Robust segmented regression with heteroscedasticity based on moving triangle

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Abstract

Robust regressions allow reducing the influence of one or more values of the sample population during the mathematical model building. This paper considers the problem of choosing the best segmented regression model for the specific example of econometric data and analyzing its advantages as a result of comparison with other options of approximations using higher-order polynomials. The model building consists of several steps. At the first step, the value of the abscissas of the switching points was optimized using the sliding setsquare method. Since the data are heteroscedastic, a new procedure of sliding triangles was proposed for its qualitative identification. This method can be considered as a generalization of the moving average, which is widely used during the analysis of time series. In general, the sliding triangle procedure makes it possible to quite reasonably construct the heteroscedasticity equation. The heteroscedasticity equation was used to calculate the confidence interval of the data variation relative to the final best regression. Thus, the proposed methodology for constructing mathematical models taking into account heteroscedasticity has the property of robustness in terms of reducing the influence of samples with large values.

Keywords

mathematical model building, ordinary least squares, minimization of absolute deviations, minimization of the range of the cumulative residual curve, robust regression, moving average, outlier correction

1. Introduction

The problem of mathematical models building for detection of regularities between natural phenomena, various processes and parameters is widely used today in different industries [1, 2, 3]. An accurate mathematical model is the basis for development of new technologies [4, 5], optimization of technological processes [6, 7], forecasting of possible events and phenomena [8, 9], support of decision-making regarding control and corrective actions [10, 11], and others.

While mathematical models building, many factors are taken into account. On the one hand, the model should not be too simple, since in such cases it may have unsatisfactory accuracy [12, 13]. Significant complication of the model may lead to overfitting and unsatisfactory forecasting results [14, 15]. Therefore, it is necessary to find a trade off between the accuracy of the model in the range of observed values and the forecasting properties.

One of the main tools for mathematical models building from a statistical point of view is regression analysis [16]. Well-known and widely used algorithms are least squares regression, lasso, ridge, least absolute deviation regression, and others [17, 18]. In these cases, a single function for approximation is usually used.

When approximating using a single general function over the entire range of data variation, the standard deviation is determined as a result of its averaging. In this case, the assumption is made that the standard deviation is constant for all dataset. Very often in econometric problems, data are described by non-stationary random processes, so it is incorrect to assume the constant standard deviation. Usually, such processes in econometrics have the property of heteroscedasticity [19, 20, 21].

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2. Literature review and problem statement

Regression models give possibility to establish a correlation between an outcome variable and one or more explanatory variables [22, 23]. Often, these models are used to solve forecasting problems, detect trends (including changes in the geometric structure of data), assess the information content of the influence of explanatory variables on the outcome variable, and in some cases, to solve a classification problem [24, 25, 26]. Regression model is an integral part of machine learning systems and artificial intelligence [27, 28].

The widespread use of regression models is explained by its main advantages, among which are simplicity, interpretability, the possibility of using various approximating functions, and ease of implementation using software [29, 30].

A new direction in the field of regression analysis today is robust regression [31, 32]. This approach to regression models building is due to the possibility of the presence of highly noisy values, outliers, and non-stationarities in the analyzed datasets [33, 34]. In addition, robust regression makes it possible to build models in conditions of non-Gaussian errors and the presence of heteroscedasticity.

Heteroscedasticity occurs when different explanatory variables have different standard deviations [35, 36]. Failure to take heteroscedasticity into account may result in unsatisfactory forecasting results due to an erroneous decision about the significance of a particular explanatory variable. The main methods for mathematical models building in conditions of heteroscedasticity are the weighted least squares method, variable transformation (including taking the logarithm of the outcome variable), and other alternative methods of regression analysis.

In case of heteroscedasticity, the first thing to do is to decide on its presence. Today, the literature provides a large number of tests, including the Goldfeld-Quandt test [37], Breusch-Pagan test [38], White test [39], and others. In the paper [40], a numerical measure of heteroscedasticity was also proposed. The next step after detection is the calculation of weighting coefficients, which make adjustments to the regression model.

The aim of this paper is to justify the use of robust segmented model in conditions of heteroscedasticity, as well as compare it with classical regression models based on the ordinary least squares method. To achieve this aim, the paper will solve specific objectives: a) justification of the method for optimizing the switching points of the regression model segments, b) development of the moving triangle method as a generalization of the moving average method, c) presentation of the new methodology using a specific numerical example.

3. Materials and methods

3.1. Sliding setsquare method

Consider methodology on a specific numerical example. The initial data for the analysis are given in Table 1.

In the book [41] two absolutely different processes (one decreasing and one increasing) were approximated by a single straight line. This approach is too simplified and gives an overestimated approximation error.

As a result of visual data analysis, an assumption was made about rationally dividing the data into four segments. Therefore, approximate values were adopted for the abscissas of three switching points.

To accurately determine the abscissas of the switching points, a heuristic approach was used. In this case, the dataset was divided into sections, on each of which only one switching point was optimized. This method can be considered as a sliding setsquare method, which sequentially moves along the entire range of data.

Let's consider the step-by-step procedure for applying the sliding setsquare method.

Step 1. We will approximate 9 points using two-segment linear regression. To do this, for five variants of the abscissa values of the switching point (located from the third to seventh points), data approximations were performed using the ordinary least squares (OLS) method. The equation of the

Table 1
Initial Dataset

#	x	y	#	x	y
1	8.8	0.36	10	15.5	0.59
2	9.4	0.21	11	16.7	0.9
3	10	0.08	12	17.7	0.95
4	10.6	0.2	13	18.6	0.82
5	11	0.1	14	19.7	1.04
6	11.9	0.12	15	21.1	1.53
7	12.7	0.41	16	22.8	1.94
8	13.5	0.5	17	23.9	1.75
9	14.3	0.43	18	25.2	1.99

Table 2
The Setsquare Parameters at the First Step

The abscissa of switching point	Parameter a	Parameter b	Parameter c	Standard deviation
10	2.507	−0.244	0.343	0.045
10.6	1.416	−0.125	0.237	0.057
11	1.129	−0.094	0.224	0.054
11.9	0.553	−0.034	0.199	0.074
12.7	0.035	0.017	0.13	0.121

Table 3
The Setsquare Parameters at the Second Step

The abscissa of switching point	Parameter a	Parameter b	Parameter c	Standard deviation
12.7	−0.9092	0.0964	0.0305	0.062
13.5	−0.9786	0.1027	0.0273	0.06
14.3	−0.9088	0.0972	0.0485	0.055
15.5	−0.9749	0.1029	0.0662	0.054
16.7	−1.1478	0.1171	0.0257	0.062

two-segment regression is:

$$y(x) = a + bx + c(x - x_1)_+, \tag{1}$$

where $(x - x_1)_+$ is modular function, x_1 is abscissa of switching point for the sliding setsquare. In this case

$$(x - x_1)_+ = \frac{(x - x_1) + \mid x - x_1 \mid}{2}. \tag{2}$$

The results of the calculations of the setsquare parameters are given in Table 2.

Next, we approximate the data from Table 2 (the dependence of the standard deviation on the abscissa of the switching point) with a second-order parabola using the OLS method. As a result, we obtain an equation of the following type:

$$\sigma(x_1) = 1.337 - 0.25x_1 + 0.012x_1^2.$$

The optimum of this parabola is at the point with the abscissa $x_1 = 10.294$.

Step 2. We move the setsquare so that its origin is in the next point after the abscissa of the first switching point. After that, we approximate the obtained nine points using two-segmented linear regression. The calculation results are given in Table 3.

Next, we approximate the data from Table 3 (the dependence of the standard deviation on the abscissa of the switching point) with a second-order parabola using the OLS method. As a result, we obtain an

Table 4
The Setsquare Parameters at the Third Step

The abscissa of switching point	Parameter a	Parameter b	Parameter c	Standard deviation
17.7	−0.905	0.1	0.0592	0.184
18.6	−0.781	0.0941	0.0759	0.169
19.7	−1.224	0.12	0.0506	0.179
21.1	−1.801	0.153	−0.00714	0.193
22.8	−2.032	0.166	−0.0862	0.173

equation of the following type:

$$\sigma(x_1) = 0.424 - 0.0501x_1 + 0.00169x_1^2.$$

The optimum of this parabola is at the point with the abscissa $x_1 = 14.754$.

Step 3. We move the setsquare so that its origin is in the next point after the abscissa of the second switching point. After that, we approximate the obtained nine points using two-segmented linear regression. The calculation results are given in Table 4.

Next, we approximate the data from Table 3 (the dependence of the standard deviation on the abscissa of the switching point) with a second-order parabola using the OLS method. As a result, we obtain an equation of the following type:

$$\sigma(x_1) = -0.196 + 0.0373x_1 - 0.000915x_1^2.$$

The optimum of this parabola is at the point with the abscissa $x_1 = 20.355$.

Step 4. For the obtained values of the three switching points, we perform data approximation using four-segmented regression and OLS method. As a result, we obtain the equation

$$y(x) = 1.936 - 0.181x + 0.287(x - 10.296)_+ + 0.0237(x - 14.754)_+ + 0.0401(x - 20.355)_+. \quad (3)$$

A visual representation of the obtained regression is shown in Figure 1.

Visual analysis (Figure 1) and comparison of the coefficients of the regression model allow to make an assumption about the possibility of combining the second and third segments. Merging the two segments allows to simplify the mathematical model. As a result, we obtain a three-segmented regression model based on the OLS of the form:

$$y(x) = 2.138 - 0.204x + 0.323(x - 10.296)_+ + 0.0551(x - 20.355)_+. \quad (4)$$

A visual representation of the obtained regression is shown in Figure 2.

3.2. Calculation of the heteroscedasticity equation

Visual analysis of the data does not provide a clear assumption on the heteroscedasticity presence. Therefore, it is necessary to conduct a more correct statistical analysis, which can be done using a new approach.

To assess heteroscedasticity, this paper uses a new approach based on a moving triangle. This method can be considered as a generalization of the moving average method, which is often used in the analysis of time series.

The moving triangle method expands the capabilities of statistical analysis, since it makes it possible to establish the dependence of the change in the standard deviation on the value of the approximated variable in a local observation area.

The heteroscedasticity equation is the dependence of current standard deviations on the corresponding values of the approximating function.

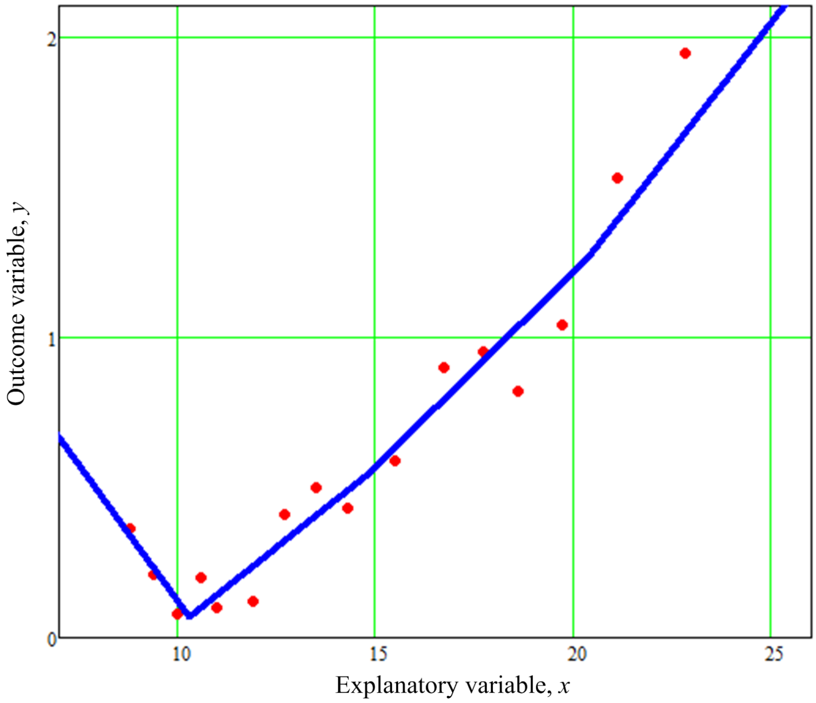

Figure 1: Data fitting using four-segmented regression.

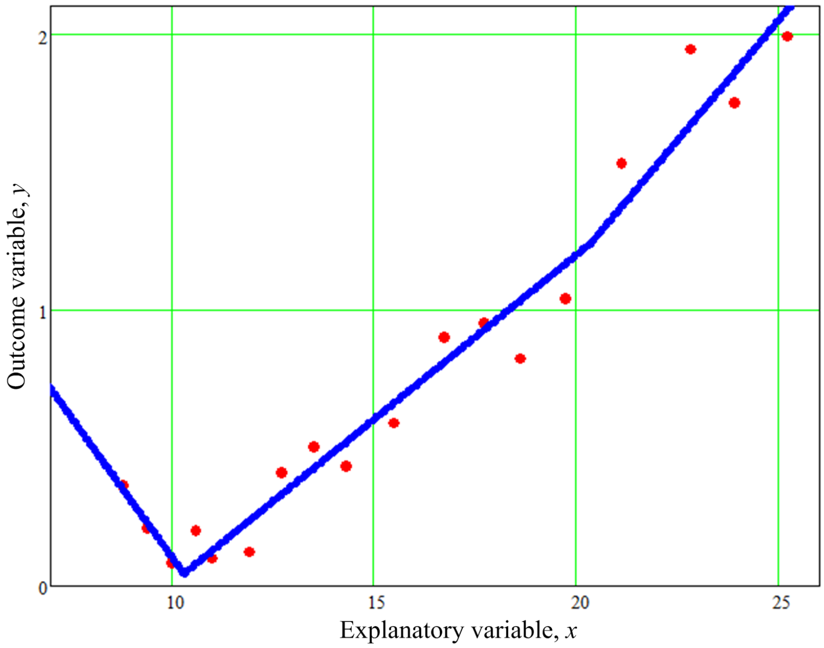

Figure 2: Data fitting using three-segmented regression.

The indicator that characterizes the current standard deviation is a segment drawn from the vertex of the triangle to its opposite side, parallel to the ordinate axis. In this paper, the designated segment will be called the measure of the triangle.

Visual analysis of the data and constructed triangles shows that in the case of large values of the approximating variable, the area and dimensions of the triangle will also be large. Accordingly, the heteroscedasticity indicator will also be overestimated.

The principle of constructing triangles is shown in Figure 3.

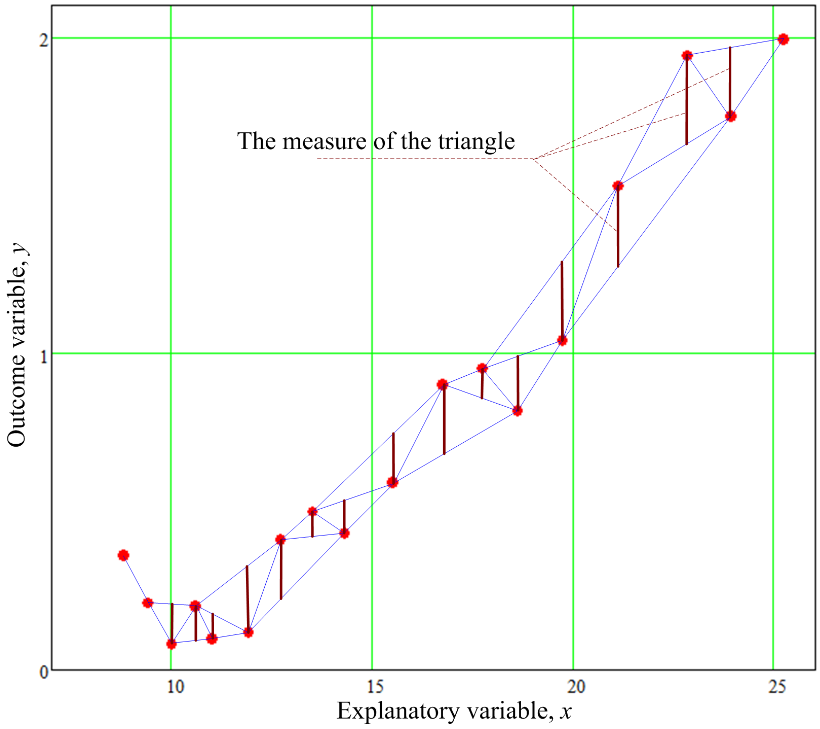

Figure 3: The principle of constructing the sliding triangle.

Table 5
The Parameters for Heteroscedasticity Equation Calculation

$\overline{y}$	σ	$\overline{y}$	σ	$\overline{y}$	σ	$\overline{y}$	σ
0.217	0.001	0.21	0.144	0.64	0.075	1.13	0.092
0.163	0.125	0.343	0.1	0.813	0.114	1.503	0.084
0.127	0.108	0.447	0.08	0.89	0.092	1.74	0.276
0.14	0.075	0.507	0.106	0.937	0.171	1.893	0.213

Some triangles may merge into a line. In this case, the measure of the triangle will tend to zero. To construct the heteroscedasticity equation, the measures of all triangles were calculated (to estimate the current standard deviation σ). In this case, to estimate the current value of the ordinate $\overline{y}$, the ordinates were averaged over the three vertices of the triangle (which is the implementation of the moving average method). The calculation results are given in Table 5.

We approximate the data presented in Table 5 using a linear function and OLS. As a result, we get an equation of the type:

$$\sigma(y) = 0.0684 + 0.0658y.$$

The equation of heteroscedasticity is shown in Figure 4.

To calculate the coefficients of heteroscedasticity, we use the formula

$$W_i = \left(\frac{y_{av}}{y(x_i)}\right)^2, \tag{5}$$

where y_{av} is expected value of outcome variable, $y(x_i)$ is the current value calculated by the best regression model (4) obtained using OLS. Each point is determined by the value of the abscissa of the current empirical value, after which it is substituted into the equation (4). Since weights are calculated for the OLS method, the obtained fraction is squared.

The values of the weight coefficients of heteroscedasticity are given in Table 6.

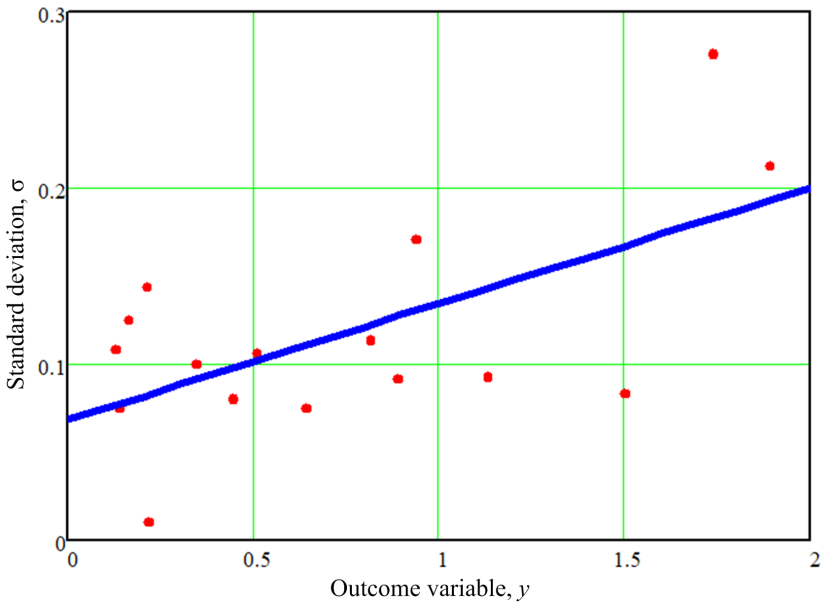

Figure 4: The heteroscedasticity equation.

Table 6
The Weight Coefficients of Heteroscedasticity

i	W_i	i	W_i	i	W_i	i	W_i	i	W_i	i	W_i
1	1.425	4	1.018	7	0.731	10	0.504	13	0.358	16	0.243
2	1.266	5	0.952	8	0.653	11	0.438	14	0.321	17	0.222
3	1.132	6	0.824	9	0.586	12	0.393	15	0.281	18	0.2

After that, we can get the final regression model with taking into account heteroscedasticity:

$$y(x) = 2.0798 - 0.197x + 0.3134(x - 10.296)_+ + 0.0663(x - 20.355)_+. \tag{6}$$

A visual representation of the obtained regression model (4) and (6) is shown in Figure 5.

Although a visual analysis of the results of approximation, taking into account and without taking into account heteroscedasticity, does not give a big difference, the accounting of heteroscedasticity still improves the quality of the model.

For the resulting version of approximation, a confidence interval was obtained. The calculation of the boundaries of the variation was based on the equation of heteroscedasticity. For each signature value, the double value of the current standard deviation was used. The result of the construction of confidence interval is shown in Figure 6. As can be seen, all points are inside the confidence range.

3.3. Calculation of the polynomial regressions

Let us consider the approximation variant using a fourth- and sixth-order polynomial. Using the OLS method, we obtain the following equations:

$$y(x) = 3.667 - 0.808x + 0.0605x^2 - 1.598 \cdot 10^{-3}x^3 + 1.412 \cdot 10^{-5}x^4. \tag{7}$$

$$y(x) = 89.553 - 32.899x + 4.878x^2 - 0.3737x^3 + 0.156x^4 - 3.385 \cdot 10^{-4}x^5 + 2.912 \cdot 10^{-6}x^6. \tag{8}$$

A visual representation of the obtained regression model (7) and (8) is shown in Figure 7.

Although the sixth-order polynomial provides better adequacy in the data variation range, it is absolutely unsuitable for forecasting purposes. Its unsuitability for forecasting is especially evident in the area on the left side.

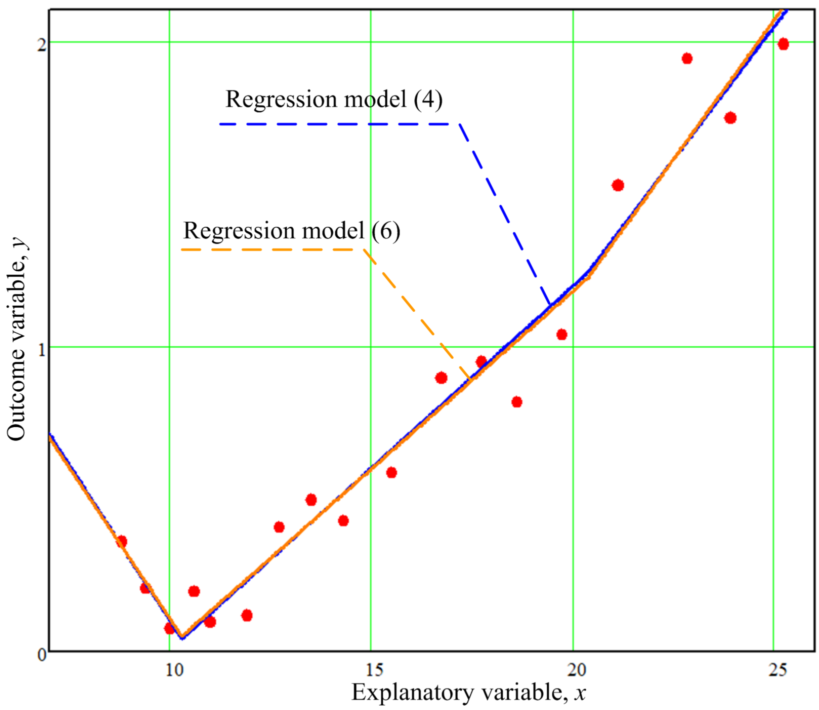

Figure 5: The comparison of models (4) and (6).

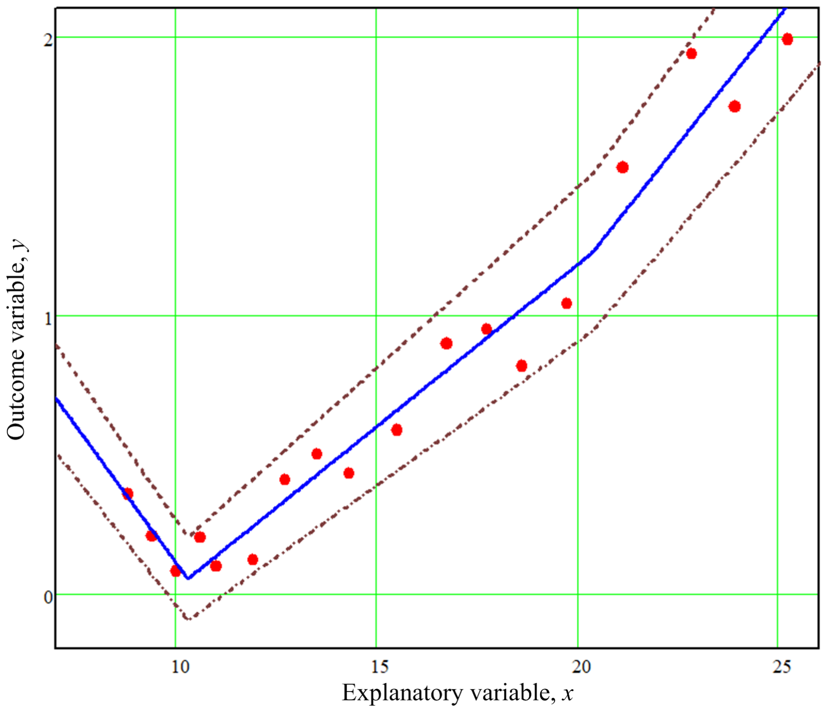

Figure 6: The regression models (6) and confidence interval.

The paper conducted a comparative analysis of all approximation options according to the standard deviation (SD) criterion, as well as according to the criterion of the maximum range (MR) of the cumulative residual curve. The calculation results are given in Table 7.

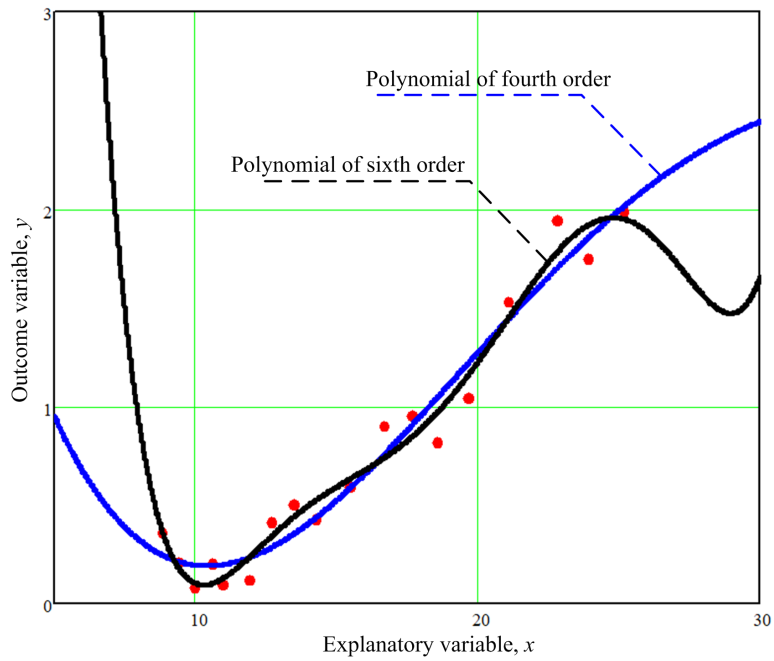

Figure 7: The polynomial regression models (7) and (8).

Table 7
The Regression Models Comparison

Parameter	Model (4)	Model (6)	Model (7)	Model (8)
SD	0.132	0.133	0.144	0.129
MR	0.4326	0.4412	0.4403	0.2808

4. Conclusions

The main goal of this paper is to construct the best segmented regression model of the studied econometric data and to substantiate its advantages as a result of comparative analysis in comparison with alternative approximations by higher-order polynomials.

At the first stage, the value of the abscissas of the switching points was optimized using the sliding setsquare method. In this case, the use of three segments was substantiated. Since the data are heteroscedastic, a new procedure of sliding triangles was proposed for its qualitative identification. This method can be considered as a generalization of the moving average, which is widely used in the analysis of time series. In general, the sliding triangle procedure makes it possible to quite reasonably construct the heteroscedasticity equation. The heteroscedasticity equation was used to calculate the confidence interval (band) of the data variation relative to the final best regression. Thus, the proposed methodology for constructing mathematical models taking into account heteroscedasticity has the property of robustness in terms of reducing the influence of samples with large values.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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