretation which, in the fuzzy case, can be proven to be a model (in a logical sense) of the weighted KB which is associated to the neural network.

The relationships between preferential and conditional approaches to non-monotonic reasoning and argumentation semantics are strong. Let us just mention, the work by Geffner and Pearl on Conditional Entailment, whose proof theory is defined in terms of “arguments” [23].

To investigate the relationships between the fuzzy multi-preferential semantics for weighted conditionals and gradual argumentation semantics [17, 36, 21, 22, 2, 5, 3, 46], we have introduced a new notion of  $\varphi$ -coherent fuzzy multi-preferential semantics [24] for weighted conditionals, besides the previously introduced notions of coherent [33] and faithful [25] fuzzy multi-preferential semantics. For weighted argumentation graphs, where positive and negative weights can be associated to pairs of arguments, we have proposed three new gradual semantics (namely, a coherent, a faithful and a  $\varphi$ -coherent semantics) inspired by the fuzzy preferential semantics of weighted conditionals, and we have studied their relationships.

The relationship of the  $\varphi$ -coherent semantics with the family of gradual semantics studied by Amgoud and Doder [2] has also been investigated, by slightly extending their gradual argumentation framework to deal with positive and negative weights to capture the strength of supports and of attacks. A correspondence between the gradual semantics based on a specific evaluation method  $M^\varphi$  and  $\varphi$ -coherent labelings has been proven [26]. Differently from the Fuzzy Argumentation Frameworks by Janssen et al. [36], where an attack relation is a fuzzy binary relation over the set of arguments, here we have considered real-valued weights associated to pairs of arguments.

While in [33] a deep neural network (possibly containing cycles) is mapped to a weighted conditional knowledge base, a deep neural network can as well be seen as a weighted argumentation graph, with positive and negative weights, under the proposed semantics. In this view,  $\varphi$ -coherent labelings correspond to stationary states of the network (where each unit in the network is associated to an argument and the activation value of the unit can be regarded as

the weight of the corresponding argument). This is in agreement with previous work on the relationship between argumentation frameworks and neural networks, first investigated by Garcez, Gabbay and Lamb [19] and recently by Potyca [45].

The work by Garcez, et al. [19] combines value-based argumentation frameworks [6] and neural-symbolic learning systems by providing a translation from argumentation networks to neural networks with 3 layers (input, output layer and one hidden layer). This enables the accrual of arguments through learning as well as the parallel computation of arguments. The work by Potyca [45] considers a quantitative bipolar argumentation frameworks (QBAFs) similar to [5] and exploits an *influence function* based on the logistic function to define an MLP-based semantics  $\sigma_{MLP}$  for a QBAF. The paper studies convergence conditions both in the discrete and in the continuous case, as well as the semantic properties of MLP-based semantics, and proves that all properties for the QBAF semantics proposed in [2] are satisfied. On the other hand, as shown in [26], the  $\varphi$ -coherent model semantics fails to satisfy some of the properties in [2].

The strong relationships between the semantics for weighted conditionals and gradual argumentation semantics also leads to an approach for defeasible reasoning over a weighted argumentation graphs, building on  $\varphi$ -coherent labelings to evaluate conditional properties of the argumentation graph [26]. In essence, a multi-preferential model can be constructed over a (finite) set of  $\varphi$ -labelling  $\Sigma$ , which allows (fuzzy) conditional formulas over arguments to be validated by model checking over a preferential model. This would, for instance, allow to verify properties like: "does normally argument  $A_2$  follows from argument  $A_1$  with a degree greater than 0.7?" This query can be formalized by a fuzzy inclusion  $\mathbf{T}(A_1) \sqsubseteq A_2 > 0.7$ , similarly to those considered for weighted knowledge bases. This approach has been exploited for the verification of defeasible properties of Multilayer Perceptrons [34]. Whether this approach can be extended to the other gradual semantics, and under which conditions on the evaluation method, requires further investigation for future work.

Observe also that, in a weighted conditional knowledge base, the concepts  $C$  and  $D$  occurring in a typicality inclusion  $\mathbf{T}(C) \sqsubseteq D$  are not required to be concept names, but can be complex concepts. In particular, in the boolean fragment  $\mathcal{L}\mathcal{C}$  of  $\mathcal{AL}\mathcal{C}$ ,  $D$  can be any boolean combination of concept names. The correspondence between weighted attacks/supports  $(A_i, A_j)$  in the argumentation graph  $G$  and weighted conditionals  $\mathbf{T}(A_i) \sqsubseteq A_j$  suggests a possible generalization of the structure of the weighted argumentation graph by allowing attacks/supports by a boolean combination of arguments. The labelling of arguments in the set  $[0, 1]$  can indeed be extended to boolean combinations of arguments using the fuzzy combination functions, as for boolean concepts in the conditional semantics (e.g., by letting  $\sigma(A_1 \wedge A_2) = \min\{\sigma(A_1), \sigma(A_2)\}$ , using the minimum t-norm as in Zadeh fuzzy logic). This also relates to the work considering "sets of attacking (resp. supporting) arguments"; i.e., several argument together attacking (or supporting) an argument. Indeed, for gradual semantics, the sets of attacking arguments framework (SETAF) has been studied by Yun and Vesic [46], by considering "the force of the set of attacking (resp. supporting) arguments to be the force of the weakest argument in the set" [46]. This would correspond to interpret the set of arguments as a conjunction, using minimum t-norm.

The correspondence between Abstract Dialectical Frameworks [12] and Nonmonotonic Conditional Logics has been studied by Heyninck, Kern-Isberner and Thimm [35], with respect to the two-valued models, the stable, the preferred semantics and the grounded semantics of ADFs. Whether the coherent/faithful/ $\varphi$ -coherent semantics developed in the paper for weighted

argumentation (as well as their two-valued and many-valued variants) can be reformulated for a (weighted) Abstract Dialectical Frameworks, and which are the relationships with the work in [35], also requires investigation for future work.

Undecidability results for fuzzy description logics with general inclusion axioms (e.g., by Cerami and Straccia [18] and by Borgwardt and Peñaloza [9]) motivate restricting the logics to finitely valued semantics [10], and the investigation of decidable approximations of fuzzy multi-preferential entailment, under the different semantics. An ASP approach for reasoning under finitely multi-valued fuzzy semantics for weighted conditional knowledge bases has been proposed in [34], by exploiting *asprin* [11] for defeasible reasoning through the computation of preferred answer sets. As a proof of concept, this approach has been experimented for checking properties of some trained Multilayer Perceptrons. A similar investigation of the two-valued and many-valued case might also be of interest for the semantics of weighted argumentation graphs introduced in this work.

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