Modeling Vegetation Index Dynamics in GIS Based on Remote Observations*

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Abstract

The primary objective of this study is to develop and implement methods for modeling vegetation index dynamics based on remote sensing data, focusing on their application in agricultural monitoring systems. This paper explores the use of the Monod system of differential equations for modeling the dynamics of vegetation indices, particularly NDVI, which allows for more accurate prediction of plant development under both normal and stress conditions. An analysis of the structural and parametric identification of the Monod model is conducted, addressing the complexity of nonlinear parameter estimation in differential equation systems. From the methodological perspective, a specialized approach for parameter identification is proposed, which accounts for the nonlinearity of the model and utilizes a combination of non-uniform and uniform grids for effective parameter space exploration. The application of the Levenberg-Marquardt gradient method for refining initial parameter estimates allows achieving high accuracy in modeling vegetation index dynamics. The findings of this study can be valuable for agricultural practitioners, researchers, and decision-makers interested in early stress detection and yield prediction in crop production systems.

Keywords

classification, vegetation indices, NDVI, remote sensing, Monod system of differential equations, parametric identification, Levenberg-Marquardt method, plant growth modeling, geographic information systems, temperature stress, yield prediction.

1. Introduction

Remote sensing is a useful technique for direct non-destructive monitoring of crop conditions [1,2]. Systematization of remote sensing results of vegetation indices in geographic information systems (GIS) is a key step for effective analysis and use of this data. For efficient utilization of remote sensing results of vegetation indices such as NDVI, NDRE, they need to be systematized in geographic information systems (GIS). This process allows converting raw data into valuable information for monitoring vegetation conditions, predicting yields, and making informed decisions in agriculture.

The aim of this study is to develop and implement methods for modeling vegetation index dynamics based on remote sensing data, particularly adapting the Monod system of differential equations for predicting NDVI and MTCI indicators. The research focuses on developing a methodology for structural and parametric identification of the Monod model, taking into account the nonlinearity of its parameters, and experimental verification of this methodology's effectiveness on real data. Such creation of tools for early detection of stresses in plant development and prediction of future yields will allow farmers to make more informed decisions regarding agronomic measures.

The first step is geospatial referencing of data. Vegetation indices are stored as raster layers, where each pixel has an index value, or as vector layers for analyzing individual areas. These layers are superimposed on other geospatial data, such as cadastral maps or meteorological data, providing a comprehensive picture. Spatial databases are used to store and manage large volumes of data, enabling spatial queries and analysis. Attribute data associated with geospatial objects are stored in tabular databases used for statistical analysis and report generation.

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An important aspect is the creation of time series that reflect the dynamics of vegetation indices for each geospatial object. This allows tracking changes in vegetation condition over time and identifying anomalies. Image classification and segmentation enable the identification of zones with different vegetation conditions and individual objects, such as damaged areas. Integration with meteorological, agrochemical, and cadastral data allows analysis of various factors' impact on vegetation condition. Using free software such as QGIS automates and simplifies the process of systematizing and analyzing remote sensing data.

Typically, vegetation indices that are widely used in crop models are calculated using spectral reflectance coefficients. These vegetation indices, evaluated from data collected by portable optical devices on unmanned aerial vehicles, have been used for grain yield prediction [3, 4, 5]. An interesting study [6] aimed to evaluate the most suitable vegetation index for determining crop response to elevated air temperature, heat stress, and herbicide damage. Spectral reflectance coefficients, yield components, and growth parameters such as plant height, leaf area index (LAI), and above-ground dry matter of rice cultivated in a temperature gradient field chamber to simulate global warming conditions were observed from 2016 to 2018.

The relationships between vegetation indices and yield parameters were evaluated considering stress conditions. NDVI, MTCI, and cumulative growing degree-days formed sigmoidal curves with high R-squared values under normal growth conditions, but their amplitudes significantly decreased with herbicide damage. Vegetation indices, particularly NDVI and MTCI, revealed slow growth and crop development caused by stress using relationships with cumulative GDD. Maximum values of these indices are often used for crop yield prediction.

These observations highlight the importance of predicting the dynamics of NDVI and MTCI indices for early diagnosis of stresses in plant development as well as their maximum values for forecasting future yields. However, modeling the accumulation of vegetation index values using logistic regression dependencies is difficult due to the weak predictive properties of logistic curves. At the same time, modeling logistic curves using Monod differential equation systems is a recognized tool for describing growth processes that are limited by resources, making it more realistic than exponential growth. Adapting the Monod model to experimental conditions and observed values requires its structural and parametric identification.

Therefore, implementing methods of structural and parametric identification of the Monod model for predicting the dynamics of vegetation indices NDVI and MTCI becomes an important tool for supporting decision-making regarding agronomic measures to ensure optimal development of agricultural plants, which is the focus of this research.

2. Literature review

The issue of identifying parameters of the Monod model has been addressed in recent years in open publications [7-11]. In the work [7], dedicated to finding a thermodynamic interpretation of the Monod equation, a key model relationship between the specific growth rate of microorganisms μ and substrate concentration S is examined. It limits the mentioned growth in a bulk solution according to the following relationship:

$$\mu = \mu_{max} \frac{S}{K_S + S} \tag{1}$$

It is shown that Ks is a constant, inverse to the equilibrium constant of the balanced microbial growth reaction. When the equilibrium constant is very large, meaning Ks is very small, μ in this case it will approach μ_{max}. On the other hand, when Ks is very large, an extremely small amount of biomass is formed, and the equilibrium position lies far from the substrate volume. This study provides insight into the large variations observed in Ks values typically reported in the literature. In the work [8], it is noted that identifying parameter values of the Monod model based on experimental data is a complex problem, which is proposed to be solved through experimental planning organization. In the work [9], explicit and implicit schemes for solving the identified system of Monod differential equations are presented. To eliminate the stiffness phenomenon when the concentration of microorganisms approaches zero, an asymptotic representation is used to construct the solution for this section.

In the work [10], a simplified Monod model is considered, presenting only the dynamics of microorganisms. Through certain transformations, it is reduced to a linear regression model, which allows for a simple parameter identification procedure. In the work [11], processes in a bioreactor are analyzed using the complete Monod model. Matlab package tools are used to construct its solutions, however, the issues of model parameter identification are not addressed.

Thus, despite their importance, the issues of structural and parametric identification of the Monod model in application to representing vegetation index dynamics are insufficiently covered.

3. Structural and Parametric Identification of Vegetation Index Dynamics Model in the Form of Monod Differential Equations System

In the structure of the vegetation index dynamics model depending on the cumulative GDD index, we will account for the irreversibility of vegetation index value accumulation, therefore the microorganism concentration reduction coefficient in the model is set to zero. Let's denote the current level of the vegetation index as X, the volume of the GDD indicator as t, and also introduce some resource value S of the agrobiological system, which can provide only a certain limited yield during one season. In this case, the Monod system will take the form

$$\begin{cases} \frac{d}{dt}X(t) = p_1 \frac{X(t)S(t)}{p_2+S(t)}, \\ \frac{d}{dt}S(t) = -p_3 \frac{X(t)S(t)}{p_2+S(t)}, \end{cases} \quad (2)$$

$$X(0) = X_0, \quad S(0) = S_0 \quad (3)$$

In this model, the parameter p_1 determines the impact of the interaction intensity between the current indicator level and the current system resource level p_2, being the aforementioned characteristic of balanced index growth. The parameter p_3 determines the intensity of yield resource depletion, X_0 determines the starting value of the modeled vegetation index, and S_0 represents the volume of potential yield.

The task of parametric identification of model (2)-(3) is to select values of its parameters that best align with experimental data. Considering that according to the nature of the analyzed processes, the modeled variables do not demonstrate sharp random fluctuations, the quality of approximation of observed values can be evaluated using the mean square criterion:

$$Q(\vec{p}) = \sum_{j=1}^{N}\left(\tilde{X}\left(t_j,\vec{p}\right) - X_j^e\right)^2 \quad (4)$$

When the model parameters were known, according to relationships (2)-(3), to construct the values of model variables, it would be necessary to solve a system of nonlinear differential equations.

In the general case, the system parameter values are unknown. The parameter p_2 enters the differential equations of the system nonlinearly and can vary over a wide range. These changes lead to cardinal changes in the nature of the system solution. Therefore, on a sufficiently coarse grid of parameter values p_2, the sequence of values of the model identification quality functional demonstrates the property of unimodality.

As parameter p_2 values increase, their influence on the result decreases. This means that for an effective search for optimal values of this parameter, it is necessary to use a grid with different steps. In particular, the step of changing values on the grid should increase with the increase of the parameter itself. For this purpose, a geometric progression can be applied

$$P_{2,j}^0 \epsilon \left\{\frac{B}{2} B^j S_0\right\}. \quad (5)$$

Parameter B is selected experimentally to ensure sufficient shift in the process activity maximum when parameter p_2 values move along grid nodes (5). Other model parameters are determined based on the selected limiting parameter and approximate difference representation of differential equations for individual time points. The minimum of this function with a single extremum outlines the search zone for parameters of the model being identified.

In the defined area, minimization of the quality criterion depends not only on changes in one parameter p_2, but also on the interaction of all parameters. Therefore, the search zone for parameter p_2 values is covered with a uniform grid. For each parameter value, corresponding values of other parameters are calculated using difference relations. These values are then refined by a modified gradient method. Among the refined parameter sets, the one that minimizes the maximum relative error between modeled and actually observed data is chosen.

Let's now proceed to the analysis of the system equations (2)-(3). They contain only one unknown parameter. Therefore, by constructing an approximate representation of the equation at one point, we can estimate the value of this parameter. For construction, we will select the point of reaching the mean value of the vegetative indicator variable, where its change tendency is close to linear. First, let's estimate the derivatives of the indicator and yield reserve at the selected point using the following relationships:

$$D_{X,J} = (X_{J+1}^e - X_{J-1}^e)/(t_{J+1}^e - t_{J-1}^e), \quad (6)$$

$$D_{S,J} = (S_{J+1}^e - S_{J-1}^e)/(t_{J+1}^e - t_{J-1}^e). \tag{7}$$

The obtained values allow constructing an approximate representation of the differential equations at the moment of reaching the mean value of the vegetative indicator variable:

$$D_{X,J} \approx p_1 X_J^e S_J^e/(p_2 + S_J^e)\,, \tag{8}$$
$$D_{S,J} \approx -p_3 X_J^e S_J^e/(p_2 + S_J^e). \tag{9}$$

Based on the constructed relationships, we build an estimation of model parameters:

$$p_1 \approx \frac{(p_2+S_J^e)D_{X,J}}{X_J^e S_J^e}, \qquad 10)$$
$$p_3 \approx -\frac{(p_2+S_J^e)D_{S,J}}{X_J^e S_J^e}. \tag{11}$$

The basis of the Monod model identification method lies in iterating through values of parameter p_2 on a certain uniform grid, for each value of which corresponding initial values of other model parameters are constructed using established difference relationships. Subsequently, all initial model parameter values are refined by the gradient method according to the minimum criterion of functional, presented by relationship (4).

Let's examine in more detail the process of constructing the mentioned uniform grid. Based on the proposed progression (5), we build a non-uniform grid for selecting the base point for constructing the next uniform grid of refined search. The non-uniform grid, built on the basis of geometric progression, is described by the representation:

$$W_2(k_{min}, k_{max}) = \left\{\frac{B}{2} B^j S_0\right\} \tag{12}$$

The construction of the non-uniform grid begins from the point associated with half of the initial yield reserve, since such a parameter p_2 value is a satisfactory initial value for a large number of practically important processes.

$$W_2(k_{min}^0, k_{max}^0) = \{P_{4,k_0}^0\}\,, \quad k_{min}^0 = k_{max}^0 = k_0 = -1, \tag{13}$$

Further, we supplement the non-uniform grid by decreasing and increasing the parameter p_2 value, provided that the relative decrease of the minimum value on the expanded grid relative to the minimum value of the previous grid configuration obtains a relative decrease greater than the value δ_Q, which is established experimentally.

Which means that under the following condition,

$$\frac{\min\limits_{p_2 \in W_2(k_{min}, k_0)} Q(p_2) - \min\limits_{p_2 \in W_2(k_{min}-1, k_0)} Q(p_2)}{\min\limits_{p_2 \in W_2(k_{min}, k_0)} Q(p_2)} > \delta_Q \Rightarrow k_{min} \coloneqq k_{min} - 1, \tag{14}$$

the lower limit of the orders of elements of the geometric progression that form the grid decreases by one, and when the condition is met

$$\frac{\min\limits_{p_2 \in W_2(k_{min}, k_{\max})} Q(p_2) - \min\limits_{p_2 \in W_2(k_{min}, k_{max}+1)} Q(p_2)}{\min\limits_{p_2 \in W_2(k_{min}, k_{\max})} Q(p_2)} > \delta_Q \Rightarrow k_{max} \coloneqq k_{max} + 1, \tag{15}$$

the upper limit of the orders of elements of the geometric progression increases by one. After completing the process of filling the non-uniform grid, the minimum value of the quality functional on it indicates the base value of the nonlinear parameter of the Monod model $P_{2,k_{base}}$

$$P_{2,k_{base}} = \underset{p_2 \in W_2(k_{min}, k_{\max})}{\operatorname{argmin}} \{Q(p_2)\} \tag{16}$$

The selected parameter p_2 value determines the base point and step for constructing a set of points for the purpose of more accurately selecting the initial values of the model parameters. This set is defined using a uniform grid of the form

$$P_{2,k_{base},l} \epsilon W_{4,k_{base}} \equiv \{B^{-k_{base}} l S_0\}_{l=1}^{\left[\frac{B}{3}\right]B} \tag{17}$$

first value of which is less than the value of the base point (for the base point $l = [B/2]$),), and the last value approaches the node of the non-uniform grid, which comes after the base one (for the point following the base one $l = [B/2]$)B).

Each value from the grid is assigned to parameter p_2, with which approximate estimates of other model parameters are constructed based on established difference relationships (10)-(11). Subsequently, they are refined using the modified Levenberg-Marquardt gradient method based on criterion (4). Among these initial model parameter values, the one that provides the minimum of the smallest of the maximum relative errors at the model identification points is selected.

4. Numerical experiments

The effectiveness of the proposed approach in modeling the dynamics of the NDVI vegetation index is clearly demonstrated by the graphs in Figure 1.

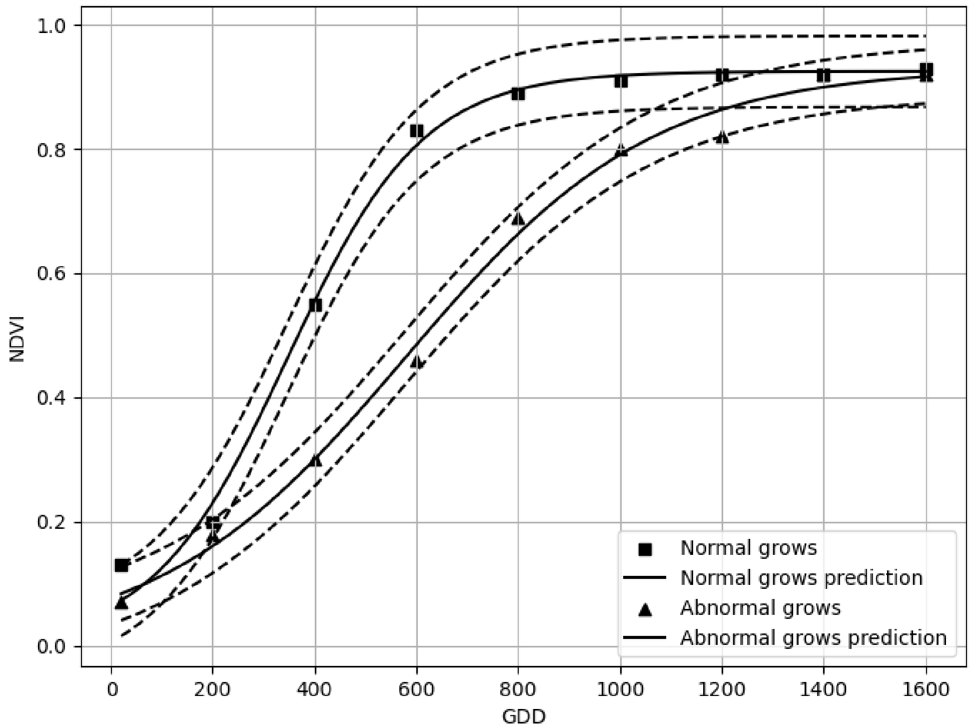

Figure 1: Modeling the dynamics of the NDVI vegetation index in normal and anomalous growing conditions

Data for modeling were taken from work [6], which presented observations of vegetation index dynamics when growing rice under normal conditions and under conditions with excess average daily temperatures throughout the season. As a result of applying the described Monod model identification methodology, acceptable levels of maximum relative modeling errors were achieved, which were 6.15% for the case of normal growth and 4.70% for the case of temperature shock.

Conclusions

The paper analyzes the role and characteristics of monitoring vegetation index dynamics based on remote observations. It proposes modeling such dynamics using the Monod system of differential equations. The structure of this system has been established to correspond to the nature of the observed data. A method for identifying the constructed system based on the minimum mean square error criterion is proposed, with the main element being an algorithm for forming initial model parameter values taking into account its nonlinearity. Subsequently, the initial model values were refined using the Levenberg-Marquardt gradient method. The effectiveness of the proposed methodology has been confirmed experimentally. A maximum relative error level of approximately 5-6% was recorded.

The effectiveness of the proposed methodology has been confirmed experimentally, with a maximum relative error level of approximately 5-6% recorded. Despite the limitations of the study to data for rice cultivation under normal and temperature stress conditions, the developed methods demonstrate significant potential for further expansion to other vegetation indices and integration into geographic information systems. Promising directions for further research include studying the influence of different types of stresses on model parameters, developing methods for early diagnosis of plant stresses, and creating software to automate the process of modeling vegetation index dynamics and making agronomic decisions.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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