

Control, diagnostics and error correction in the modular number system

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Abstract

This article discusses methods that allow controlling, diagnosing and correcting errors that occur in a computer system (CS) operating in a modular number system (MNS). A feature of using the MNS is the possibility, in some cases, of correcting errors even if there is only one control base, using the concept of an alternative set of numbers. The article deals with the control, diagnostics and error correction in the dynamics of the data processing process. Currently, to correct errors in the dynamics of the computational process, the CS uses the projection method and its modifications. The projection method requires the calculation of all projections \tilde{A}_i of the distorted number \tilde{A} , which leads to a large number of additional operations for each correction of an individual result. The article discusses a method for correcting errors in the CS, based on the use of a conditional alternative set of numbers in the MNS. Two methods for determining an alternative set $W(\tilde{A})$ are considered. The disadvantage of the first method is the large hardware and time costs for determining an alternative set of numbers. As a working method, the second method is proposed, in which the initial number \tilde{A} is first reduced to the form $\tilde{A}^{(Z)} = (0, 0, \dots, 0, \gamma_{n+1})$, i.e. the operation of zeroing the initial number \tilde{A} is performed. Examples of specific execution of modular operations for a given MNS are considered.

Keywords1

Alternative set, corrective modular code, error correction, information processing cycle, modular number system, non-positional code structure, projection method, zeroing process.

1. Introduction

One of the promising areas for ensuring the fault-tolerance of CSs is the widespread use of corrective codes that can detect and correct errors that occur in the dynamics of the data processing process. A characteristic feature of such codes is the presence in the construction structure of corrective codes of interdependent parts: informational and control. The analysis of known positional codes showed that these parts of the code are not equal in relation to arithmetic operations. The non-arithmetic nature of the procedures for obtaining the check digits of the corrective code doesn't allow controlling the results of performing arithmetic operations [1-4]. Thus, it is obvious that the use of positional corrective codes when implementing arithmetic operations in a CS operating in a positional number system (PSN) is impossible.

Non-positional codes, in particular, codes in the modular number system (MNS), are deprived of this drawback. A number of works, both domestic and foreign, are devoted to the construction of corrective modular codes [5-7]. The equality of residues in the structure of the correcting code in the MNS is the basis for constructing codes capable of detecting and correcting errors in the process of

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implementing modular operations. In addition, this property of modular codes serves as the basis for the implementation of exchange operations between the accuracy, fault tolerance and speed of the implementation of arithmetic operations in the dynamics of the computational process of the CSs. This is due, first of all, to the fact that each residue of the modular code carries information about the entire original number [8]. Then, by varying the number of information and control bases of the MNS, it is possible to achieve the required values of the main indicators of the quality of the functioning of the CSs. One of the promising directions for ensuring the fault tolerance of the CSs is the widespread use of codes capable of detecting and correcting errors that occur in the dynamics of the data processing process. A characteristic feature of such codes is the presence in the structure of the corrective code of two interdependent parts: informational and control [9].

The specificity of the representation of numbers in the MNS allows [10], in a number of cases, not only to detect an error, but also to find the place of its occurrence, using only a single control base, which is impossible with existing methods for monitoring and correcting errors in the MNS. In some cases, it is possible to carry out error correction with a minimum code distance $d_{\min} = 2$ either by the projection method or by using the concept of an alternative set (AS) of numbers.

The projection method requires the calculation of all projections \tilde{A}_i of the distorted number \tilde{A} , which leads to a large number of additional operations for each correction of an individual result. Hardware and especially software implementation of the projection method leads to a large expenditure of time. In addition, this method fundamentally doesn't allow unambiguous detection of the place of occurrence of any single errors, i.e. errors in one of the residues of a non-positional code structure in the MNS.

Much more effective is the method developed and researched further for correcting errors in the CSs, based on the use of a conditional alternative set (CAS) of numbers in the MNS.

2. Development and research of methods for monitoring, diagnosing and correcting errors in CS based on the use of an alternative and conditional alternative set of numbers

A distinctive feature of the MNS is the possibility, in some cases, of correcting errors even if there is only one control base, using the concept of an alternative set of numbers [5].

The set of bases: $m_{i_1}, m_{i_2}, \dots, m_{i_k}$ on which the numbers A_1, A_2, \dots, A_k differ from the wrong (distorted) number \tilde{A} , this is called an alternate number set of the number A_1, A_2, \dots, A_k and denote it as $W(\tilde{A}) = \{m_{i_1}, m_{i_2}, \dots, m_{i_k}\}$. The basic principle of determining the erroneous residue a_i is that for the set of incorrect (distorted) numbers $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_\rho$ obtained as a result of operations, during the execution of the program, CASs are determined sequentially in time:

$$W_{\wedge}(\tilde{A}) = W(\tilde{A}_1) \wedge W(\tilde{A}_2) \wedge \dots \wedge W(\tilde{A}_\rho) \quad (1)$$

where $W(\tilde{A}_l) = \{m_{i_1}, m_{i_2}, \dots, m_{i_\rho}\}$ – alternative set of l -th wrong (distorted) number.

Let's consider the error correction time in the dynamics of the information processing process of the CS. It is known that the correction time is determined by the following expression:

$$T_{cor} = T_{det} + T_{fix} \quad (2)$$

where T_{det} – errors detection time; T_{fix} – errors fix time.

For MNS, the error detection time in the dynamics of the data processing process is determined by the following expression:

$$T_{det} = k_1 \cdot T_{det_{\gamma_{n+1}}} + k_2 \cdot T_{det_{AS}} + k_3 \cdot T_{det_{CAS}} \quad (3)$$

where $T_{det_{\gamma_{n+1}}}$ – time to determine and check the value of γ_{n+1} ;

γ_{n+1} – the value of the residue of the number \tilde{A} to the base m_{n+1} during the zeroing process;

$T_{det_{AS}}$ – time of determination and verification of AS $W(\tilde{A})$;

$T_{det_{CAS}}$ – time of determination and verification of CAS $W_{\wedge}(\tilde{A})$;

k_1, k_2, k_3 – multiplicity factors for determining, respectively, the processes of zeroing numbers ($\tilde{A}^{(Z)}$), the number of operations to determine the AS of numbers ($W(\tilde{A})$) and the number of operations to determine the CAS of numbers ($W_{\wedge}(\tilde{A})$).

Thus, the error correction time in the dynamics of the information processing process in the MNS is determined by the final expression:

$$T_{cor} = k_1 \cdot T_{det_{n+1}} + k_2 \cdot T_{det_{AS}} + k_3 \cdot T_{det_{CAS}} + T_{fix} \quad (4)$$

Taking into account the fact that the operation of determining AS, CAS and fix (correction) errors in the MNS can be performed in a tabular version, i.e. in one cycle, get that:

$$T_{cor} \approx k \cdot T_Z \quad (5)$$

where k – number of stages of AS determination;

T_Z – zeroing time, which is necessary to convert the original number $A = (a_1, a_2, \dots, a_n, a_{n+1})$ into a number of the form $\tilde{A}^{(Z)} = (0, 0, \dots, 0, \gamma_{n+1})$.

It can be seen from the last expression that there are two main ways to reduce the error correction time T_{cor} . The first way is to reduce the number of stages k in determining AS. This is achieved through the use of the developed error correction methods in the MNS [9]. The second way is to reduce the zeroing time T_Z . This can be achieved by using the method of pairwise zeroing of numbers with a preliminary sample of digits [11].

Let's consider the necessary and sufficient condition for error correction in the dynamics of the computational data processing process.

On Figure 1 schematically shows the contraction (pull together) process of the AS of numbers in the process of processing CS information, where:

Δt_i – the duration of the implementation of the i -th operation of the information processing cycle;

S – the number of the operations in the considered information processing cycle;

S' – the number of the operation in the CS information processing cycle, at which the presence of errors is recorded;

t_0 – start time of the information processing cycle;

t_k – end time of the information processing cycle;

Δt_{AS_i} – the duration of the determination of the AS of the i -th number to be checked;

Δt_{\wedge_i} – the duration of the determination of the i -th CAS;

t' – the moment of time of the error detection;

Δt_{Δ_i} – the duration of time from the end of finding the i -th CAS to the start of determining the next AS;

Δt_{det} – the error detection time;

k – the number of numbers to be checked (the number of stages in determining the AS).

Let in the process of monitoring the processing of information by the CS at time t' , a distortion of the number \tilde{A} is detected. In this case, the information processing process is not interrupted, and for the distorted number for the period Δt_{AS_1} , AS $W(\tilde{A}_1)$ is determined. After determining $W(\tilde{A}_1)$, after a period of time Δt_{Δ_1} , AS $W(\tilde{A}_2)$ is determined, where: \tilde{A}_2 – the result of the operation of the next cycle of information processing of the CS. After that, the CAS $W_{\wedge}(\tilde{A}) = W(\tilde{A}_1) \wedge W(\tilde{A}_2)$ is determined. If $W(\tilde{A}) \leq 2$, then finding the AS stops. When $W(\tilde{A}) > 2$ the process of finding CAS continues. At the end of the information processing cycle ($t = t_k$), on the basis of the received AS $W(\tilde{A}) \leq 2$, the result is corrected.

3. The third option consists of a synthesis of the first and second. The CAS of the numbers obtained as the information processing program is implemented up to their contraction (pull together) in the number \tilde{A} to two bases m_i and m_{n+1} are determined. Further, the hypothesis of the error of the residue \tilde{a}_i is accepted and its correction is carried out. If the hypothesis turns out to be untenable, then the residue \tilde{a}_{n+1} will be wrong.

Thus, with any existing version of error correction in the dynamics of the information processing process, it becomes necessary to determine the AS, i.e. the effectiveness of any of the possible options for correcting errors depends on the method of determining the alternative set of numbers [12].

As a rule, the duration of the information processing cycle, for solving this algorithm, is a constant value $t_k - t_0 = \text{const}$. In this regard, to strengthen the fulfillment of condition (6), it is necessary to strive to reduce the time Δt_{det} . This can be achieved by performing the following operations.

1. Creation at the beginning of the information processing cycle of complicated modes of operation of the CS [13], which can lead to a shift of the time axis (error detection time) to the left to t_0 . In other words, the error is detected almost immediately after the start of the information processing cycle. However, this path does not guarantee high reliability of error detection at the beginning of the data processing cycle.

2. Reducing the time $\sum_{i=1}^{k-1} \Delta t_{\Delta_i}$ as a result of splitting the operations of the information processing cycle into shorter ones. However, in most cases, splitting operations is either impossible or impractical.

3. Reducing the time $\sum_{i=1}^{k-1} \Delta t_{\wedge_i}$ as a result of the acceleration of the process of logical multiplication $W(\tilde{A}_i) \wedge W(\tilde{A}_{i+1})$. As a rule, the CAS definition block is built according to the tabular principle. The result of the operation is determined in one clock cycle of the CS.

4. Reducing the time $\sum_{i=1}^k \Delta t_{AS_i}$ as a result of increasing the performance of the zeroing operation [14].

5. A decrease in the number k of the numbers to be checked as a result of an increase in the information content $W(\tilde{A})$, i.e. reduction in the AS of the number of bases on which an error is possible.

Thus, studies are necessary and relevant for the development of effective methods for determining AS, which will increase the information content of $W(\tilde{A})$, which reduces the error correction time in the MNS.

3. Methods for determining an alternative set of numbers

Let's consider two methods for determining AS $W(\tilde{A})$.

The first method is that AS $W(\tilde{A})$ is established by checking each of the bases $m_i (i = \overline{1, n+1})$ as follows. A sequence of numbers is determined that has the same digits in all bases as the number \tilde{A} , except for the base m_ρ , differing only in digits in this base, i.e. numbers of the form:

$$A_{\rho_s} = (a_1, a_2, \dots, a_{\rho-1}, s, a_{\rho+1}, a_{n+1}) \quad (9)$$

where $s = \overline{(1, m_\rho - 1)}$.

Among the numbers of the form (9) there may not be a single correct number, or there may be only one correct number. In the latter case, m_ρ enters the AS of the number \tilde{A} . Having carried out similar checks for each of the bases of the MNS, should determine $W(\tilde{A}) = \{m_{i_1}, m_{i_2}, \dots, m_{i_k}\}$.

The disadvantage of the first method is the large hardware and time costs for determining the AS.

In the second method, the number \tilde{A} is reduced to the form $\tilde{A}^{(Z)} = (0, 0, \dots, 0, \gamma_{n+1})$ i.e. the so-called operation of zeroing the number \tilde{A} is performed. In accordance with the error distribution theorem, the number of the interval $(j+1)$, in which the number \tilde{A} falls, is determined by the expression:

$$j = \left\lfloor \frac{\Delta a_i \cdot \bar{m}_i \cdot m_{n+1}}{m_i} \right\rfloor \bmod m_{n+1} + \Delta^* \quad (10)$$

where Δa_i – the value of a possible error in base m_i ;

\bar{m}_i – weight of the orthogonal basis B_i ;

$[x]$ – the integer part of the number x , not exceeding the value of x .

Δ^* – takes the value 0 or 1.

In accordance with expression (10), a table of values of the correspondence of the number γ_{n+1} to possible errors Δa_i is compiled, where $j = (\gamma_{n+1} \cdot \bar{m}_{n+1}) \bmod m_{n+1}$. The desired AS is determined from the correspondence table.

4. Method of time numerical sections

The considered second method for determining an alternative set of numbers is called the method of time numerical sections, and in comparison with the first method, it allows to reduce hardware and time costs in determining the AS, however, the disadvantage of the second method is that the AS may contain redundant bases. This is due to the fact that the values of γ_{n+1} correspond to errors Δa_i related not only to the distorted number \tilde{A} , but also to the group of numbers \tilde{A}_k lying in the interval

$\left[j \frac{M_1}{m_{n+1}}, (j+1) \frac{M_1}{m_{n+1}} \right]$, where $M_1 = \prod_{i=1}^{n+1} m_i$ – the full range (including control base m_{n+1}) of numbers represented in the MNS. Excessive bases contained in AS reduce the information content of $W(\tilde{A})$.

Indeed, with an increase in the number of bases in the AS, the entropy of error detection in one of the bases of the MNS increases. An increase in the entropy of determining the erroneous base m_l increases the number k of the numbers to be checked (check cycles), and this, in turn, increases the time of contraction (pull together) of the AS to the erroneous base. This method cannot be effectively applied in a short chain of CS data processing. Thus, there is a need to develop procedures for determining the AS, with the help of which it is possible to effectively correct errors in a fairly short chain of the information processing process in the CS. Let us consider the procedure for increasing the information content of the AS in the MNS, based on obtaining additional information about the possible distorted residues of the wrong number \tilde{A} . This information is contained in all possible ASs of number \tilde{A} .

Let MNS be given by ordered bases m_1, \dots, m_{n+1} and let the wrong number \tilde{A} be determined in the process of information processing. To increase the information content about the location and magnitude of the error [15], it is proposed to additionally determine the AS of number of the form $W_{k_{\rho_i}}(\tilde{A}) = \{m_{k_1}, m_{k_2}, \dots, m_{k_{\rho_i}}\}$, i.e. set of AS of the form:

$$\begin{aligned} W_{1_{\rho_1}}(\tilde{A}) &= \{m_{1_1}, m_{1_2}, \dots, m_{1_{\rho_1}}\} \\ W_{2_{\rho_2}}(\tilde{A}) &= \{m_{2_1}, m_{2_2}, \dots, m_{2_{\rho_2}}\} \\ &\dots \\ W_{n+1_{\rho_{n+1}}}(\tilde{A}) &= \{m_{n+1_1}, m_{n+1_2}, \dots, m_{n+1_{\rho_{n+1}}}\} \end{aligned} \quad (11)$$

To determine the set of values (11), first should calculate the number of the interval j_k ($k = \overline{1, n+1}$) where the number \tilde{A} hits:

$$j_k = (\gamma_k \cdot \bar{m}_k) \bmod m_k \quad (12)$$

Note that for $k = n+1$ the equality $W_{n+1, \rho_{n+1}}(\tilde{A}) = W(\tilde{A})$ is satisfied. In accordance with expression (12), let's compile k tables, where the values of γ_k are compared to the value of Δa_i . After ASs $W_{k, \rho_i}(\tilde{A})$, which called primary, are determined, define secondary ASs in the form of vectors, the components of which are possible error values Δa_i of the form: $W_1^1(\tilde{A}) = \{\Delta a_1^{(1)}, \Delta a_2^{(1)}, \dots, \Delta a_{n+1}^{(1)}\}, \dots, W_1^{(\varphi_1)}(\tilde{A}) = \{\Delta a_1^{(\varphi_1)}, \Delta a_2^{(\varphi_1)}, \dots, \Delta a_{n+1}^{(\varphi_1)}\}, W_2^2(\tilde{A}) = \{\Delta a_1^{(2)}, \Delta a_2^{(2)}, \dots, \Delta a_{n+1}^{(2)}\}, \dots, W_2^{(\varphi_2)}(\tilde{A}) = \{\Delta a_1^{(\varphi_2)}, \Delta a_2^{(\varphi_2)}, \dots, \Delta a_{n+1}^{(\varphi_2)}\}$ and so on up to the value of vectors of the form: $W_n^{(\varphi_n)}(\tilde{A}) = \{\Delta a_1^{(\varphi_n)}, \Delta a_2^{(\varphi_n)}, \dots, \Delta a_{n+1}^{(\varphi_n)}\}$ and vector $W_{n+1}^{(\varphi_{n+1})}(\tilde{A}) = \{\Delta a_1^{(\varphi_{n+1})}, \Delta a_2^{(\varphi_{n+1})}, \dots, \Delta a_{n+1}^{(\varphi_{n+1})}\}.$

The components of vector $W_{n+1}^{(\varphi_{n+1})}(\tilde{A})$ are compared with the corresponding components of all vectors $W_i^{(\varphi_i)}(\tilde{A})$ for $i = \overline{1, n}$. In the components of the vectors coinciding in magnitude, the MNS bases are determined, the set of which will determine the desired (resulting) AS of the form: $W'(\tilde{A}) = \{m_{z_1}, m_{z_2}, \dots, m_{z_p}\}$. The AS $W_{k, \rho_i}(\tilde{A})$ always contains the base m_i , on which the error Δa_i occurred, and this base can only be among the bases common to the set (11), i.e.:

$$W(\tilde{A}) \geq W'(\tilde{A}) \quad (13)$$

where $W'(\tilde{A})$ – desired (resulting) AS.

If Δa_i is such that the number $\tilde{A} = A + \Delta A$ (where $\Delta A = (0, 0, \dots, \Delta a_i, 0, \dots, 0)$ is a single error) lies in the interval $[(m_{n+1} - 1)M, M_1]$, where $M = \prod_{i=1}^n m_i$ – the operating range of numbers represented in the MNS, then:

$$W(\tilde{A}) = W'(\tilde{A}) \quad (14)$$

Thus, the essence of the proposed procedure lies in the fact that all possible AS are determined on each of the intervals where the numbers A hit. After that, the common bases m_{z_1}, \dots, m_{z_p} for these intervals are determined, on which an error is possible. This set of bases determines the desired AS. Reducing the number of bases in AS increases the information content of AS $W(\tilde{A})$ about the place and magnitude of the error. This reduces the time of AS contraction to an erroneous base (reduces the number of stages for determining the CAS), which increases the efficiency of corrective codes in the MNS. The block diagram of the process of contraction of the AS to the erroneous base is shown in Figure 2. It is advisable to consider the block diagram of the process of contraction of the AS. Determining the number of the hit interval $(j+1)$ (under the influence of the error Δa_i) of the

distorted number \tilde{A} is equivalent to shifting this number in the interval $\left[j \frac{M_1}{m_i}, (j+1) \frac{M_1}{m_i}\right]$ to the left to the value $j \frac{M_1}{m_i}$. Let's divide the numerical segment $[0, M_1]$ into the corresponding intervals with duration: $\frac{M_1}{m_1}, \frac{M_1}{m_2}, \dots, \frac{M_1}{m_{n+1}}$. Let's determine the numbers of intervals $(j+1)$ in which the number \tilde{A} is located on each of the numerical segments:

$$\begin{aligned} T_{j_i} &= \left[j_i \frac{M_1}{m_i}, (j_i + 1) \frac{M_1}{m_i} \right] \\ &\dots \\ T_{j_{n+1}} &= \left[j_{n+1} \frac{M_1}{m_{n+1}}, (j_{n+1} + 1) \frac{M_1}{m_{n+1}} \right] \end{aligned} \quad (15)$$

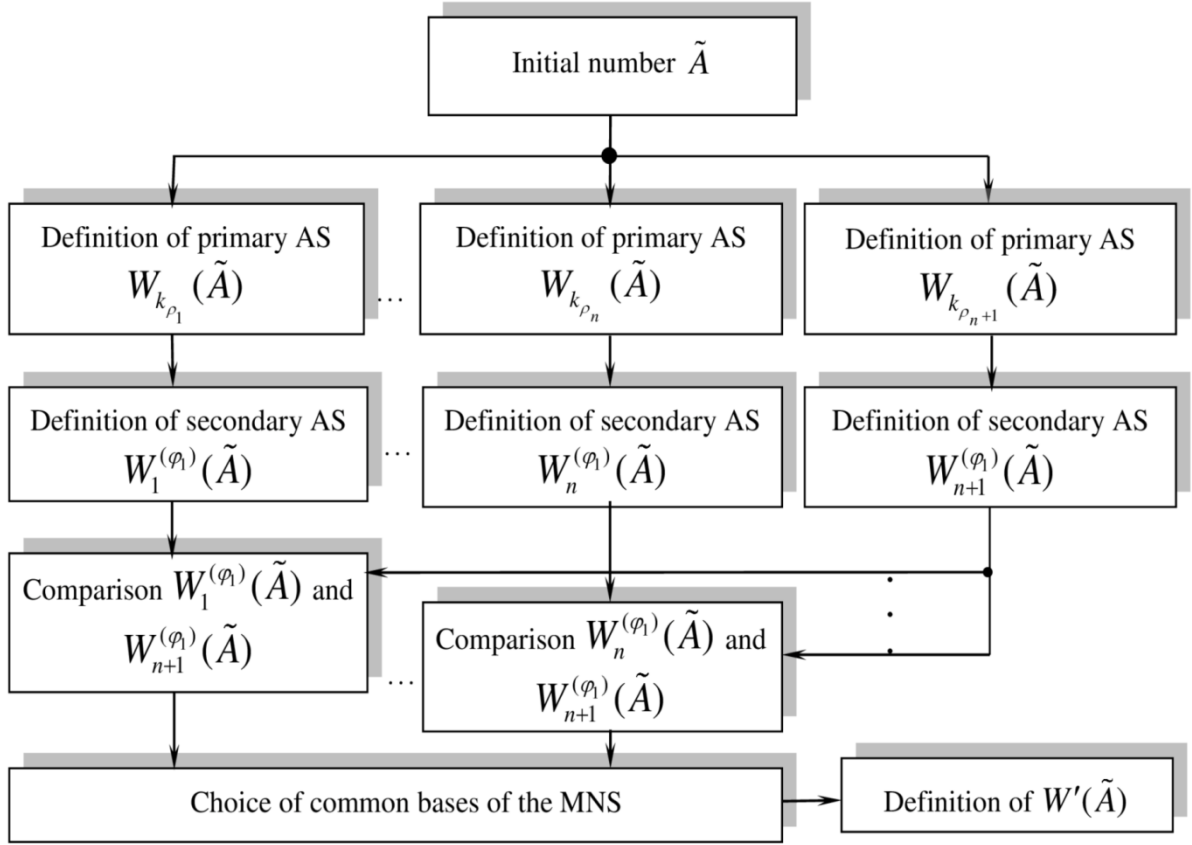


Figure 2: Structural diagram of the AS contraction (pull together) process

The definition of primary AS (11) corresponds to the definition the numbers of intervals (15). The definition of the secondary AS, geometrically corresponds to the definition of the interval $[z_1, z_2)$, where $z_1 = \max \in j_i \frac{M_1}{m_i}$. And $z_2 = \min \in (j_i + 1) \frac{M_1}{m_i}$, i.e. the desired interval is defined as the intersection of the sets of intervals (15) $T_{W'(\tilde{A})} = T_{j_1} \wedge T_{j_2} \wedge \dots \wedge T_{j_{n+1}}$. It's obvious that:

$$z_2 - z_1 = \frac{M_1}{M_{n+1}} \quad (16)$$

Condition (16) is equivalent to condition (13). If the error translates the number \tilde{A} into the interval $[(m_{n+1} - 1)M, M_1)$, then:

$$z_2 - z_1 = \frac{M_1}{M_{n+1}} = M \quad (17)$$

Condition (17) is equivalent to condition (14).

The presented structural diagram (Figure 2) of the AS contraction (pull together) process confirms the correctness of the mathematical description and more clearly demonstrates the essence of the procedure for increasing the information content of the AS – narrowing the interval for getting the distorted number \tilde{A} .

Consider an example of determining the AS of number \tilde{A} in accordance with the developed procedure. Let the MNS be given by the bases $m_1 = 2$, $m_2 = 3$, $m_3 = 5$. Table 1 shows the code words of this MNS. Thus $M = 2 \cdot 3 = 6$, $M_1 = M \cdot m_3 = M \cdot 5 = 6 \cdot 5 = 30$, $m_{n+1} = m_3 = 5$, $A = (0, 2, 2)$ according to Table 1 this number in the positional number system is 2, let equals $\Delta A = (0, 2, 0)$ according to Table 1 this number in the positional number system is 20. Let, under the influence of a single error

$\Delta A = (0, 0, \dots, \Delta a_i, \dots, 0)$, based on the i -th base ($\Delta a_i = \Delta a_2 = 2$) received number $\tilde{A} = A + \Delta A = (0, 1, 2)$.

To determine the set of primary AS, first determine the values of γ_k . To do this, let's zeroing of the number \tilde{A} in accordance with the tables of zeroing constants (see Tables 2-4), obtain the values $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma_3 = 2$.

The set of primary AS is defined as: $W_{1_{\rho_1}}(\tilde{A}) = \{m_2, m_3\}$, $W_{2_{\rho_2}}(\tilde{A}) = \{m_1, m_3\}$, $W_{3_{\rho_3}}(\tilde{A}) = \{m_1, m_2, m_3\}$.

From Table 5-7, compiled according to the values γ_k , determine the set of secondary AS: for $\gamma_3 = 2$ has that $W_3^{(1)}(\tilde{A}) = \{1, 1, 2\}$; for $\gamma_2 = 1$ has that $W_2^{(1)}(\tilde{A}) = \{1, 0, 2\}$, $W_2^{(2)}(\tilde{A}) = \{0, 0, 3\}$; for $\gamma_1 = 1$ $W_1^{(1)}(\tilde{A}) = \{0, 2, 3\}$, $W_1^{(2)}(\tilde{A}) = \{0, 0, 4\}$.

Table 1

Table of code words in the MNS

A	m_1	m_2	m_3	A	m_1	m_2	m_3
0	0	0	0	15	1	0	0
1	1	1	1	16	0	1	1
2	0	2	2	17	1	2	2
3	1	0	3	18	0	0	3
4	0	1	4	19	1	1	4
5	1	2	0	20	0	2	0
6	0	0	1	21	1	0	1
7	1	1	2	22	0	1	2
8	0	2	3	23	1	2	3
9	1	0	4	24	0	0	4
10	0	1	0	25	1	1	0
11	1	2	1	26	0	2	1
12	0	0	2	27	1	0	2
13	1	1	3	28	0	1	3
14	0	2	4	29	1	2	4

Table 2

Table of zeroing constants

m_1	m_2
(1, 1, 1)	(0, 1, 4)
	(0, 2, 2)

Table 3

Table of zeroing constants

m_1	m_3
(1, 1, 1)	(0, 0, 1)
	(0, 2, 2)
	(0, 0, 3)
	(0, 1, 4)

Table 4

Table of zeroing constants

m_2	m_3
(0, 1, 0)	(1, 0, 3)
(1, 2, 0)	(0, 2, 2)
	(1, 1, 1)

Table 5

Table of zeroing constants of the secondary AS

γ_3	Errors	$W_i^{(\varphi_i)}$
0	no	—
1	$\square a_2 = 1,$ $\square a_3 = 1$	$W_3^{(1)}(\tilde{A}) = 0, 1, 1$
2	$\square a_1 = 1,$ $\square a_2 = 1,$ $\square a_3 = 2$	$W_3^{(1)}(\tilde{A}) = 1, 1, 2$
3	$\square a_1 = 1,$ $\square a_2 = 2,$ $\square a_3 = 3$	$W_3^{(1)}(\tilde{A}) = 1, 2, 3$
4	$\square a_2 = 2,$ $\square a_3 = 4$	$W_3^{(1)}(\tilde{A}) = 0, 2, 4$

Table 6

Table of zeroing constants of the secondary AS

γ_2	Errors	$W_i^{(\varphi_i)}$
0	$\square a_3 = 1$	$W_2^{(1)}(\tilde{A}) = \{0, 0, 1\}$
1	$\square a_1 = 1,$ $\square a_2 = 1,$ $\square a_3 = 1$	$W_2^{(1)}(\tilde{A}) = \{1, 0, 2\},$ $W_2^{(2)}(\tilde{A}) = \{0, 0, 3\}$
2	$\square a_3 = 4$	$W_2^{(1)}(\tilde{A}) = \{0, 0, 4\}$

Table 7

Table of zeroing constants of the secondary AS

γ_1	Errors	$W_i^{(\varphi_i)}$
0	$\square a_1 = 1,$ $\square a_2 = 2,$ $\square a_3 = 1$	$W_1^{(1)}(\tilde{A}) = \{0, 0, 1\},$ $W_1^{(2)}(\tilde{A}) = \{0, 0, 2\}$
1	$\square a_2 = 2,$ $\square a_3 = 3$	$W_1^{(1)}(\tilde{A}) = \{0, 2, 3\},$ $W_1^{(2)}(\tilde{A}) = \{0, 0, 4\}$

It is convenient to implement the choice of common MNS bases in the form of tables (see Tables 8-11), where the “+” sign indicates the coincidence of the components of the secondary AS, and the “-” sign indicates a mismatch. From Tables 8-11 it can be seen that the components of the vectors coincide in the bases m_1, m_3 , i.e. the desired AS has the form $W'(\tilde{A}) = \{m_1, m_3\}$. Thus,

$W(\tilde{A}) \geq W'(\tilde{A})$. Therefore, the described procedure is guaranteed to increase information about the location of the error in number \tilde{A} .

In geometric interpretation, this procedure, for a given MNS, will be implemented as follows (Figure 3). Let's divide the segment $[0,30)$ into the corresponding numerical intervals $[15,30)$, $[10,20)$ and $[12,18)$. Determine the numbers of intervals in which the number $\tilde{A} = (0,1,2)$ is located: $T_{j_1} = [15,30)$, $T_{j_2} = [10,20)$, $T_{j_3} = [12,18)$. The desired interval is determined as follows $T_{W'(\tilde{A})} = [15,18)$.

The interval $T_{W'(\tilde{A})}$ is reduced compared to T_{j_3} by three units (by 50%), which leads to a reduction in the number of possible error options. This procedure is most effectively used in a chain of calculations that doesn't allow all the planned procedures to be carried out before the AS is contracted (pulled together) to an erroneous base, i.e. in a long chain of data processing CS [16-18].

Consider the procedure for probabilistic evaluation of the choice of the working hypothesis about the fallacy of the residue on an arbitrary i -th base. To do this, it is advisable to determine the relationship between the reduced error distribution coefficient $\xi_{k_i} = F(\gamma_{n+1})$ and the value of γ_{n+1} .

The reduced distribution coefficient of errors will be the ratio of the number of possible errors on the basis of m_k in the i -th interval to the number of possible errors of the number \tilde{A} in the entire range $[0, M_1)$. It is numerically equal to the share of errors in the i -th interval according to the k -th base of the MNS.

Table 8

Table of zeroing constants

m_1	m_2	m_3
1	1	2
1	0	2
+	-	+

Table 9

Table of zeroing constants

m_1	m_2	m_3
1	1	2
0	0	3
-	-	-

Table 10

Table of zeroing constants

m_1	m_2	m_3
1	1	2
0	2	3
-	-	-

Table 11

Table of zeroing constants

m_1	m_2	m_3
1	1	2
0	0	4
-	-	-

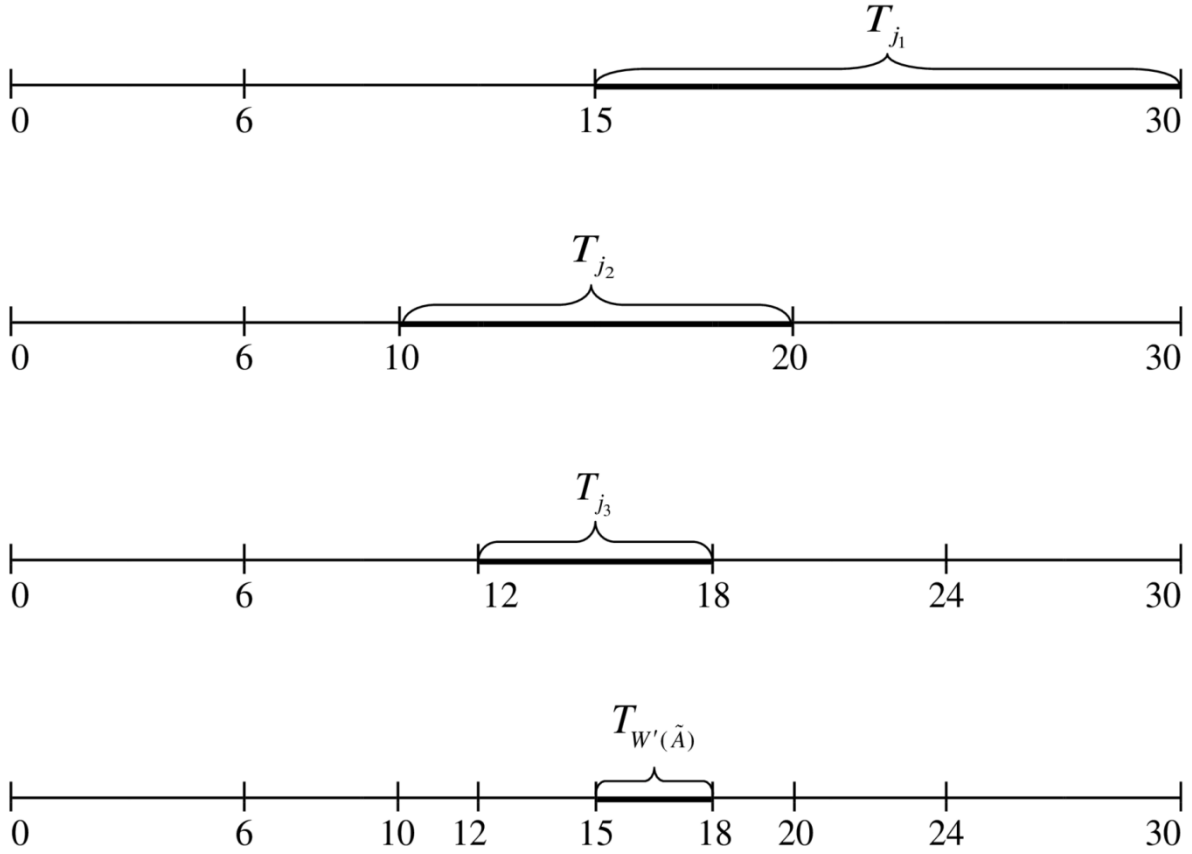


Figure 3: Selection scheme of time interval

When calculating ξ_{k_i} , it is necessary to consider the distribution of errors on the intervals $[jM, (j+1)M)$ for $j=1,2,\dots,m_{n+1}-1$. Based on the distribution, a table of values is compiled, according to which histograms n are built $\xi_{k_i} = F(\gamma_{n+1})$. The criterion for choosing a working hypothesis about the error of the residue based on m_z is the maximum value of the reduced error distribution coefficient, i.e.:

$$\xi_{z_i} = \max, \text{ for } i = \text{const} \quad (18)$$

Let's compose an algorithm for determining an erroneous base when implementing the procedure for probabilistic evaluation of the choice of a working hypothesis.

1. The distorted number \tilde{A} zeroable out. Getting the value of $\tilde{A}^{(Z)} = (0, 0, \dots, 0, \gamma_{n+1})$.
2. According to the value of γ_{n+1} , determine AS $W(\tilde{A}) = \{m_{i_1}, m_{i_2}, \dots, m_{i_p}\}$.
3. By the value of γ_{n+1} , let's turn to the tables (histograms) $\xi_{k_i} = F(\gamma_{n+1})$. In the i -th interval ($i = (\gamma_{n+1} \cdot m_{n+1}) \bmod m_{n+1}$), the largest of the values ξ_{z_i} is determined. Base m_z , for which the value of $\xi_{z_i} = \max$ at $i = \text{const}$ is the desired one. Thus, the working hypothesis is that the error is assumed in the residue to the base $m_z \in W(\tilde{A})$. If it turns out that the hypothesis is erroneous, then as the second working hypothesis should choose the base $m_i \in W(\tilde{A})$, for which $\xi_{k_i} < \xi_{z_i}$, and so on. As a rule, the choice of the primary working hypothesis according to the criterion (18) gives a reliable result about the location of the error.

As an example of choosing a working hypothesis, let's determine the number of the erroneous residue for the number $\tilde{A} = (0, 1, 2)$ specified in the MNS with bases $m_1 = 2, m_2 = 3, m_3 = 5$ ($m_{n+1} = m_3 = 5$).

1. Zeroing of the number $\tilde{A} = (0,1,2)$ is performed, get $\tilde{A}^{(Z)} = (0,0,3)$.
2. According to the value of $\gamma_{n+1} = \gamma_3 = 3$, an appeal is made to Table 5, where AS $W_3^{(1)}(\tilde{A}) = \{1,2,3\}$ is defined.
3. Table 12 is compiled for $\xi_{k_i} = F(\gamma_3)$, in accordance with the distribution of errors over the intervals of the range $(0, M_1]$ (Figure 4). On Figure 4, the sign $\uparrow k$ shows that the error in the base m_k translates the correct number A into the wrong (distorted) number \tilde{A} . For the value $\gamma_3 = 3$ (fourth interval), according to the criterion (18), determine $\xi_{2_4} = 0,3$.

Table 12

Table of values $\xi_{k_i} = F(\gamma_3)$

γ_3	i	ξ_{1_i}	ξ_{2_i}	ξ_{3_i}	Working hypothesis of fallibility
0	1	0	0	0	-
1	2	0	0,25	0,75	m_2, m_3
2	3	0,23	0,3	0,47	m_2, m_3
3	4	0,23	0,3	0,47	m_2, m_3
4	5	0	0,25	0,75	m_2, m_3

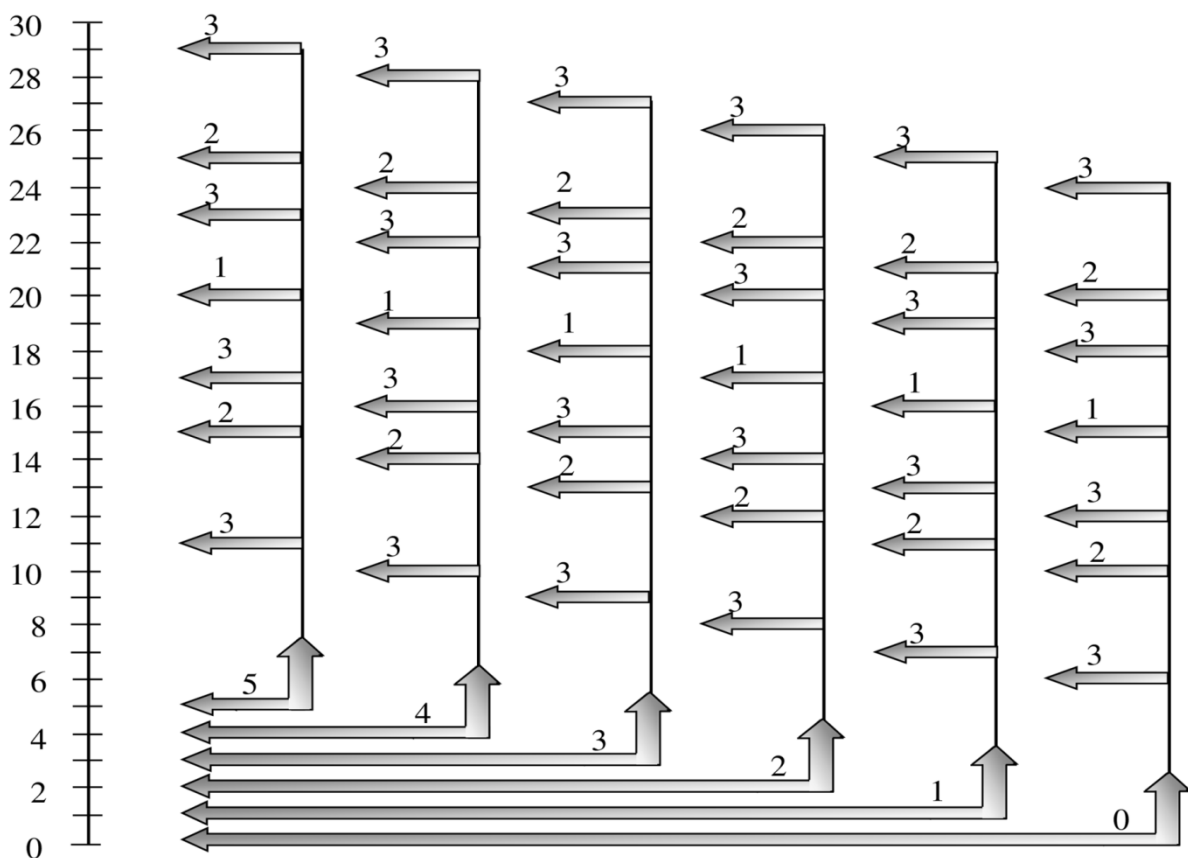


Figure 4: Scheme of distribution of errors over intervals of the numerical range $[0,30)$

Thus, in that the error is assumed in the residue of the base m_2 (an error in the residue of the control base m_3 is not taken into account). Verification confirms the correctness of the choice of hypothesis. So $A = \tilde{A} - \Delta A = (0, 1, 2) - (0, 2, 0) = (0, 2, 2)$, as a result it was received the original number (see Table 1, $A = (0, 2, 2)$ corresponds to the value of 2 positional number system, which was at the beginning).

The use of a probabilistic estimate makes it possible to reduce the number of check numbers, which in turn makes it possible to reduce the error correction time [19].

5. Conclusions

The article considers the process of monitoring, diagnosing and correcting errors in the MNS in the dynamics of the computational process of the CS. In some cases, this is possible in the presence of a minimum code distance by using the concepts of an alternative set of numbers and a conditional alternative set of numbers. A scheme for monitoring and correcting errors in the MNS has been developed. The necessary and sufficient condition for error correction in the dynamics of the data processing process is formulated and substantiated. When correcting errors in the dynamics of the information processing process, it is assumed that the implemented chain of computational operations has a sufficient length to allow the CAS to be reduced to one erroneous residue.

Three variants of error correction are defined. With any existing option for correcting errors in the dynamics of the data processing process, it becomes necessary to determine the AS, i.e. the effectiveness of any of the possible error correction options depends on the method for determining an alternative set of numbers. In this aspect, the article conducted research on the development of effective methods for determining the AS, which can increase the information content of the AS, which reduces the error correction time in the MNS. The analysis of methods for determining AS $W(\tilde{A})$ was carried out.

A procedure has been developed for monitoring and diagnosing errors in the CS based on a probabilistic assessment of the choice of a working hypothesis about the error of the residue based on the i -th base of the MNS. An algorithm for determining an erroneous base in the implementation of the procedure for probabilistic assessment of the choice of a working hypothesis has been implemented. This procedure allows, in some cases, to reduce the number of check numbers, which in turn reduces the error correction time.

It is shown that the proposed method for correcting errors in the MNS (the method of time numerical sections) makes it possible to simplify the implementation of the process of determining AS in the MNS and correct not only multiple errors in one residue, but also, in some cases, multiple errors in different residues. This method is most effectively used in a short data processing chain of the CS. An example of error correction for a specific MNS given by the bases $m_1 = 2$, $m_2 = 3$, $m_3 = 5$ is given. The results of the presented example confirm the main provisions of this article.

6. References

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