

Some Techniques for Simplifying the Solution of Linear Optimization Problems in Project Management

Serhii Chernov^a, Serhii Titov^a, Liudmyla Chernova^a, Liubava Chernova^a, Antonina Trushliakova^a, Natalia Kunanets^b and Serhii Lupenko^c

^a Admiral Makarov National University of Shipbuilding, 9, Heroiv Ukrainy ave., Mykolaiv, 54025, Ukraine

^b Lviv Polytechnic National University, 12, St. Bandera str., Lviv, 79013, Ukraine

^c Opole University of Technology, Mikołajczyka 16, Opole, Poland

Abstract

Management of IT projects becomes a topical and important task of each company and its personnel makes all possible efforts and applies the full scope of knowledge and experience to select efficient methods and tools to form a project with an objective to achieve the best result. The authors propose to use modern mathematic models of project management processes description reducing them to linear optimization (LO) problems containing standard steps of an algorithm. The program library of such known software packages as Mathematica[®], Maple[®], MathCad[®] is used for study and solution of LO problems. This allows solving complicated types of combinatory integer optimization problems and using solution of big-dimension problems, which contributes to finding the most efficient way of an IT project implementation. The methods of exact or approximate solution of such problems are used with account taken of their belonging to so-called P or NP class problems (algorithms of polynomial or exponential realization of the solution). The modern combinatory computer methods of LB problems solution help to develop algorithms for receiving an approximate solution with guaranteed target value assessment that is important for the project implementation. The paper proposed an approach to simplifying a mathematic model prepared for computer realization. This approach contributes to developing efficient and improving existing algorithms of preparing for computer calculations, which significantly saves the time of computer calculation and reduces the scope of hardware requirements to computer. This paper is concerned with construction of a chain of efficient algorithms to simplify the primary mathematic model of problem and realization its computer-aided calculation. The research objective consists in construction of efficient algorithms and common principles of preparing for computer-aided solution of LO problems that will contribute to formation of efficient alternatives to IT projects implementation.

Keywords¹

IT projects management, project implementation, products, services, linear optimization, polyhedron, target function, simplex method, basis vectors, primary plan, reference plan, polyhedron apex, reduction, duality.

1. Introduction

An essential part of the modern business in the whole world is project-oriented. In Ukraine, it is approaching 50% and this percentage value is much higher in the IT industry. The most IT companies are oriented at development of principally new products or services. Projects are aimed at

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EMAIL: 19chsk56@gmail.com (Sergiy Chernov); ss1-ss10@ukr.net (Sergiy Titov); lyudmylachernova@gmail.com (Ludmila Chernova); 19chls92@gmail.com (Lubava Chernova); holdon000@gmail.com (Antonina Trushliakova); nek.lviv@gmail.com (Nataliia Kunanets); serhii.lupenko@gmail.com (Serhii Lupenko).

ORCID: 0000-0002-9069-0409 (Sergiy Chernov); 0000-0001-8772-9889 (Sergiy Titov); 0000-0002-0666-0742 (Ludmila Chernova); 0000-0001-7846-9034 (Lubava Chernova); 0000-0003-1958-3472 (Antonina Trushliakova); 0000-0003-3007-2462 (Nataliia Kunanets); 0000-0002-6559-0721 (Serhii Lupenko).

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implementation of clearly formulated purposes. Efficient management of these projects becomes a topical and important task of each company and its personnel makes all possible efforts and applies the full scope of knowledge and experience for selecting efficient methods and tools to form a project for achieving the best result. Modern mathematic models of project management processes description will contribute to solution of these tasks subject to reducing to linear optimization (LO) problems containing standard steps of an algorithm.

For study and solution of these problems, researchers use a subprogram library of such known software packages as Mathematica®, Maple®, MathCad®. Computer-aided calculations help to solve complicated types of combinatory linear integer optimization problems and to solve big-dimension problems that is important in project management procedures.

For solving such problems, it is of great importance to improve the mathematic model itself which is prepared for beginning of computer-assisted realization. This practical reasonability for solving a wide LO problems range promotes development of new efficient and improvement of already existing algorithms of preparing a model for computer calculation. Application of these algorithms will allow for reduction of time for computer calculation and lowering of the level of hardware requirements to computer.

The paper is concerned with construction of a chain of efficient algorithms to simplify the primary mathematic model of problem and realization its computer-aided calculation, the use of which will contribute in its turn to achieving the best result while implementing a project.

2. Research paper study and problem statement

In most cases, linear optimization uses canonical classic algorithms for solution of problems [1,2,3,4,5,6]. Typical problems contain standard algorithm steps: obtaining the primary reference plan, construction of a chain of reference plans, evaluation of their optimality, improvement of the plan and of the target function value [7,8,9,10]. Thus the linear programming algorithm finds a point on the polygon where this function acquires the biggest or the smallest value. However, this algorithm requirement may lead in some linear optimization problems to appearance of too many iterations if we have a transition not to the neighbor apex, but to another one that is determined by additional requirements. Each of the reference plans has a set of linearly independent basis vectors. Transition to the new basis that is a move along the edge to the neighbor polyhedron apex is carried out within a rigorous algorithm. As per the simplex method algorithm theory, such transition is made towards the best change of the target function values [11,12]. In multiple cases, mathematic models of active systems management are interpreted in the form of linear optimization problems [1,2,3,10].

Simplification algorithms provide an efficient method of searching for optimization problem solution. If we project a multidimensional process onto a two-dimensional plane, such method will enable visual representation of a set of problem solutions in graphic form while forming efficient systems of a project management at each stage of its implementation, particularly while calculating the budget and optimizing the cost[15–28]. While forming the project implementation stages, the number of alternatives requiring consideration increases extremely fast with augmentation of the project dimension. In this case, realization of exhaustive search variants, even with use of high-speed computers, requires significant input of time and resources. This research has proposed a method of simplifying this procedure as combinatorial solution of a discrete optimization problem. It is based on decomposition of the system representing a system of constraints of a five-dimensional initial problem into the two-dimensional coordinate plane. Such method allows obtaining a simple system of graphic solutions to a complicated linear discrete optimization problem. From the practical point of view, the method proposed allows simplifying the calculation complexity of optimization problems belonging to this class.

Solution of linear optimization problems is based on algorithm of the classic or a common simplex method. It consists in intellectual iteration over polyhedron apexes Ω_i (allowable area of optimization problem) that enables the movement over the reference plans till finding the optimum solution to the problem. The plan or an apex of polyhedron Ω_i is specified by a system n of basis vectors $\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n$. The number of possible apexes of polyhedron equals to the number of combinations

C_n^m (n – problem measurability, and $m = \text{rang}(\Omega_i)$). Real linear optimization problems that interpret models of management are characterized by big values of m . In view of this, we had to develop an algorithm ensuring ordered iteration over angular points of the polyhedron. Such a method was developed by George Danzig, American mathematician [1, 2] and is called simplex method. It allows obtaining the optimum optimization problem solution from the known primary reference plan X_0 , within a finite number of steps. Each iteration step of a simplex method corresponds to a new plan improving the target function value, and therefore, can contribute to selecting the best variant of the budget or of the project implementation approach. The algorithmic process continues until finding the optimum value of target function or the absence of optimization problem solution.

The number of simplex method iterations is determined by the primary reference plan X_0 and the number of angular points Ω_i . As there are several “ways” of transition from X_0 to the optimum X_{opt} , we encounter the need to find the shortest (in terms of the number of apexes) “way” of iteration. Now there are not any publications with such assessments and their correlation to the classic simplex method algorithm.

3. The purpose and objectives of the research

The research objective provides for use and development of efficient algorithms and preparing mathematic models of the linear optimization theory with further realization of their solution with the help of computer, which will contribute to formation of efficient IT projects implementation alternatives. For achievement of the objective stated, the following tasks were specified: provide the general problem statement for construction of efficient algorithms; analyze model examples illustrating the efficiency of algorithms operation at the stage of computer calculation.

4. LO-aided construction of recommendation systems

An up-to-date approach to project management is impossible without using modern information technology. A convenient tool for developing a project is provided by recommendation systems as software products for generating recommendations to users based on analysis of alternatives. Recommendation systems are based on the algorithms aimed at proposing respective alternatives to users. The recommending systems are of critical importance in the project management as they can bring a huge profit when they are efficient, and they can be a competitive advantage at the same time.

There is a rather wide set of methods to formulate recommendations, but all of them have their merits and demerits. That is exactly why researches in this field keep being topical. The quality of recommendations provided by the system depends on multiple criteria that are normally evaluate the productivity and reflect imagination of the system behavior. Most frequently, developers use to evaluate the accuracy of recommendations with the help of metrics measuring the ability of recommendation system to exactly present the set of known advantages. The accuracy is measured by reserving a part of the set of these ratings in the form of a set of known preferences and using other ratings for recommendation system’s study and recommendations stating. The advantages can be characterized by some metrics, particularly by such as the accuracy. For determining the assessment providing the accuracy of recommendations, we use an algorithm of analyzing a certain set of data. In this case, we should take into account that the accuracy metrics just provide a rough approximation to the image of a recommendation system productivity and that they are restricted to several methods. They give quite poor information on behavior of items with unknown preferences. And when the set of elements characterizing the preferences is small, only low values can be normally obtained as it is hardly possible that recommended elements are included in the set of known preferences. The accuracy cannot reflect the prospects of behavior for the items not included in the set of known preferences, therefore, the conclusions stating that one system relatively prevails over the other may not be accurate.

Recommendation systems gather various information on a person using several methods under which the recommended systems are classified. Thus, the first type provides for evident data

acquisition. In this case, the user provides necessary information for stating recommendations. Should the user refuse to provide information, the following procedure is applied. The second type is based on latent data acquisition. This approach is similar to spying as the necessary data is fixed by a special software and subjected to further processing. The said software shall recognize purchases and assessments on websites, gather information on browsing and comments. This procedure is to be used in compliance with ethic rules and personal data protection requirements. There are also such types of recommendation systems that are determined by combining different approaches.

Using other recommendation methods with content filtering is a rather frequent case as well. For instance, a simple Bayes classifier, various methods of machine-aided education including neural networks, decision tree and clustering. As opposed to collaborative filtering methods based only on the interaction between the elements and the users, content-based approaches use additional information on the users and elements.

In a recommendation system developed, it is reasonable to use linear optimization problems containing standard algorithm steps (Figure 1).

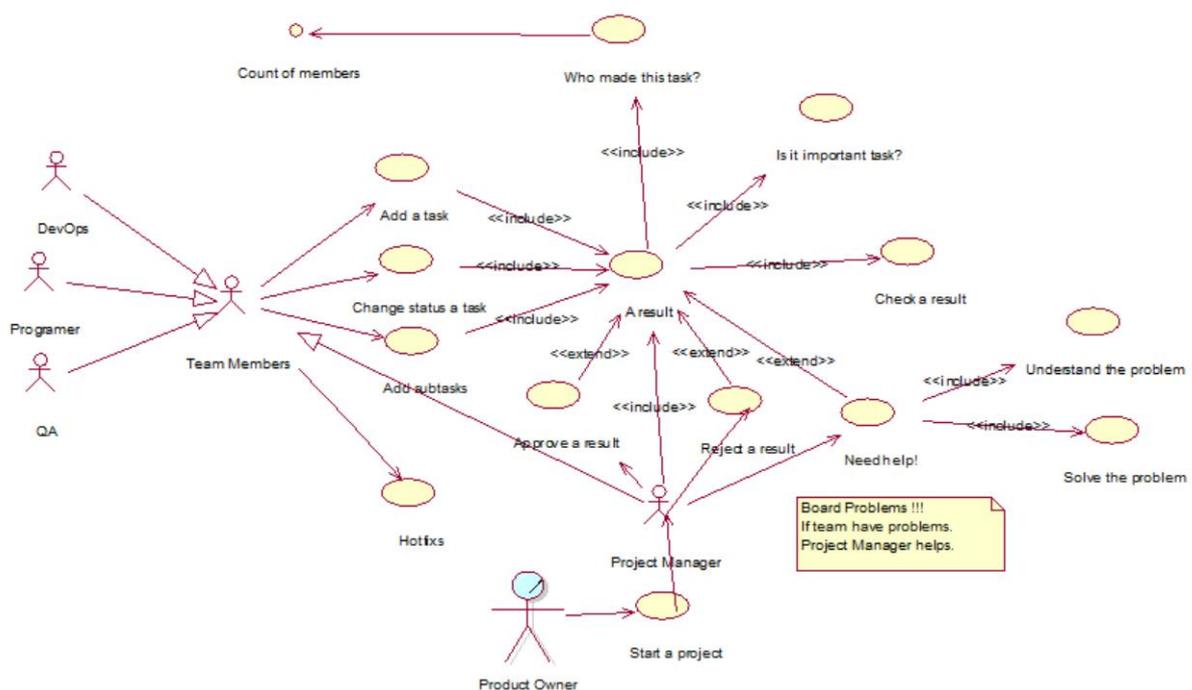


Figure 1: Diagram of options in implementing a project on creation of a recommendation system

In most cases, recommendation systems development consists in improving recommendation algorithms. The purpose of this progress is to provide project managers with the most exact recommendations meeting their needs at particular moment. For achieving this purpose, mathematic algorithms constituting the basis of recommendation services have to be permanently improved.

The process of study by the algorithm may repeat several times for avoidance of mistakes connected with using a small subset instead of the whole initial data. After creation of a training and test samplings, the model of recommendation system is to be adjusted with subsequent evaluation of its accuracy. Then a new couple of samplings is created and the training and testing process is repeated k times. Finally, we can use the averaged value of k models. This approach is called cross-validation. It can be performed by several methods. In the case with repeated random samplings, subsets for training are randomly selected k times consecutively. The other method may consist in selection of n subsets of initial data. For optimizing productivity of providing recommendations, it is important to normalize assessment to calculation of a similarity matrix. This can be achieved by calculation of the basic forecast in which deviations of the user and of the element are encapsulated.

The prototype architecture has been simplified as far as possible. It is not resistant to failures, it cannot support a big number of users. The main task of the prototype is to provide potential users the possibility to evaluate the utility of recommendation system based on isochrones, as well as to

evaluate reasonability of developing a more complicated architecture and the prospects of technology. The prototype can only support the desktop version of the web application.

While implementing the system for recommending alternatives in project development, let us analyze its functions. The set of functional possibilities is based on the expected scenario of the recommendation system usage.

The user's interface is realized through a mobile application. In this way, we have a realized mobile application for iOS, based on the fact that today it is one of the two most popular mobile operation systems.

The expected scenario of using the developed application consists of several stages. In the beginning, users need to authorize themselves in the iOS application, then, having got the access to the system, they can enter information as to alternatives selecting them like from the general process catalogue of the project actions.

Recommendations are to be based on customer's requirements, i.e. in our case on scenarios of the previously implemented projects on creation of software products.

Therefore, we can formalize and distinguish the basic functions of the recommendation system:

- providing user with a list of recommended alternatives
- carrying out mathematic calculations
- possibility of authorization and authentication of user in the system
- viewing the user's access history
- taking account of recommendations previously given.

Having determined the functions, we also need to set requirements to the system. It is planned first that the system will consist of three modules, namely:

- recommendations module;
- server part;
- mobile application.

Based on the above, we have formulated separate requirements to each part of the system. Let us analyze the requirements to the recommendations module. The main functions of this module consist in generating recommended alternatives based on calculations made.

Therefore, the basic requirements are as follows:

- the module is to have access to the data base of information entered by the user, about possibilities of using the resources;
- recommended alternatives must be based on the history of implementing similar projects;
- calculation of recommendations is to take up to 5 seconds;
- interaction with the recommended module is to be based on REST API;
- the module must have possibility of being easily scaled horizontally.

5. Module of calculations. Reduction in common LO problems

Let us consider the task of optimization while planning project resources as a linear discrete optimization problem presented in canonical form:

$$\begin{aligned} W_j &= CX \rightarrow \max \\ \Omega_j : AX &= B, \\ X &\geq 0, \end{aligned} \tag{1}$$

where the rank of the system of constraints coefficient matrix is equal to $\text{rang } A = m$.

In this case, solving the system by Gauss-Jordan method with arbitrary basis combination of variables, we obtain a projection of n -dimensional initial problem to $(n-m)$ -dimensional space. In case $n - m = 2$, we have projecting to the two-dimensional plane. Solution of the most problems related to selection tasks in project management consists in construction of a mathematic model reflecting relationship of the most important project components at all stages of its implementation. Let us consider a model example of solving a five-dimensional linear optimization problem based on such projecting of a multidimensional space onto the two-dimensional one.

5.1 Model example No. 1

The linear optimization problem is to be solved by method of projecting onto two-dimensional coordinate planes

$$\begin{aligned}
 &W_1 = 4x_1 + 14x_2 + 2x_3 - 100 \rightarrow \max, \\
 \Omega_1 : &\begin{cases} 5x_1 + 11x_2 + x_3 + x_4 + x_5 = 118, \\ x_1 - 5x_2 - x_3 + 2x_5 = -28, \\ 7x_1 + 6x_2 + x_4 + 5x_5 = 101, \\ x_j \geq 0, \quad j = 1, 2, 3, 4, 5. \end{cases} \quad (2)
 \end{aligned}$$

Solution.

The method of projection or simplification of a linear optimization problem (LOP) consists in transition from the canonical form of linear optimization problem presentation to the standard one. This transition is to be effected with solution of the system by Gauss-Jordan method. Let us select arbitrary variables to be the basic ones. In the beginning, we take these three – x_3, x_4, x_5 . As a result of elimination, we obtain the solved system

$$\begin{cases} x_1 + x_5 = 11, \\ x_1 + 5x_2 + x_3 = 50, \\ 5x_1 + 6x_2 + x_4 = 79. \end{cases} \quad (3)$$

Dropping the nonnegative basic variables, we can project the initial multidimensional problem onto a two-dimensional coordinate plane Ox_1x_2 :

$$\begin{aligned}
 &W_1 = 2x_1 + 4x_2 \rightarrow \max, \\
 \Omega_1^{Ox_1x_2} : &\begin{cases} x_1 \leq 11, \\ x_1 + 5x_2 \leq 50, \\ 5x_1 + 6x_2 \leq 79, \\ x_1 \geq 0, x_2 \geq 0. \end{cases} \quad (4)
 \end{aligned}$$

Graphic solution is shown on Figure 2.

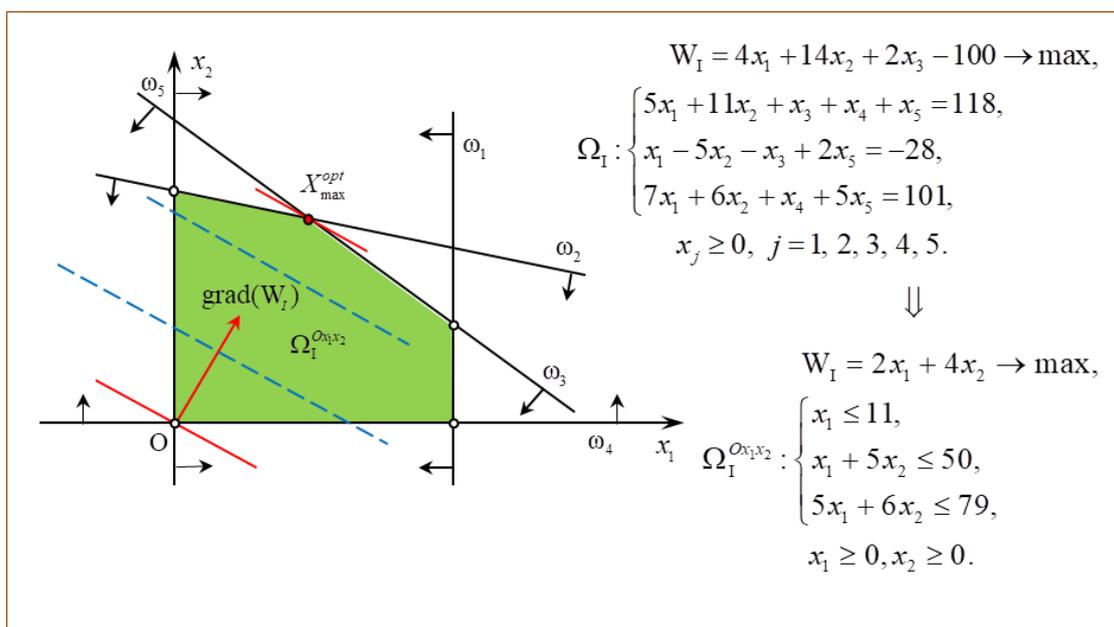


Figure 2: Graphic solution method. Projection onto Ox_1x_2

We find the optimum apex coordinates from the system solution

$$X_{\max}^{\text{opt}} : \omega_2 \times \omega_3 \Leftrightarrow \begin{cases} x_1 + 5x_2 = 50, \\ 5x_1 + 6x_2 = 79, \end{cases} \Leftrightarrow \begin{cases} x_1 = 5, \\ x_2 = 9. \end{cases} \quad (5)$$

The optimum solution of the initial problem is calculated with (2):

$$X_{\max}^{\text{opt}} = [5, 9, 0, 0, 6] \quad (6)$$

The maximum target function value will be $W_1^{\max} = 46$.

5.2 Acceleration of a linear optimization problems solution convergence

In most cases, solution of linear optimization problems is searched for by the simplex method. However, this classic algorithm of linear optimization problems solution may create additional iterations in the immediate process of calculation. If we break some components of the standard simplex method algorithm, we can accelerate the convergence of simplex calculation – reduce the number of simplex tables. For acceleration of the simplex method convergence, it is proposed to deviate from the canonical algorithm. It is required to choose not the neighbor apex as the next problem plan, but the verified apex selected according to evaluation of the biggest and the smallest target function values. Let us consider the general approach to a linear optimization problem solution by classic simplex method algorithm [1, 2, 3].

The main idea of using the simplex method algorithm is sequential iteration over allowable reference plans. One vector is excluded from and another included to the basis by the Gauss-Jordan method [13]. Subject to compliance with these criteria, we have to build a chain. The beginning of the chain is located at the starting apex \mathbf{X}_0 of polyhedron Ω_i and corresponds to the first simplex table of calculation. Moving to the next reference plan \mathbf{X}_1 by following the classic algorithm corresponds to transition to the neighbor apex. Actually, each table is a numeric description of apexes Ω_i . The process is to be continued till finding the optimum apex \mathbf{X}_{opt} or confirming its absence. At the arbitrary step of calculation by following the common simplex method algorithm, we have the possibility to move not to the neighbor apex, but to the arbitrary apex located around the optimum apex. Such an apex can be selected based on multiple evaluation methods, e.g. the half-interval method. For this selection, the alternative chain of simplex calculation may have a much smaller number of iterations. Let us consider a model example of a two-dimensional linear optimization problem solution to confirm this case, first by following the standard procedure and then by breaking the rule of basis vectors combination selection.

5.3 Model example No. 2

Solution of the linear optimization problem for selecting a budget variant is given by Formula (7):

$$\begin{aligned} & W_1 = 3x_1 + 4x_2 \rightarrow \max, \\ & \Omega_1 : \begin{cases} -x_1 + 2x_2 \leq 12, \\ x_1 + 4x_2 \leq 36, \\ 2x_1 + 3x_2 \leq 37, \\ 4x_1 - 5x_2 \leq 19, \\ x_1 - 6x_2 \leq 0, \\ x_1 \geq 0, x_2 \geq 0. \end{cases} \quad (7) \end{aligned}$$

Solution.

Primary reference plan $\mathbf{X}_0 = [0, 0, 12, 36, 37, 19, 0] \in \Omega_1$.

We make an initial simplex table (Table 1).

Table 1
Initial simplex table

<i>Basis</i>	<i>C</i>	<i>B</i>	a_1	a_2	a_3	a_4	a_5	a_6	a_7	$\{b/a_{ij}\}$	X_i
			3	4	0	0	0	0	0		
a_3	0	12	-1	2	1	0	0	0	0	6	
a_4	0	36	1	4	0	1	0	0	0	9	
a_5	0	37	2	3	0	0	1	0	0	37/3	X_0
a_6	0	19	4	-5	0	0	0	1	0		
a_7	0	0	1	-6	0	0	0	0	1		
Δ_j	$W_1(\mathbf{X}_0) =$	0	-3	-4	0	0	0	0	0		

After further respective calculations, we obtain a simplex table at step five (Table 2).

Table 2
Simplex table

<i>Basis</i>	<i>C</i>	<i>B</i>	a_1	a_2	a_3	a_4	a_5	a_6	a_7	$\{b/a_{ij}\}$	X_i
			3	4	0	0	0	0	0		
a_2	4	5	0	1	0	0	2/11	-1/11	0		
a_1	3	11	1	0	0	0	5/22	3/22	0		
a_3	0	13	0	0	1	0	-3/22	0	0		X_{\max}
a_4	0	5	0	0	0	1	-21/22	5/22	0		
a_7	0	19	0	0	0	0	19/22	-15/22	1		
Δ_j	$W_1(\mathbf{X}_{\max}) = 53$	0	0	0	0	0	31/22	1/22	0		

All estimates are nonnegative: $\Delta_j \geq 0$. This means that we have found the optimum solution.

$$\mathbf{X}_{\max} = [11, 5], W_1(\mathbf{X}_{\max}) = 53. \quad (8)$$

Therefore, the calculation within the classic simplex method contains the following chain $\mathbf{X}_0 \rightarrow \mathbf{X}_1 \rightarrow \mathbf{X}_2 \rightarrow \mathbf{X}_3 \rightarrow \mathbf{X}_{\max}$ of polyhedron Ω_1 apexes sequential iteration.

Let us prove that breaking the simplex method algorithm can significantly reduce the length of calculation chain. Among negative estimates, we select not the smallest as at the common simplex method, but the biggest one. Respective calculations are given in Table 3. For reducing the number of iterations, we break this algorithm and select not the smallest, but the biggest negative estimate $\Delta_1 = -3$ in the initial simplex table. Further calculations are given in Table 3. The number of simplex tables decreased from five to three: $\mathbf{X}_0 \rightarrow \mathbf{X}_4 \rightarrow \mathbf{X}_{\max}$.

Table 3
Respective calculations

<i>Basis</i>	<i>C</i>	<i>B</i>	a_1	a_2	a_3	a_4	a_5	a_6	a_7	$\{b/a_{ij}\}$	X_i
			3	4	0	0	0	0	0		
a_3	0	12	-1	2	1	0	0	0	0	□	
a_4	0	36	1	4	0	1	0	0	0	□	

a_5	0	37	2	3	0	0	1	0	0	<input type="checkbox"/>	X_0
a_6	0	19	4	-5	0	0	0	1	0	<input type="checkbox"/>	
a_7	0	0	1	-6	0	0	0	0	1	<input type="checkbox"/>	
Δ_j	$W_1(X_0) = 0$	-3	-4	0	0	0	0	0	0	<input type="checkbox"/>	

a_3	0	13	0	0	1	0	-3/22	7/22	0	<input type="checkbox"/>	X_{\max}
a_4	0	5	0	0	0	1	-21/22	5/22	0	<input type="checkbox"/>	
a_7	0	19	0	0	0	0	19/22	-15/22	1	<input type="checkbox"/>	
a_2	4	5	0	1	0	0	2/11	-1/11	0	<input type="checkbox"/>	
a_1	3	11	1	0	0	0	5/22	3/22	0	<input type="checkbox"/>	
Δ_j	$W_1(X_{\max}) = 53$	0	0	0	0	0	31/22	1/22	0	<input type="checkbox"/>	

The calculation chain got significantly shorter - $X_0 \rightarrow X_5 \rightarrow X_{\max}$.

Therefore, for solving the problem by classic algorithm, we need to create five simplex tables.

6. Research findings

The paper has considered examples of solving project management problems by improving the efficiency of algorithms for preparing to computer calculations of an optimization problem based on problem reduction and the method of breaking the standard simplex algorithm. These examples confirm reasonability of such methods in solving linear optimization problems. From the practical point of view, this approach allows simplifying the complexity of initial problems in this class. Simplification of initial problems solution has been shown based on comparative solutions of model problems.

7. Conclusions

The scientific result obtained allows arriving at the conclusion that generally it is reasonable to search for approaches to breaking the algorithm of standard algorithmic diagrams formed by the present moment. The applied value of the approach proposed consists in using the obtained scientific result for creating the possibility of improving canonical methods of optimization problem solution and, respectively, simplification of computer calculation with use of libraries of standard subprograms of known mathematic packages. Therefore, it has been proved that in project management procedures, it is reasonable to use linear optimization problems for which it makes sense to search for more efficient algorithms to prepare LO problems for computer calculation. The example of project management problem solution showed that the proposed approach can essentially simplify the solution of LO problems.

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