# Distribution of coupled thermoelastic waves in the different rocks using MatLab GUI application

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**Abstract**

GUI MATLAB application developed offers a robust platform for both graphical and numerical analysis of wave numbers and wave velocities within coupled thermoelastic systems. It enables the examination of how frequency affects the behavior of P-primary and S-secondary waves across different rock types, utilizing physical properties from Kazakhstan's geological deposits. This method yields critical insights into the impact of geological conditions on wave propagation, which is essential for evaluating seismic risks and designing resilient structures. Furthermore, the application enhances the visualization and interpretation of results by providing detailed graphs and tables. This feature facilitates a thorough analysis of the relationship between wave numbers, velocities, and frequency, thereby deepening the understanding of thermoelastic material behavior. The tool's capabilities are valuable for scientific research and practical geophysical and engineering applications, aiding in precise geological risk assessment and the development of effective risk management strategies.

**Keywords**

Coupled thermoelasticity, P and S waves velocity, rocks, thermoelastic characteristics, wave numbers, MatLab GUI application

## 1. Introduction

Thermoelasticity problems in solids investigate the interaction between temperature and elastic deformations in materials when they are subjected to temperature changes. The basic idea is that a change in temperature not only causes thermal expansion or contraction of the material but can also create additional mechanical stresses that affect its elastic properties. This leads to the need to consider the relationship between the temperature field and the deformation field in the equations of mechanics and thermodynamics. For example, sudden changes in temperature can cause the development of cracks or other defects in solids, which is important to consider when designing and operating materials in various engineering applications.

Coupled thermoelasticity problems involve considering how the rate of change in the first invariant of the strain tensor affects the first law of thermodynamics, linking temperature and strain fields and thus integrating elastic and thermal fields. This interplay becomes significant when rapid changes in thermal boundary conditions or other thermal loads trigger the propagation of thermal stress waves [1-7].

Mathematically handling coupled thermoelasticity problems analytically is quite complex, and as a result, only basic cases have been explored in the literature. Historically, analytical solutions have been limited to problems in infinite spaces, half-spaces, and layers. To address more complex scenarios, numerical methods such as finite element and boundary element techniques have been employed. Additionally, there have been several published analytical solutions for one-dimensional

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coupled thermoelasticity problems in both rectangular and cylindrical geometries, often utilizing Laplace transforms [8,9].

MATLAB provides powerful tools for solving thermoelastic problems in solids with its built-in functions and numerical simulation tools. With MATLAB, you can use finite element methods (FEA) and boundary element methods (BEA) to analyze complex thermoelastic problems. MATLAB includes specialized packages such as Partial Differential Equation Toolbox and Thermal Analysis Toolbox, which allow you to model and analyze the interaction of temperature and mechanical fields. Users can create and solve models considering various geometries, boundary conditions, and physical properties, and visualize the results for a deep understanding of the temperature and stress distribution in the material.

The purpose of this article was to show the creation and use of a computer GUI MATLAB application, which has been developed to address the issue of wave generation and propagation in thermoelasticity problems for different types of rocks, specifically sourced from deposits in Kazakhstan [10].

## 2. Physical characteristics of rocks

A thermoelastic medium refers to a material that exhibits thermoelastic properties, meaning it can return to its original shape after being deformed due to temperature changes. When exposed to varying temperatures, such materials experience thermal expansion or contraction, resulting in thermal deformation. However, their inherent elastic properties enable them to recover their shape when subjected to thermal stress. This characteristic is crucial in fields such as engineering and construction, where thermoelastic materials help mitigate thermal deformation and avert structural damage. Examples include specialized alloys, polymers, and composites used in industries such as aerospace, construction, and electronics.

An isotropic body, on the other hand, is characterized by uniform properties in all directions. In materials science and physics, isotropy refers to consistent mechanical, thermal, electrical, and optical properties throughout the material. For thermal conductivity, an isotropic body has an identical capacity to conduct heat in all directions, meaning its thermal conductivity is direction independent.

Each rock type, including those from Kazakhstan's deposits, possesses a unique set of characteristics that are crucial for understanding and predicting the future behavior of geological formations. Knowledge of these properties is essential for accurate calculations and assessments related to rock stability, deformation, and overall response to various geological and mechanical stresses. Properly accounting for these specific characteristics ensures more reliable predictions and effective management of geological resources and challenges [11].

Thus, an isotropic thermoelastic medium is characterized by a finite number of positive thermodynamic parameters (Table 1): mass density $\rho$, E – Young's modulus, ν - Poisson's ratio. Thermal diffusivity coefficient κ ¿ $\frac{\lambda_0}{b_\varepsilon}, [\kappa] = m^2/sec$- a physical parameter characterizing the rate of temperature equalization in a substance, $\lambda_0$- thermal conductivity coefficient, $b_\varepsilon$ - specific heat capacity at constant deformation. The constant $\gamma \equiv (3\lambda + 2\mu) a_t$, having the dimension $[\gamma]$=N/$m^2$*K, is associated with the property of expansion of a free element of an isotropic body with increasing temperature, $a_t$ is the coefficient of linear thermal expansion. The quantity $\eta = \gamma T_0 / \gamma_0$ has dimension $[\gamma]$=K*sec/m², where $T_0$ is the current absolute temperature of the environment in natural (initial) state, measured in degrees Kelvin (K). Key thermoelastic constants include Lamé constants:

$$\lambda = \frac{vE}{(1+v)(1-2v)}, \mu = \frac{E}{2(1+v)} \tag{1}$$

Lamé constants (also known as Lamé coefficients) are material constants that describe the elastic properties of a material in terms of elasticity theory. These constants are used in theory to describe the response of a material to mechanical stress and strain. Constant $\lambda$ is associated with volumetric changes in the material under the influence of mechanical pressure. It is often called the "volumetric elastic deformation coefficient". Constant $\mu$ is called the "transverse modulus of elasticity" or "Lamé coefficient" and is associated with the change in shape of a material under the influence of mechanical stress [2,3].

**Table 1**
Thermal and elastic characteristics of rocks.

| | Granite | Sandstone | Limestone | Siltstone | Shale |
|---|---|---|---|---|---|
| $\rho_{ave}\cdot 10^{-3}$, [kg/m³] | 2,61 | 2,69 | 2,72 | 2,69 | 2,77 |
| $E\cdot 10^{-10}$, [N/m²] | 4,02 | 4,13 | 5,2 | 3,87 | 5,25 |
| $\nu$ | 0,26 | 0,09 | 0,21 | 0,29 | 0.15 |
| $\lambda_0$, [W/m • K] | 2,4 | 1,66 | 2,4 | 1,49 | 2,46 |
| $a_T$, [J/N • K] | 946 | 972 | 887 | 880 | 866 |
| $\alpha_T\cdot 10^5$, [1/K] | 0,8 | 0,64 | 0,53 | 0,5 | 0,68 |
| $c_1$, [m/sec] | 5600 | 3272 | 3243 | 2390 | 4493 |
| $c_2$, [m/sec] | 2750 | 1293 | 1808 | 1204 | 2879 |
| $\mu\cdot 10^{-10}$, [kg/m sec²] | 1,974 | 0,45 | 0,89 | 0,39 | 2,296 |
| $\lambda\cdot 10^{-10}$, [kg/m sec²] | 4,24 | 1,98 | 1,08 | 0,756 | 1 |
| $\gamma\cdot 10^{-5}$, [Pa/K] | 13,3 | 2,5 | 2,65 | 1,5 | 5,2 |
| $\eta\cdot 10^{-8}$, [K • sec/m²] | 1,7 | 0,44 | 0,32 | 0,29 | 0,6 |
| $\kappa\cdot 10^{7}$, [m²/sec] | 9,27 | 9,86 | 11,27 | 10,28 | 9,46 |

Initial temperature $T_0 = 293\,K = 20\,C$.

In particular, the following rocks from Kazakhstan deposits were considered: cataclastic granite, medium- and coarse-grained (Bolshoy Karatau deposit); sandstone (Central Karatau); silicified shales (Maly Karatau); limestone between ore layers (Akatuevskoe deposit); siltstone (Karaganda basin).

## 3. Mathematical statement of the problem of coupled thermoelasticity

Under certain assumptions of continuity and homogeneity of the medium the dynamics of a thermoelastic medium (rock) is defined by the system of differential equations of mixed hyperbolic–parabolic type [12-15]. In Cartesian coordinate system, it has the next form:

$$(\lambda+\mu)u_{j,ji}+\mu u_{i,jj}-\gamma\theta_{,i}+F_i=\rho\ddot{u}_i$$
$$\Delta\theta-\frac{1}{\kappa}\dot{\theta}-\eta\dot{u}_{j,j}+\frac{1}{\kappa}Q=0 \tag{2}$$

Thus, $u_i(x,t)$ are the components of a displacement vector; $\theta(x,t)$ is temperature; $F(x,t)=F_i e_i$ are surface forces; $e_i$ are unit basis vectors; $Q(x,t)$ is a heat source; at i,j=1,..., N (N=2 by plane deformations and N=3 in the 3D case).

The stress tensor $\sigma_{ij}(x,t)$ is related to displacements $u(x,t)$ and temperature $\theta(x,t)$ by the Duhamel–Neumann law:

$$\sigma_{ij}=(\lambda u_{k,k}+\gamma\theta)\delta_{ij}+\mu(u_{i,j}+u_{j,i}) \tag{3}$$

Here $\delta_{ij}$ is Kronecker symbol.

Everywhere the symbol $u_{i,j}$ denotes partial derivatives with respect to coordinates: $u_{i,j}\equiv\partial u_i/\partial x_j$, the symbol $\dot{u}$ denotes differentiation with respect to time t: $\dot{u}=\partial u/\partial t$. Tensor convolution applies in formulas to repeated indices (summation from 1 to N).

Thus, governing equations of motion of thermoelastic media (2) will be described taking into consideration (3) as follows:

$$\sigma_{ij,j}+\rho F_i=\rho\ddot{u}_i$$
$$\dot{\theta}=\kappa\Delta\theta-\eta\,\mathrm{div}\dot{u}+Q, \qquad i,j=1,\ldots,N \tag{4}$$

- the dot above the symbol denotes the differentiation with respect to time.

Now system of equation (4) is reduced to the form:

$$\left(c_1^2-2c_2^2\right)u_{j,ji}+c_2^2u_{i,jj}-\gamma\theta_{,i}+F_i=\ddot{u}_i,$$
$$\Delta\theta-\kappa^{-1}\dot{\theta}-\tilde{\eta}\dot{u}_{j,j}+\kappa^{-1}Q=0, \tag{5}$$

where γ=ν/ρ, $\tilde{\eta}=\eta/\kappa$, $c_1=\sqrt{(\lambda+2\mu)/\rho}$ – speed propagation of an elastic irrotational wave (compression - expansion wave) in corresponding $(\lambda,\mu,\rho)$ elastic medium, $c_2=\sqrt{\mu/\rho}$ – speed propagation of an elastic wave of distortion (rotations), causing a change in shape without a change in volume (the velocities of longitudinal (P wave) and transversal (S wave) waves in the thermoelastic media).

## 4. Thermoelastic waves: P and S waves. Wave numbers

Let us consider the distribution of shock thermoelastic waves in the thermoelastic media and conditions on the fronts. The system of equations (5) of mixed hyperbolic-parabolic type. Its characteristic equation has the form [16-18]:

$$det\left\{L_{ij}^2\left(\nu,\nu_t\right)\right\}=det\left\{L_{ij}^e\left(\nu,\nu_t\right)\right\} \quad\leftrightarrow\quad \sum_{i=1}^3\nu_i^2\ , \tag{6}$$

here $L_{ij}^{e}$ is the differential operator of the equations of motion of the (λ,μ,ρ)-elastic body. $(\nu,\nu_t)=(\nu_1,\dots,\nu_3,\nu_t)$ is the normal vector to the characteristic surface in $R^4=\{(x,t)\}$. $L_{ij}^{e}$ is the main part of the differential operator $L_{ij}(\partial_x,\partial_t)$, moreover $L_{ij}^{e}$ contains only the highest derivatives of the second order. It follows from (6) that

$$\text{either} \quad \sum_{i=1}^{3}\nu_i^2=0 \qquad \text{or} \qquad \det\left\{L_{ij}^{e}(\nu,\nu_t)\right\}=0$$

The first correlation describes the characteristic surface of the classical parabolic equation, which does not determine the wave front in $R^4$. The second correlation describes the wave fronts $F_t$ moving in $R^3$ with the velocity

$$c=-\nu_t\Big/\sqrt{\sum_{i=1}^{N}\nu_i^2} \qquad c=c_j \qquad j=1,2 \tag{7}$$

That is, the wave fronts (thermal shock waves) in a thermoelastic medium move with the velocity of elastic waves.

The characteristic equation can denote the equation derived from solving linear differential equations with constant coefficients. This equation yields solutions expressed as exponential functions, which determine the system's behavior over time.

For a system of differential equations (5) that models the stress-strain state of a material, the characteristic equation typically has six roots (wave numbers): four complex roots $\zeta_1^2,\zeta_2^2$ and two real roots $\zeta_3^2$.

$$\begin{gathered}
\zeta_1^2=\frac{1}{2}\left[\frac{\omega^2}{c_1^2}-\frac{i\omega(1+\varepsilon)}{\kappa}+\sqrt{\left(\frac{\omega^2}{c_1^2}-\frac{i\omega(1+\varepsilon)}{\kappa}\right)^2+\frac{4i\omega^3}{\kappa c_1^2}}\right]\\
\zeta_2^2=\frac{1}{2}\left[\frac{\omega^2}{c_1^2}-\frac{i\omega(1+\varepsilon)}{\kappa}+\sqrt{\left(\frac{\omega^2}{c_1^2}-\frac{i\omega(1+\varepsilon)}{\kappa}\right)^2+\frac{4i\omega^3}{\kappa c_1^2}}\right]\\
\zeta_3^2=\frac{\omega^2}{c_2^2}\ \varepsilon=\frac{\gamma\eta\kappa}{\lambda+2\mu}
\end{gathered} \tag{8}$$

Here ω is the frequency of waves (P and S waves) (Figure 1) [19].

By delving into the details of wave propagation in various rocks, researchers can gain deeper insights into their behavior under different conditions, ultimately improving our understanding of their physical properties. In the context of linear thermoelasticity, where small deformations and temperature changes are assumed, the wave equations can be derived for both mechanical (elastic) and thermal waves. The wave numbers, which represent the spatial variation of the waves, are related to the frequency of the waves through the dispersion relation (8).

Body waves can be further sub-categorized into:

P waves (Primary waves)

S waves (Secondary waves)

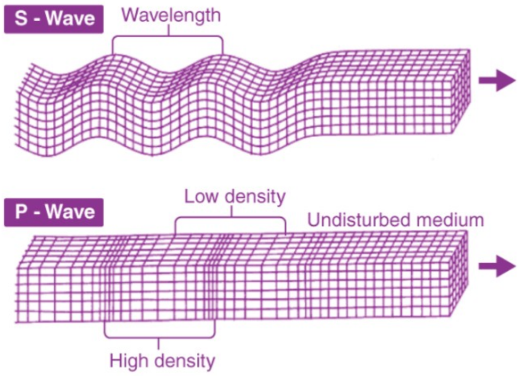


**Figure 1:** P and S waves distribution.

Wave numbers in the context of thermoelastic waves in rocks refer to the spatial frequency of the wave, essentially describing how the wave's amplitude varies with position. In linear thermoelasticity, where both small deformations and temperature changes are considered, the wave numbers help determine how waves, both mechanical (elastic) and thermal, propagate through the rock. The relationship between wave numbers and wave frequency is governed by the dispersion relation, which links these parameters and reveals how waves of different frequencies spread in the medium.

## 4.1. P Waves

P waves, or Primary waves (their wave numbers are four complex roots $\zeta_1^2, \zeta_2^2$ from (8)), are the earliest seismic waves recorded by seismographs and are distinguished by their high propagation speed. These waves can traverse solids, liquids, and gases, generating a sequence of compressions and rarefactions in the medium. Consequently, P waves are also known as pressure waves. Certain animals, such as dogs, can detect these waves before the surface waves of an earthquake reach the area, whereas humans typically perceive the effects only after the waves have interacted with the Earth's crust.

Longitudinal waves, such as P waves, are distributed through the Earth's interior during an earthquake, propagating as compressional waves that move parallel to the direction of wave travel. These waves travel through various layers of the Earth, including the crust, mantle, and core, and their propagation is influenced by the material properties of these layers, such as density and elasticity. As P waves encounter different geological formations, their speed and direction can change, leading to complex wave patterns. The distribution of these longitudinal waves is crucial for understanding the internal structure of the Earth and for interpreting seismic data, as variations in wave behavior provide insights into subsurface features and the nature of the seismic event [20,21].

## 4.2. S Waves

S waves, or secondary waves (their wave numbers are two real roots $\zeta_3^2$ from (8)), , are the second type of seismic waves to reach a seismograph during an earthquake. They travel more slowly than P waves and are restricted to movement through solid materials only. By analyzing the path of S waves as they traverse different layers of the Earth, scientists have determined that the Earth's outer core is in a liquid state. This insight was pivotal in understanding the composition and structure of the Earth's interior. It is after studying the trajectory of S waves through the layers of earth, scientists were able to conclude that the earth's outer core is liquid.

Transverse waves, such as S waves, exhibit a distinct propagation pattern during an earthquake. Unlike longitudinal waves that move in the direction of wave travel, transverse waves oscillate perpendicular to the direction of propagation. In the context of seismic activity, S waves create shear forces as they travel through the Earth's crust and upper mantle. These waves move more slowly compared to P waves and can only propagate through solid materials, not through liquids or gases. The nature of transverse waves allows them to cause significant ground shaking, which can contribute to the overall impact and damage of an earthquake. Their behavior and propagation are crucial for understanding the internal structure of the Earth and assessing seismic hazards.

Here we have calculated the dependence of the roots of the characteristic equation (wave numbers) (8) on the frequency $\varpi$ - 1 to 10 Hz for various rocks (GUI MatLab program code snippet is shown on Figure 2, dependence of wave numbers on frequency for different rocks - Figure 3-6).

```
epsilon = gamma_table.*eta_table.*k_table./(lambda_table+2*mu_table);
for j = 1:1:5
    % get complex and numeric values of out diff eq
    zeta1 = (1/2)*(w.^2/c1(j).^2 - 1i*w.*(1+epsilon(j))/k_table(j) + sqrt((w.^2/c1(j).^2 - 1i*w.*(1+epsilon(j))/k_table(j)).^2 + 4*1i*w.^3/(k_table(j)*c1(j).^2)));
    zeta2 = (1/2)*(w.^2/c1(j).^2 - 1i*w.*(1+epsilon(j))/k_table(j) - sqrt((w.^2/c1(j).^2 - 1i*w.*(1+epsilon(j))/k_table(j)).^2 + 4*1i*w.^3/(k_table(j)*c1(j).^2)));
    zeta3 = (w.^2/c2(j).^2);
    zeta1 = sqrt(zeta1);
    zeta2 = sqrt(zeta2);
    zeta3 = sqrt(zeta3);
    % display the graph
    figure;
    plot(w, zeta1, 'r', w, zeta2, 'b', w, zeta3, 'g');
    xlabel('Frequency (Hz)');
    ylabel('ζ_1 ζ_2 ζ_3');
    legend('ζ_1', 'ζ_2', 'ζ_3');
end
```

**Figure 2:** GUI MatLab program code snippet.

Different rocks have varying material properties that affect their wave propagation characteristics, such as density and thermal conductivity. These properties influence the wave numbers and, consequently, how waves travel through the rock. Accurate knowledge of these properties is crucial for predicting wave behavior and understanding the rock's response to thermal and mechanical stresses.

It's important to note that the specific values of these parameters for different rocks would need to be known or estimated to analyze the dependence of wave numbers on frequency for those particular rocks (Table 1). Additionally, the above equations are simplified for isotropic materials; anisotropic materials would involve more complex expressions. The study of coupled thermoelasticity in rocks is crucial for understanding their response to thermal and mechanical loading, which is relevant in geophysics and geomechanics.

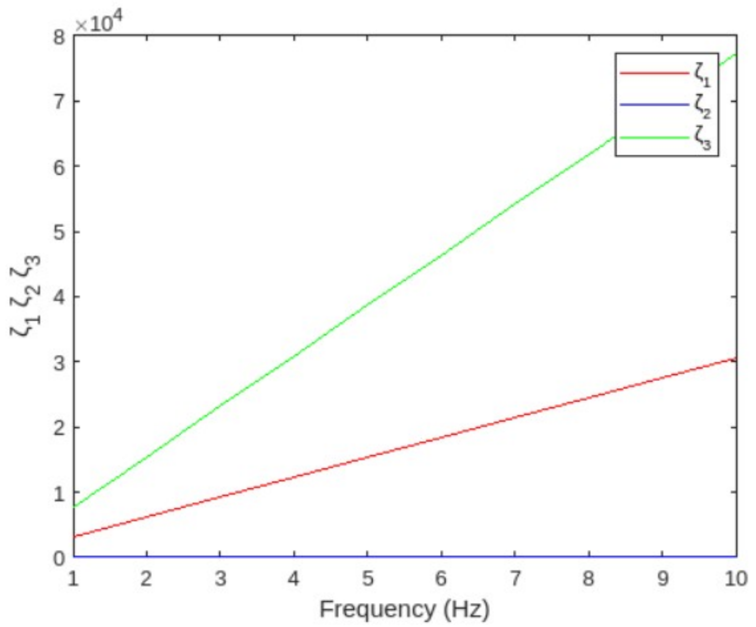


**Figure 3:** Dependence of wave numbers on frequency for shale.

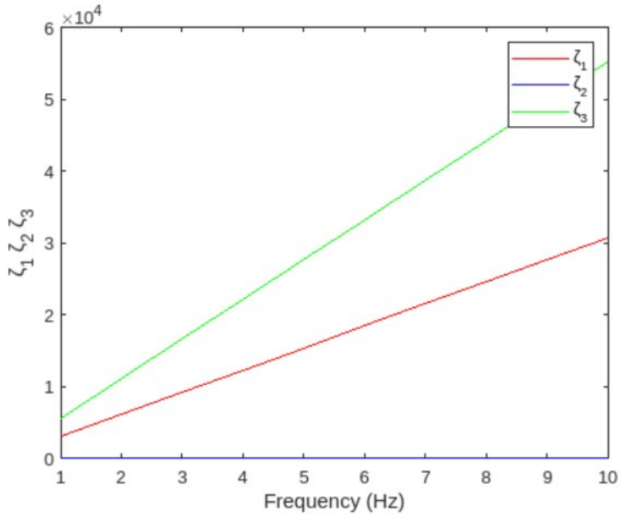


**Figure 4:** Dependence of wave numbers on frequency for limestone.

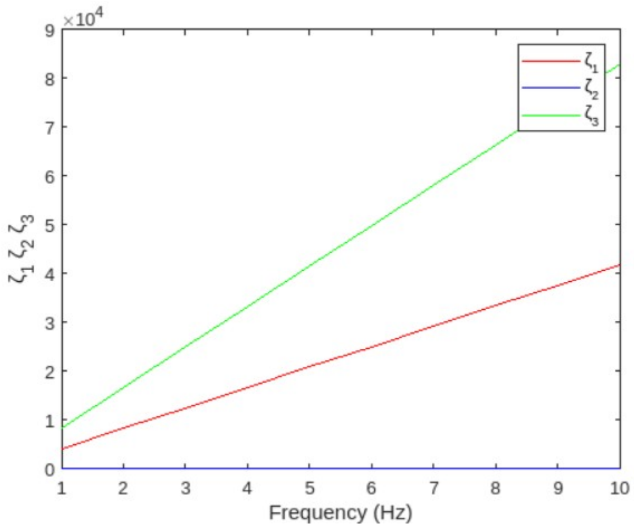


**Figure 5:** Dependence of wave numbers on frequency for granite.

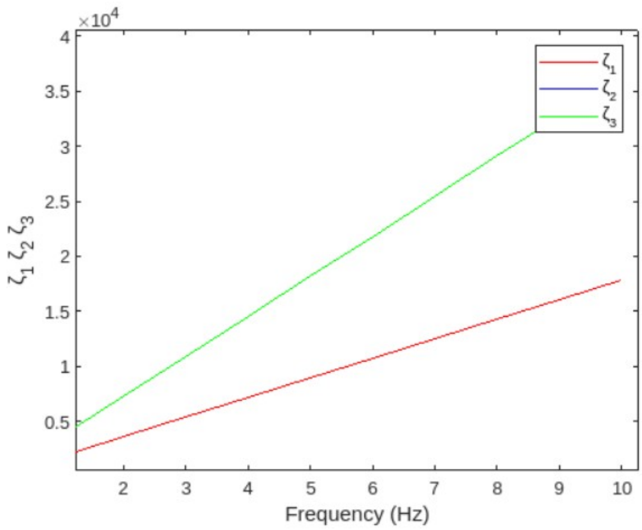


**Figure 6:** Dependence of wave numbers on frequency for siltstone.

Wave numbers, which describe the spatial frequency of seismic waves, are fundamental in characterizing earthquake behavior, particularly at a frequency of 10 Hz. At this frequency, the wave numbers help determine how waves propagate through different geological materials, influencing their speed and amplitude. By analyzing wave numbers, scientists can gain insights into the earthquake's impact on various rock types and the underlying structure of the Earth's crust. Understanding these characteristics is crucial for accurate seismic modeling, hazard assessment, and designing structures resilient to earthquake-induced stresses.

# 5. Computer implementation of mathematical model of coupled thermoelastodynamics in form of GUI MatLab Application

MATLAB, with its extensive computational capabilities, is widely used in thermoelastodynamics to model and analyze complex interactions between thermal and elastic phenomena. Its powerful mathematical toolbox allows for the efficient solving of partial differential equations governing thermoelastic behavior, enabling researchers to simulate how materials deform and respond to thermal changes. MATLAB’s scripting and programming environment provides flexibility for customizing simulations and performing in-depth analyses of thermoelastic systems.

The graphical user interface (GUI) in MATLAB further enhances its utility by offering a user-friendly platform for visualizing and interacting with simulation results. Through GUIs, users can create interactive tools to manipulate parameters, visualize thermoelastic wave propagation, and interpret data in real-time. This interface simplifies the process of model setup, execution, and analysis, making it accessible to both experienced researchers and those new to the field. By integrating MATLAB’s computational strength with intuitive GUI features, users can efficiently explore and understand complex thermoelastodynamic phenomena.

Here we have created the GUI MatLab application to calculate the problem thermoelastic waves distribution in the coupled thermoelastic media. The specific numerical methods is employed here to solve the boundary value problems of coupled thermoelasticity - method boundary integral equations using generalized functions [12,19].

These numerical methods work together within the MATLAB GUI to provide a robust framework for simulating coupled thermoelastic behavior in rocks. By integrating these techniques, users can obtain accurate predictions and insights into the thermal and mechanical interactions in geological contexts.

## 5.1. Input data for GUI MatLab application

In MATLAB's GUI for coupled thermoelasticity in rocks, input data is crucial for accurately simulating and analyzing the interactions between thermal and mechanical stresses. Users must provide detailed material properties such as thermal conductivity, elasticity, density, and specific heat capacity for the rocks under study. Additionally, boundary conditions, initial temperature distributions, and loading conditions must be specified to reflect real-world scenarios. The GUI facilitates the input of these parameters through user-friendly forms and data entry fields, allowing for easy customization and adjustment. This structured input process ensures that the simulations are based on precise and relevant data, enabling accurate predictions of thermoelastic behavior and helping to interpret the impact of thermal and mechanical interactions in geological contexts (Figure 7).

```
rho_table = [2.61,2.69,2.72,2.69,2.77];
rho_table = rho_table*0.001;
w = 1:1:10;

% input
lambda_table = [4.24,1.98,1.08,0.756,1];
lambda_table = lambda_table*10.^(-10);
mu_table = [1.974,0.45,0.89,0.39,2.296];
mu_table = mu_table*10.^(-10);
T = 20;
gamma_table = [13.3,2.5,2.65,1.5,5.2];
gamma_table = gamma_table * 10.^(-5);
eta_table = [1.7,0.44,0.32,0.29,0.6];
eta_table = eta_table * 10.^(-8);
k_table = [9.27,    9.86,   11.27,  10.28,  9.46];
k_table = k_table * 10.^(7);
% Calculation of wave speeds
c1 = sqrt((lambda_table + 2 * mu_table)./ rho_table);
c2 = sqrt(mu_table ./ rho_table);
```

| | Гранит | Песча-ник | Извест-няк | Алевро-лит | Сланцы |
|---|---|---|---|---|---|
| $\rho \cdot 10^{-3}$, [$кг/м^3$] | 2,61 | 2,69 | 2,72 | 2,69 | 2,77 |
| $E \cdot 10^{-10}$, [$Н/м^2$] | 4,02 | 4,13 | 5,2 | 3,87 | 5,25 |
| $\nu$ | 0,26 | 0,09 | 0,21 | 0,29 | 0.15 |
| $\lambda_T$, [$Вт/м \cdot K$] | 2,4 | 1,66 | 2,4 | 1,49 | 2,46 |
| $C_T$, [$Дж/Н \cdot K$] | 946 | 972 | 887 | 880 | 866 |
| $\alpha_T \cdot 10^5$, [$1/K$] | 0,8 | 0,64 | 0,53 | 0,5 | 0,68 |
| $v_p$, [$м/с$] | 5600 | 3272 | 3243 | 2390 | 4493 |
| $v_s$, [$м/с$] | 2750 | 1293 | 1808 | 1204 | 2879 |
| $\mu \cdot 10^{-10}$, [$кг/м\ с^2$] | 1,974 | 0,45 | 0,89 | 0,39 | 2,296 |
| $\lambda \cdot 10^{-10}$, [$кг/м\ с^2$] | 4,24 | 1,98 | 1,08 | 0,756 | 1 |
| $\gamma \cdot 10^{-5}$, [$Па/K$] | 13,3 | 2,5 | 2,65 | 1,5 | 5,2 |
| $\eta \cdot 10^{-8}$, [$K \cdot с/м^2$] | 1,7 | 0,44 | 0,32 | 0,29 | 0,6 |
| $\kappa \cdot 10^7$, [$м^2/с$] | 9,27 | 9,86 | 11,27 | 10,28 | 9,46 |

**Figure 7:** Input data program code snippet.

Input data for application will be implemented from (1) and (7) and calculated as:

$$\kappa = \lambda_0 / b_\varepsilon, \qquad \gamma \equiv (3\lambda + 2\mu)\alpha_t \tag{9}$$

$$\eta \equiv \frac{\gamma T_0}{\lambda_0}, \ \mu > 0, 3\lambda + 2\mu > 0, \gamma/\eta > 0, \kappa > 0.$$

In MATLAB's GUI for coupled thermoelasticity, inputting data is streamlined to ensure both efficiency and accuracy in simulations. Users can quickly enter material properties, boundary conditions, and loading scenarios using intuitive data entry forms. Once the data is inputted, MATLAB processes it rapidly, leveraging its computational power to perform complex calculations and simulations. The results are then swiftly visualized through dynamically generated graphs and numerical outputs, providing immediate feedback on thermoelastic behavior. This seamless integration of data input, calculation, and visualization allows for efficient exploration of various scenarios and facilitates a deeper understanding of the coupled thermal and mechanical responses in geological materials.

## 5.2. GUI MatLab Application

The creation of a MATLAB GUI application (Figure 8) for calculating wave numbers as a function of frequency for various rocks within the framework of coupled thermoelasticity involves several key steps. Initially, the GUI is designed to facilitate user input for critical parameters such as material properties, including density, thermal conductivity, and elastic moduli of different rocks. Users can specify these parameters through intuitive input fields and drop-down menus, allowing for flexible and accurate modeling of various geological scenarios. The application is programmed to incorporate these inputs into computational models that solve the relevant partial differential equations governing wave propagation in coupled thermoelastic systems.

Once the input data is provided, the GUI facilitates the rapid computation of wave numbers across a range of frequencies. The application uses MATLAB's robust numerical algorithms to perform these calculations, ensuring high precision and efficiency. Results are then displayed in real-time through dynamically updated graphs and tables, enabling users to visualize how wave numbers vary with frequency for the specified rock types. This interactive and visual approach not only aids in the immediate interpretation of results but also supports iterative analysis, making it easier for researchers to explore different scenarios and refine their models based on the computed data.

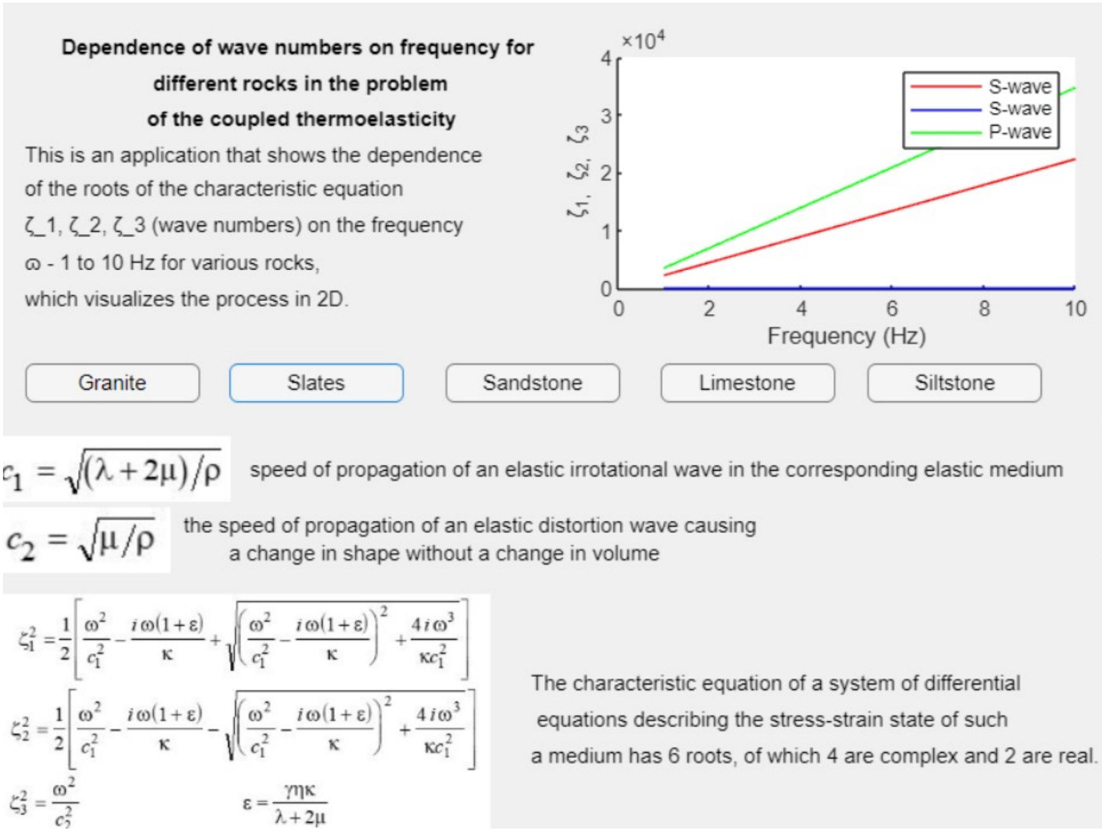


**Figure 8:** Interface of GUI MatLab Application.

This application focuses on graphical and numerical calculations of wave numbers as a function of frequency. The application analyzes wave numbers corresponding to primary (P) and secondary (S) waves of thermoelastic waves at frequencies ranging from 0 to 10 Hz. This allows one to study how wave behavior changes with frequency, which is key to understanding the interaction of wave processes with geological conditions.

The application provides sets of physical properties for five different rocks extracted from deposits in Kazakhstan. These data include density, thermal conductivity, and elastic moduli, which are used to model the behavior of waves in different types of rocks. Such sets of properties allow one to conduct a detailed study that takes into account the specifics of each type of rock and their influence on the propagation of thermoelastic waves.

The calculation process in the application includes not only the determination of wave numbers, but also the study of the velocities of longitudinal and transverse waves. These velocities are important indicators for assessing the behavior of rocks under earthquakes and other dynamic effects. Wave velocity analysis allows identifying the features of wave propagation in various geological environments and assessing their impact on the stability and safety of structures. Such problems have been discussed in various sources [22-25]; however, specific issues related to particular rock types have not been addressed, with a primary focus on the theoretical aspects of these types of problems.

In addition, the application provides the user with the ability to visualize the calculation results in the form of graphs and tables, which simplifies the interpretation of data and allows for comparative analysis. Such graphical representations help to better understand the dependence of wave numbers and velocities on wave frequency and facilitate a more in-depth analysis of the behavior of rocks under various conditions of seismic and thermoelastic effects.

## 6. Conclusion

In conclusion, the developed application for MATLAB provides an effective tool for graphical and numerical analysis of wave numbers and wave velocities in thermoelastic systems. It allows to study the influence of frequency on the behavior of primary and secondary waves in various rocks using the physical characteristics of rocks from Kazakhstan deposits. This approach provides valuable information on how geological conditions affect wave propagation, which is important for seismic risk assessment and design of sustainable structures.

The application also facilitates visualization and interpretation of calculation results, providing the user with clear graphs and tables. This allows for a more in-depth analysis of the dependence of wave numbers and velocities on frequency, improving the understanding of the thermoelastic behavior of materials. Such capabilities make the tool useful for both scientific research and practical applications in the field of geophysics and engineering, contributing to a more accurate assessment of geological risks and the development of effective solutions for their management.

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## Declaration on Generative AI

The author have not employed any Generative AI tools.

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