

Logics for Reasoning about Auctions

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Abstract

In this paper, I summarize the results obtained in my thesis, whose aim was to investigate the use of formal methods for the specification, design, and evaluation of mechanisms, with a focus on auctions. We address such challenges by introducing logic-based approaches for representing and designing auction-based markets with strategic players. Firstly, for providing a foundation for general and automated auction playing in MAS, we propose a framework for representing auctions, denoted Auction Description Language (ADL). ADL addresses important dimensions of auction-based markets and is general enough to represent most auction settings. Second, we propose a novel approach for reasoning and designing new auctions (and, in general, mechanisms) based on formal methods. Such an approach for Automated Mechanism Design aims to automatically generate mechanisms from their specifications and verify them in relation to target economical properties. We demonstrate how this approach can express key concepts from Economic Theory (including Nash equilibrium and strategyproofness) and how it enables automatically generating optimal mechanisms from a quantitative logical specification.

Keywords

Strategic Reasoning, Auctions, Formal Methods, Multi-Agent Systems, Mechanism Design

1. Introduction

An auction is a popular mechanism that aggregates participants' bids into a social decision, usually expressed in terms of allocations and payments. Automated agents are widely used in auction-based markets but software agents are usually designed to act on a specific context, which prevents them from switching between different kinds of markets. For doing so, they should be able to "understand" the auction rules and reason about their own valuations and also about other players private information valuations. This limitation inspires the development of a lightweight logic-based language for representing the rules of an auction market.

More than describing, the design and evaluation of new auctions (and, more generally, mechanisms) is a central problem in multiagent settings [1]. In such setting, we need to be able to aggregate individual preferences, which are conflicting when agents are self-interested. More importantly, the mechanism should choose a socially desirable outcome and reach an equilibrium despite the fact that agents can lie about their preferences [2]. Although logic-based languages have been widely used for verification [3] and synthesis [4]

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of Multi-Agent Systems (MAS), the use of formal methods for reasoning about auctions under strategic behavior as well as automated mechanism design has not been much explored yet. An advantage in adopting such perspective lies in the high expressivity and generality of logics for strategic reasoning [5]. Moreover, by relying on precise semantics, formal methods provide tools for rigorously analysing the correctness of systems, which is important to improve trust in mechanisms generated by machines. The problem of formally reasoning about mechanisms requires to consider quantitative information (e.g., utilities and payments), private information about the participant's preferences, and complex strategic concepts (such as strategy dominance and equilibria).

My thesis [6] addressed such challenges by introducing logic-based approaches for representing and designing auction-based markets with strategic players. Our motivation is two fold: first, we aim to provide a foundation for general and automated auction playing in MAS, by establishing a logical framework to create a good balance between expressive power and computational efficiency. Second, we propose a novel approach based on formal methods for (i) reasoning about auctions under strategic behavior and (ii) Automated Mechanism Design. Such approach aims to automatically generate auctions from their specifications and verify them in relation to target economical properties.

2. Background and Context

Representation of Auctions There are numerous variants of auctions depending on the parameters considered, including the number of distinct items and copies as well as the number of sellers and buyers [7, 8]. For a fixed set of parameters, the protocol, i.e., the bidding, payment and allocation rules, may also differ. Building intelligent agents that can switch between different auctions and process their rules is a key issue for building automated auction-based marketplaces. In this scenario, the auctioneer should at first allow participants to express their preferences and second describe the rules governing the market. In relation to the formal representation of auction rules, we recall the descriptive auction language [9], which allows the specification of auctions by allowing players to bid using the XOR language. Rule-based approaches have also been used for representing single-dimensional auctions [10] and negotiation protocols [11]. Similarly, negotiation protocols have been handled with meta-languages [12], the Extensible Markup Language [13] and rule-markup languages [14]. Since the above languages lack a precise semantics, Wooldridge and Parsons (2000, 2000) motivate and compare the use of different logical languages for specifying negotiation protocols. In the context of General Game Playing (GGP), the Game Description Language (GDL) was designed for specifying game rules while maintaining a tractable complexity [17]. This language has been extended to deal with integer values [18] and epistemic operators [19]. The use of languages inspired on GDL for describing market-based protocols have been studied in the context of negotiation [20, 21] and single-item markets [22]. Such approaches lack a clear link between the language, the mechanism formalization and the agents' preferences, which is a key aspect for enabling reasoning about auctions.

Automated Mechanism Design Designing an auction in such manner to ensure features of the outcome alongside with a desirable behavior of the participants is a key challenge in Economics. In fact, this problem is known as Mechanism Design: the formulation of game-like systems whose equilibria satisfy some desired properties, usually expressed in terms of incentive, utility or social welfare. Traditionally, mechanisms have been formulated and verified by human specialists, who use their knowledge and experience for defining the game rules. Creating and verifying mechanisms which will be played by strategic agents can be a very difficult and time-consuming task. Sandholm (2003) introduced Automated Mechanism Design (AMD), whose goal is to automatically create mechanisms for solving a specific preference aggregation problem. AMD is usually handled as an optimization and domain-oriented problem. Most solutions used on the literature are based on machine learning, which include, for instance, neural networks [24, 25] and statistical techniques [26]. A number of works explore computed-aided verification of auctions [27, 28, 29], where the process is assisted by a reasoner. In the context of fully-automatic verification, Pauly and Wooldridge (2003) and Wooldridge et al. (2007) advocate the use of Alternating-time Temporal Logic (ATL) [32] to reason about mechanisms. The main limitation in these works is the purely qualitative setting and the impossibility of expressing key strategic concepts such as dominance in the logic.

Logics for Strategic Reasoning This work is also related to the long-established logical approach to systems verification [3] and synthesis [4]. In the recent years much progress has been made in the field of logics for strategic reasoning. Pioneering work includes the Alternating-time Temporal Logic ATL [32], which uses coalition modalities to specify strategic abilities of groups of agents, an important notion in Mechanism Design. The Strategy Logic (SL) [33, 34] subsumes ATL and considers explicit manipulation of strategies. A recent quantitative extension of SL, denoted $SL[\mathcal{F}]$ [35], introduces values in models and functions in the language, enabling the reasoning about key game-theoretic concepts such as utilities and preferences. Several works have also considered extensions of SL with imperfect information [36, 37, 38], which is also an important feature when modeling auctions with private valuations.

3. Contribution

We addressed the problem of modeling and analyzing auction mechanisms for MAS using logics and strategic reasoning. First, we provided a framework for representing auctions, denoted Auction Description Language (ADL) [39, 40, 41]. ADL addresses important dimensions of auction-based markets and is general enough to represent most auction settings. We illustrated the generality of ADL by modeling a number of representative auctions. We demonstrated how this language can be used for the automated verification of direct mechanisms and for automatically checking well-formedness of auction descriptions. We then extended ADL with knowledge operators and an action modality [42, 43] (denoted Epistemic ADL, or simply ADLK) for providing a ground for the design of general auction players and characterizing their rational behavior when reasoning about the effect of

actions and other players rationality. We show that the model-checking problem for ADL-formulas belongs to Ptime when it involves functions that be computed in polynomial time. By the other hand, we show that the model-checking problem for ADLK is in Exptime.

In relation to Automated Mechanism Design, we investigate logical frameworks for strategic reasoning about mechanisms. We first propose a new variant of Strategy Logic with quantitative features, imperfect information and epistemic operators, denoted $SLK[\mathcal{F}]$ [44]. We demonstrated how $SLK[\mathcal{F}]$ can express the implementation of social choice functions and be used for automatically verifying a number of important concepts and properties often required in auctions, or more generally in mechanism design. We also show how we can express properties relating agents' revenues with respect to their beliefs about other agents' preferences. We showed that verifying a mechanism in relation to classical properties boils-down to model checking a $SLK[\mathcal{F}]$ formula and we prove it can be done in Pspace for memoryless strategies.

We then introduced a quantitative semantics for SL with natural strategies and imperfect information (denoted $NatSL[\mathcal{F}]$) [45], which provides a new perspective for formal verification and design of novel mechanisms based on the complexity of strategies. We show how to model popular strategies for repeated keyword auctions using $NatSL[\mathcal{F}]$ and prove properties pertaining to this game; We proved that the model-checking problem for $NatSL[\mathcal{F}]$ is Pspace-complete and that $NatSL[\mathcal{F}]$ has incomparable distinguishing and expressive power to $SL[\mathcal{F}]$. Finally, we offered a novel perspective on the design of mechanisms by rephrasing the AMD problem in terms of synthesis from $SL[\mathcal{F}]$ specifications [46, 47]. This approach enables automatically generating optimal mechanisms from a quantitative logical specification, which may include not only game rules but also requirements over the strategic behavior of participants and quality of the outcome. We solved the synthesis problem for $SL[\mathcal{F}]$ by investigating the related satisfiability problem in two cases: action-bounded and turn-based mechanisms.

4. Perspectives and Future Work

In a recent work [48], we considered the use of the probabilistic extension of SL [49] to handle stochastic features often present in auctions. Going from deterministic setting to a more general and probabilistic one is challenging due to several aspects. First, the wide and heterogeneous range of settings considered in the literature obscures the path for a general and formal approach to verification. The setting may consider deterministic or randomized mechanisms, incomplete information about agents' types (Bayesian mechanisms), mixed or pure strategies and iterative protocols (indirect mechanisms). Second, considering Bayesian mechanisms brings out different methods for evaluating a mechanism according to the time-line for revealing the incomplete information as the game is run.

We studied the verification of mechanisms under memoryless combinatorial strategies and Natural Strategies with bounded recall. This setting is enough to capture many kinds of auctions (such as one-shot or English auctions) where memoryless strategies are sufficient to represent the bidders behaviour since all the relevant information can be

encoded in a state. However, when participating in repeated auctions, agents could gather information from other agents behaviour and act based on what happened in previous instances of the game. An interesting direction is, then, to investigate the use of strategies with recall for learning other players' valuations based on their behavior. For such situations we can study the model-checking problem for $\text{SLK}[\mathcal{F}]$ with memoryful strategies. In the qualitative setting already, imperfect information yields undecidability, but known decidable cases exist [37, 36], which should be considered also in the quantitative case.

We believe the automated synthesis of mechanisms is a promising and powerful tool for AMD. However, the high expressiveness of $\text{SL}[\mathcal{F}]$ may not always be needed for simple classes of mechanisms, and one may consider fragments of it to achieve better complexity. Therefore, an interesting direction for future work is to study the complexity of synthesizing from $\text{SL}[\mathcal{F}]$ -fragments, inspired from the SL -fragments One-Goal SL [50, 51] and Simple-Goal SL [52], for instance. These fragments are usually computationally easier than full SL , and we can hope that similar results can be established in the quantitative setting. On a related vein, we can study the translation of ADL to $\text{SL}[\mathcal{F}]$ -formulae, so as to include the auction description in the mechanism specification. In this setting, $\text{SL}[\mathcal{F}]$ formulae can be used to express requirements of well-formed auction descriptions.

The problems contemplated in my thesis are also worth investigating from an empirical perspective. One direction is to explore the interplay between agents' bounded rationality and the auctioneer revenue so as to understand the impact of bounded rationality on mechanism design. An implementation of a model checker for $\text{NatSL}[\mathcal{F}]$ would enable the empirical evaluation of natural strategies and auctions played by participants with restricted memory. Finally, experimental results could be used to assess the practical relevance of our proposed approaches, especially in relation to mechanism synthesis from $\text{SL}[\mathcal{F}]$ specification due to the high theoretical complexity of the problem.

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