Optimization of the electrical networks reconstruction strategy by the criterion of functioning quality

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Abstract
The intensive development of renewable energy sources is one of the main reasons that necessitates the reconstruction or modernisation of existing power distribution networks. The study proposes a method based on the use of dynamic programming and an integral indicator of the quality of functioning as an optimality criterion. This method allows developing an optimal strategy for the reconstruction of power grids, taking into account the quality of electricity, reliability and efficiency.

Keywords
electric network, renewable sources, operation quality, integral index, dynamic programming

1. Introduction

In modern conditions, the requirements for the quality of electricity supply to enterprises, residential areas, etc. are constantly increasing. Therefore, the reconstruction of existing electrical networks with the construction of new power transmission lines, their transfer to a higher nominal voltage, and the use of Smart Grid technologies are the main measures to improve the quality of electricity supply, especially in conditions of intensive development of renewable energy sources.

Due to the limited resources of electricity supply companies, the task of rational distribution of funds arises in order to select priority areas for reconstruction and modernization of equipment [1]. Decision-making on financed measures should be based on data on the quality of functioning of the electricity network economy as a whole, and not only on individual components. That is, when evaluating the measures taken, it is necessary to analyze the quality of functioning of electrical networks.

2. Setting the task of optimizing the development of electrical networks with renewable energy sources

Objective assessment of promising technical solutions for the development of electrical networks is possible with full consideration of the temporal (dynamic) and spatial relationships operating in the systems [2]. This requires the construction of appropriate dynamic models of optimal development of electrical networks. However, the development of dynamic models of optimal development that meet the requirements of researchers and designers of real systems is a complex task that does not have a general solution method. In the work, a dynamic model of optimal development is understood as a method and algorithm for choosing a development strategy (terms of implementation of measures

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for the development of electric networks) according to the selected optimality criterion. It provides a minimum or sufficiently close value of the integral over the calculation period and over the object as a whole of the objective function while observing the imposed restrictions. The work considers only such dynamic models that implement multi-step solution processes using the optimality principle of dynamic programming [3].

The implementation of dynamic methods of models for optimizing the development of electric power systems with renewable energy networks is associated with significant difficulties. This is determined by the nature of dynamic problems, namely:

- the need to consider multi-step development processes;
- the discreteness of variables;
- the presence of discrete nonlinear and linear constraints;
- the complex nature of the dependence of the integral in time and space of the objective function on the variables.

When setting a dynamic problem, the existing practice of comparing options during planning and designing the development of electric power systems according to the technical and economic criterion is taken as a starting point. A work [4] uses a multi-criteria approach to evaluating solutions.

For the mathematical formulation of the dynamic problem of optimizing the development of the electric network, we will introduce a number of concepts. Event – reflects a certain set of specific works carried out on a certain set of network elements. This definition does not include the time of implementation of the event. The specific meaning of the concept of "event" can be very diverse. For example, a specific event can be the construction, reconstruction or dismantling of lines or substations, transferring lines to a higher nominal voltage, etc. However, any event, regardless of its specific meaning, is characterized by:

a) capital investments associated with the implementation of the event;
b) changes in the graph network, which are described by a set of branches that are added to or disconnected from the graph.

The set of alternative events, regarding the development of networks, contains various alternative n events. The notation N^n is introduced for this set. Alternative events in the dynamic problem act as variables.

Development step is a time interval characterized by the sequence number t, the coefficient of reduction of multi-temporal costs for this step - $\sigma(t)$, the design loads of nodes for the considered network modes - $P(t, j),\ j = 1..r$, where r - the number of design modes in the step. It is assumed that no network development measures are implemented during the step. The moment of implementation of the measure is attributed to the beginning of a certain step.

Network development strategy G is a sequence $g(1),\ g(2),\ ..., g(t), .., g(T)$ in which $g(t)$ is a set of measures implemented before the start of the t-th development step. The set $g(t)$ has the following properties:

a) the set $g(t)$ in the general case can include from 1 to m implemented measures, i.e. in limiting cases or in special cases, restrictions can be imposed on the number of implemented measures;
a) for any pair of development steps t' and $t'' - g(t') \cap g(t'') = 0$ since any measure can be implemented no more than once;
a) the relation holds $\bigcup^{T} g(t) \in N^n$, since in the process of network development measures can be implemented only from the set .

The state of network development $N(t)$ at a step t is a set of measures implemented at the development steps $\tau = 1, 2, ..., t$:$N(t) = \bigcup^{\tau=1} g(t)$. Note that $N(t) \subset N^n$. Each state of the network uniquely corresponds to:

1) capital investment;
2) network graph;
3) the level of quality of operation of the electric network.

The estimated costs of the t-th step $\varepsilon(t, N)$ have the property that they are uniquely determined by the state N and step t , since the measures implemented in N, determine the capital investments and changes in the graph network (its scheme), and the loads of the step t - the network mode and the indicators dependent on it - currents, losses, etc.

Admissible development strategy. Based on the fact that development can also be represented as a sequence of states , admissible development is determined by two conditions:

1) $N(t-1) \subset N^n$
2) at all development steps the given constraints must be satisfied.

In the above definitions, the dynamic problem of optimizing the development of the electric network can be formulated as follows.

It is necessary to find such admissible strategy of the network development G_0, which has the minimum total reduced costs for the calculation period T. Hereinafter, G_0 is called the optimal development strategy.

The objective function characterizing the technical and economic criterion, according to [4], is usually a sum. For the applied model of the development process, it has an important property: each term of the sum depends only on the state of development and the parameters of this step. Under these conditions, it is possible to use the dynamic programming method.

The solution of the dynamic problem of optimizing the development of electrical networks is carried out using the appropriate dynamic models. Such models implement a systematic approach to the problem of optimizing the development of electrical networks, which is expressed in solving the problem of the development of the object in space and time.

The development space model is a network graph containing branches that reflect both existing and prospective network elements. The development time model is an ordered sequence of development steps $t = 1, 2, ..., T$.

The structure of the dynamic model, designed to solve specific problems of designing and planning the development of the electric network, is shown in Figure.1

3. Modeling the problem of optimizing the development of the electric network

As shown above, the dynamic development problem has a multi-step structure and can be formulated as determining the optimal sequence of network states in time according to the criterion of the minimum of the objective function of the form:

$$F = \sum_{t=1}^{T} g(t, N(t)) \tag{1}$$

where $g(t, n0)$ is the component of the objective function at step t for the development state N. The solution of the dynamic problem consists of two stages. At the first stage, the optimal development of the network for the calculation period T is determined from the initial state $N(0)$ to the final state $N(t)$. At the other stage, its optimal state is determined. Therefore, the task of optimizing the network development process is to determine

$$\min F = \min_{N} \min_{[G(T,N)]} F[G(T, N)] \tag{2}$$

where N – the state of network development at time T; $[N]$– the set of possible states of development at time T; – the network development strategy for the period T from the initial state to state N; $G(T, N)$

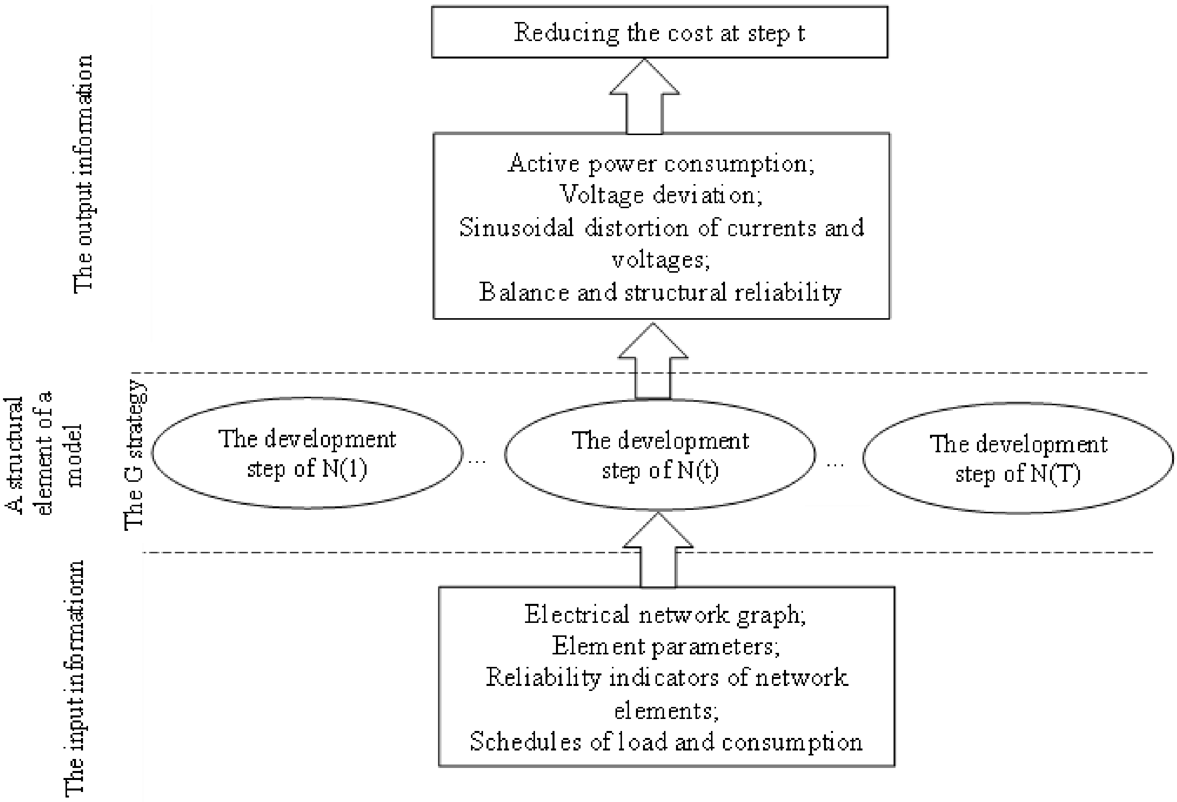

Figure 1: Structure of the dynamic model of optimal development of electrical networks.

– the set of possible development strategies for the period to state N; $F[G(T,N)]$ – the value of the integral objective function for the development strategy $G(T,N)$.

Expression (2) is a general mathematical expression of the dynamic problem of optimizing the development of the electric network.

Each t-th term of the criterion depends on the loads and other indicators at step t, on the state $N(t)$ at step t and on the state $N(t-1)$ at the previous step. Reduction to the form (1) is possible only if the definition of the state used in the work is used. If the cost reduction criterion is used, then its components are determined by the expression:

$$g\left(t, N\left(t\right)\right) = \left(E_H K\left(N\left(t\right)\right) + C\left(t, N\left(t\right)\right)\right) \cdot E_\Pi \left(1+E_\Pi\right)^{-(t+1)}, \ t = 1, 2, ..., T-1 \tag{3}$$

$$g\left(T, N\left(T\right)\right) = \left(E_H K\left(N\left(T\right)\right) + C\left(T, N\left(T\right)\right)\right) \cdot E_\Pi \left(1+E_\Pi\right)^{-T}, \tag{4}$$

where $K\left(N\left(t\right)\right)$– total capital investments for the event, implemented in the state $N\left(t\right)$;
$C\left(t, N\left(t\right)\right)$ – expenses of the t -th year; E_H– normative coefficient of efficiency of capital investments; E_Π– normative coefficient of reduction of different-time costs.

In this way, it is shown that the technical and economic criterion used in practice can be reduced to the form (1). The objective function (1) has the following properties:

- it is expressed as a sum,
- each component $g\left(t, N\right)$ depends only on N and t and does not depend on the development strategy before N.

These properties are sufficient for the application of the optimality principle of dynamic programming. For network development optimization problems, the optimality principle is formulated as follows: the optimal development of the network over time $t+1, ..., T$ from $N\left(t\right)$ state to state $N\left(t+1\right)$ does not depend on the transition strategy to $N\left(t\right)$ from the initial state $N\left(0\right)$. Such a formulation of the optimality principle allows us to obtain a recurrent expression by which the minimum value of the objective function is determined - the functional $f(t,N)$, for development from $N(0)$ to state N over time t (see Figure2).

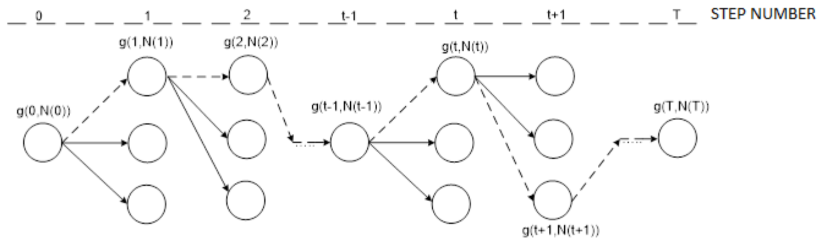

Figure 2: Graphical interpretation of the process of choosing the optimal strategy.

Taking into account (1) and (2)

$$f(t,N) = \min_{\{G(t,N)\}} \left[g(0,N(0)) + g(1,N(1)) + ... + g(t,N(t))\right], \tag{5}$$

where $\{G(t,N)\}$ is the set of available network development strategies one hour t before state N.

The component of the objective function for the zero-development step can be taken as constant:

$$g(0,N(0)) = const \tag{6}$$

The notation $\min\limits_{\{G(t,N)\}}$ shows that the objective function is minimized for all network development strategies from $N(0)$ to this state N.

Let us introduce the notation $\{N(t-1) \subset N\}$ for the set of states $N(t-1)$ from which a transition to is possible, as well as the notation $\{G(t-1,N(t-1))\}$ for the set of development strategies one hour $(t-1)$ before state $N(t-1)$.

Minimization of the objective function can be performed in the following sequence

$$f(t,N) = \min_{\{N(t-1)\subset N\}} \left\{ g(t,N) + \min_{\{G(t-1,N(t-1))\}} \left[g(0,N(0)) + ... + g(t-1,N(t-1))\right] \right\}. \tag{7}$$

In the model of the development process of the electrical network under consideration, $g(t,N)$ does not depend on $N(t-1)$. In addition,

$$f(t-1,N(t-1)) = \min_{\{G(t-1),N(t-1)\}} \left[g(0,N(0)) + g(1,N(1)) + ... + g(t-1,N(t-1))\right], \tag{8}$$

therefore

$$f(t-1,N(t-1)) = g(t-1,N(t-1)) + \min_{\{N(t-1)\subset N\}} f(t-1,N(t-1)). \tag{9}$$

Expression (9) is a recurrent dynamic programming formula that allows for a multi-step procedure for optimizing the network development strategy in an hour T from the initial to any state $N(t)$.

Minimizing the objective function by (2) is reduced to solving a certain set of dynamic programming problems by (9), in which the objective function is minimized for development from the initial to some given state. This procedure is performed by induction, starting from the first step $t=1$ and ending with the last $t=T$. For $t=1$ can be taken

$$f(t=1,N) = g(t,N).$$

The minimization of the objective function for the step t is carried out according to the following scheme. The possible states $\{N(t)\}$ of the network at the step t are considered. For each $N(t)$:

1) calculations of the technical and economic indicators of the network are carried out, which are necessary for the calculation of $g\left(t, N\left(t\right)\right)$;
2) the set of states $\{N\left(t-1\right)\}$ is determined, from which a transition to the state $N\left(t\right)$ is possible. In the model under consideration, verification of possible transitions can be carried out under the condition

$$N\left(t-1\right) \cap N\left(t\right) = N\left(t-1\right);$$

3) according to (9) is calculated $f\left(t, N\left(t\right)\right)$, which corresponds to the minimum value of the objective function for the development of the network from $N\left(0\right)$ to $N\left(t\right)$.

To perform the calculation according to this scheme, it is necessary to tabulate the values of $f\left(t-1, N\left(t-1\right)\right)$ and $f\left(t, N\left(t\right)\right)$ for the possible states $\{N\}$.

3.1. Method for choosing the optimal strategy for developing electric networks with renewable energy sources

The task of minimizing the costs of reconstruction and operation of the electric network is solved when choosing the optimal option for its development. The task is formulated as follows: it is necessary to ensure a given level of quality of functioning of the electric network at the minimum reduced costs for its reconstruction and operation. The consumer can receive power according to the variants of the electric network scheme (the variants of the scheme depend on the state in which the electric network is as a result of the failure of certain of its elements). It is necessary to ensure a given level of quality of functioning of the so that the resulting costs for reconstruction and operation are the smallest. Therefore, the task is reduced to minimizing

$$Z = \sum_{i=1}^{m} Z_i\left(E_i^{'}\right) \tag{10}$$

for restrictions

$$1 - E_{setpoint}^{'} = \prod_{i=1}^{m}\left(1 - E_i^{\prime}\right), \text{ or } \ln\left(1 - E_{setpoint}^{'}\right) = \sum_{i=1}^{m}\ln\left(1 - E_i^{'}\right) \tag{11}$$

where $Z_i\left(E_i{'}\right) = Z_r i + Z_o i$ – costs for reconstruction and operation of the i-th scheme of the distribution network; Z_i and Z_{oi}– costs for reconstruction and operation, respectively; i – variant of the power supply scheme, $E_i^{\prime}$– indicator of the quality of functioning for the i-th scheme of the distribution network; m – total number of operating states of the power network.

Since the study of the relative change in the value of costs has a number of advantages over the study of absolute values [5], let us proceed to

$$\delta Z_i(E^{\prime}) = \frac{Z_i(E_i^{\prime}) - Z_{i0}(E_i^{\prime})}{Z_{i0}(E_i^{\prime})} = \frac{Z_i\left(E_i^{\prime}\right)}{Z_{i0}\left(E_i^{\prime}\right)} - 1 = \overline{Z_i} - 1,$$

where ${}^{\prime}_{i0}$ – indicator of the quality of functioning of the original scheme.

Using the base point method, let us proceed from (17) to the criterion equation of the form:

$$y = \sum y_i\left(E^{\prime}\right) \tag{12}$$

where $y\left(E_i{'}\right) = \Theta_r^{'} F_{ri}\left(E_i^{'}\right) + F_{ri}\left(E_i^{'}\right)$– dimensionless function to be minimized is equal to $\frac{\delta Z(')}{A_o Z_o}$,

$\Theta_r{'} = \frac{A_r \overline{Z_r}}{A_o \overline{Z_o}}$ – similarity criteria; $F_{ri}\left(E_i^{'}\right)$, $F_{oi}\left(E_i^{'}\right)$ – functions that determine the physical connections in the object for reconstruction and operation, respectively; A_{ri}, A_{oi} – functions that include the initial specific cost indicators for reconstruction and operation, respectively.

Equation (12) is invariant with respect to any changes in cost and technical indicators that do not change the value of Θ'_{ri}. This allows the solution to be generalized to a number of other cases.

Taking into account (12), the problem is rewritten as minimize

$$y = \sum_{i=1}^{m} y\left(E'\right)$$

for the constraint

$1 - E'_{setpoint} = \prod_{i=1}^{m}\left(1 - E'_i\right)$ or $\ln\left(1 - E'_{setpoint}\right) = \sum_{i=1}^{m} \ln\left(1 - E'_i\right)$,

To solve the problem, we will use the method of indefinite Lagrange multipliers. The Lagrange function is represented in the form

$$\partial L\left(E'_1, ..., E'_n\right) = \sum_{i=1}^{m} y_i + \lambda \sum_{i=1}^{m} \ln\left(1 - E'_i\right),$$

where λ is the indefinite Lagrange multiplier.

To determine λ, we will solve the system of equations:

$$\begin{cases} \frac{\partial L\left(E'_1, ..., E'_n\right)}{\partial E'_i} \\ \ln\left(1 - E'_{setpoint}\right) = \sum_{i=1}^{n} \ln\left(1 - E'_i\right) \end{cases};$$

$$\frac{\partial}{\partial E'_i}\left[\sum_{i=1}^{m} y_i\left(E'_i\right) + \lambda \sum_{i=1}^{m} \ln\left(1 - E'_i\right)\right] = \frac{\partial y_i}{\partial E'} - \lambda \frac{1}{1 - E'_i} = 0.$$

Then $\partial y_i\left(E'_i\right) = \lambda \frac{\partial E'_i}{1 - E'_i}$; $y_i\left(E'_i\right) = -\lambda \cdot \ln\left(1 - E'_i\right) + C_i$.

The constant of integration C_i is determined from the boundary conditions: for $E'_i = E'_{i0}$, $y_i = y_{i0} = 1$, where E'_{i0} and y_{i0} are the quality indicator of the functioning of the original scheme and the relative increase in costs for this scheme, respectively:

$$C_i = 1 + \lambda \cdot \ln\left(1 - E'_{i0}\right);$$

$$y_i = 1 - \lambda \cdot \ln\left(1 - E'_i\right) + \lambda \cdot \ln\left(1 - E'_{i0}\right);$$

$$\ln\left(1 - E'_i\right) = \frac{1 - y_i}{\lambda} + \ln\left(1 - E'_{i0}\right). \tag{13}$$

Substituting (13) into (11), we obtain

$$\frac{1}{\lambda} \sum_{i=1}^{m} (1 - y_i) + \sum_{i=1}^{m} \ln\left(1 - E'_{i0}\right) = \ln\left(1 - E'_{setpoint}\right).$$

Whence

$$\lambda = \frac{\sum_{i=1}^{m} (1 - y_i)}{\ln\left(\frac{1 - E'_{setpoint}}{\prod_{i=1}^{m}\left(1 - E'_{i0}\right)}\right)} \tag{14}$$

After substituting (14) into expression (13), we arrive at a system of m equations with unknowns E'_i:

$$\ln\left(1 - E'_i\right) = \frac{1 - y_i}{\sum_{i=1}^{m} (1 - y_i)} \ln\left(\frac{1 - E'_{setpoint}}{\prod_{i=1}^{m}\left(1 - E'_{i0}\right)}\right) + \ln\left(1 - E'_{i0}\right). \tag{15}$$

According to (15), $E_i^{'}$ the schemes of the distribution electric network are determined, which correspond to its possible states and will provide the general specified level of quality of functioning $E_{setpoint}^{'}$.

With the change in the relations between electricity supply companies and consumers, the requirements for the quality of electricity supply have reached a significantly higher level. Therefore, taking into account the quality of functioning as an indicator by which the reliability and quality of electricity are assessed when assessing the ways of reconstruction of distribution electric networks is necessary and relatively easy to implement [6].

According to the material presented above, it is possible to form the first stage of the iterative method for determining the optimal strategy for reconstruction of the electric network. That is, at the first stage, when considering the possible states of the electric network in which it is as a result of changes in the state of its elements, those most probable schemes of the electric network are determined, which have the greatest impact on the value of the integral indicator of the quality of functioning. Based on the sensitivity analysis, the components of the quality of functioning with a determining impact on the integral indicator for the corresponding schemes are determined. Based on this analysis, a set of measures is formed N^n to achieve $E_{setpoint}^{'}$. That is, as a result of the first stage, the general problem of obtaining a given value of the integral indicator of the quality of functioning is decomposed into subproblems, in which individual cases are analyzed according to the graph of possible states of the electrical network.

The next stage of the method is the selection of the optimal strategy for the reconstruction of a separate variant of the electrical network scheme with a significantly smaller set of possible strategies. However, to select the optimal strategy for the development of the electrical network, it is necessary to obtain conditions under which it is possible to select the optimal one from the set of strategies in terms of costs for achieving a given value of the integral indicator of the quality of functioning.

So, at the first stage, the value of the integral indicator of the quality of functioning $E_i^{'}$ was obtained, the value of which we must achieve as a result of performing certain measures for the reconstruction of the i -th scheme.

Let us use the approach that was used in the first stage. Let us formulate the optimization problem for a separate circuit in the form of (10), but taking into account that $E_i^{''}$ obtained in the first stage corresponds to $E_{isetpoint}^{''}$ in the second stage, that is, the given value of the integral indicator of the quality of functioning for a certain circuit of the electrical network. Taking this into account, (11) is rewritten as:

$$\ln\left(1 - E_{isetpoint}^{''}\right) = \sum_{i=1}^{N^n} \ln\left(1 - E_i^{''}\right),$$

where N^n is the set of alternative measures for the development of networks.

To solve this problem, we will use the method of indefinite Lagrange multipliers (we assume the continuity of variables).

The Lagrange function for this problem will have the form:

$$L\left(E_1^n, ..., E_{N^n}^n\right) = \sum_{i=1}^{N^n} Z_i + \lambda \prod_{i=1}^{N^n} \left(1 - E_i^n\right) \tag{16}$$

Differentiating (16) with respect to the variables and equating them to zero, we obtain a system of equations of the form:

$$\frac{\partial Z}{\partial E''} - \lambda \prod_{S=1,\ S\neq i}^{N^n} \left(1 - E_S^n\right) = 0;$$
$$\frac{\partial Z_k}{\partial E''} - \lambda \prod_{S=1, S\neq k}^{N^n} \left(1 - E_S^n\right) = 0.$$

The solution to this system of equations will be:

$$\frac{\frac{\partial\varepsilon}{\partial_i}\prod\limits_{S=1,\ S\neq k}^{N^n}(1-E_k)}{\frac{\partial\varepsilon_k}{\partial_k}\prod\limits_{S=1,\ S\neq i}^{N^n}(1-E_S)}=\frac{\dfrac{\partial\varepsilon}{\partial\ln(E_i)}}{\dfrac{\partial\varepsilon_k}{\partial\ln(E_k)}}=1 \text{ or } \frac{\partial\varepsilon}{\partial\ln(E_i)}=\frac{\partial\varepsilon_k}{\partial\ln(E_k)} \tag{17}$$

The graphical interpretation of (17) is shown in Figure 3.

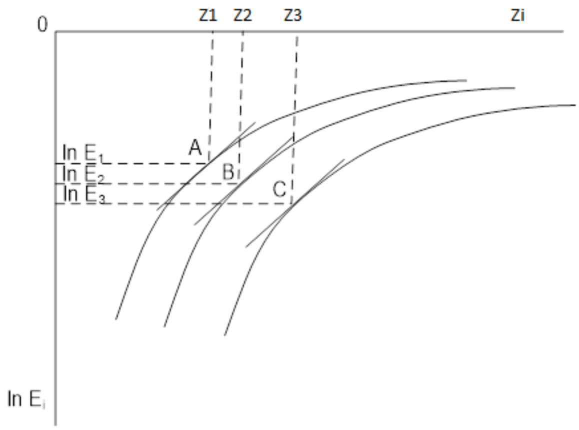

Figure 3: Graphical representation of the increments of the function $\ln(E_i) = f(Z)$.

Expression (17) is a generalized condition for choosing the optimal strategy for the development of the electric network. Taking into account the discreteness of the unknowns, the optimality condition is rewritten as $\frac{\partial Z}{\partial\ln(_i)}\approx\frac{\partial Z_k}{\partial\ln(_k}$.

At the second stage, the optimal strategy is selected according to condition (17) by comparing the ratio of increments for possible schemes of the electric network that correspond to the graph of the states of its functioning (see Figure 4, the numbering of the schemes is adopted in accordance with Figure 3).

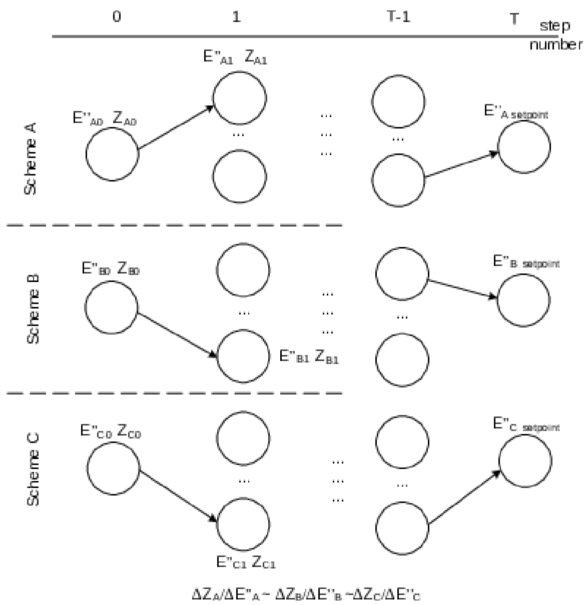

Figure 4: Scheme of choosing the optimal strategy under the optimality condition (17).

This approach does not require a return move. However, due to the discreteness of the variables, the method requires a number of iterations, which may require refinement of the calculations at the first stage.

3.2. Analysis of a given level of quality of operation of electrical networks

Let us illustrate the assessment of possible options for supplying consumers using the criterion of quality of operation of the network using the example of a diagram of a fragment of a 10 kV distribution electrical network, shown in Figure5.

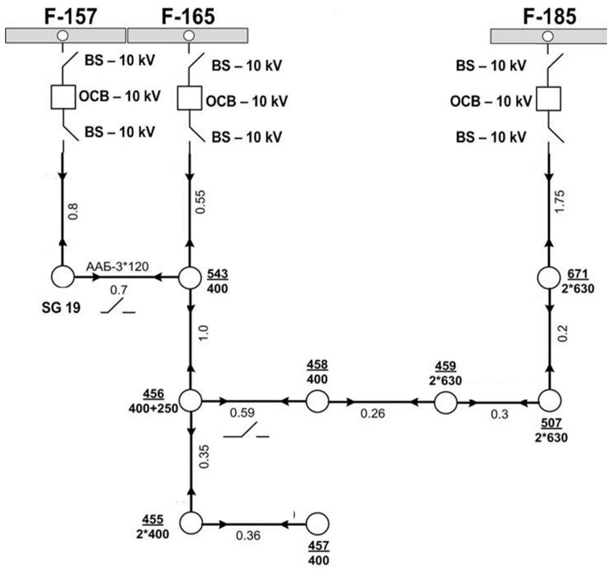

Figure 5: Scheme of the F-165 feeder of the electrical network substation.

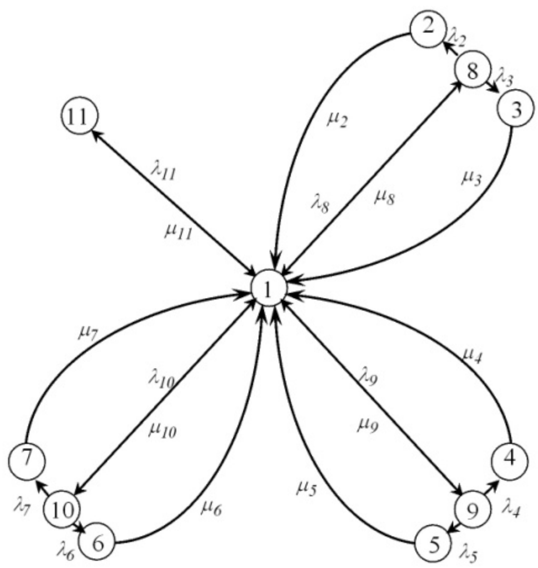

Figure 6: System state graph for the F–165 power supply option from substation buses.

For the scheme, the state change graph, taking into account hierarchical transformations, will have the form shown in Figure 6. The physical essence of the states of the research object is as follows:

Table 1
Reliability Indicators of Individual Elements of the Electrical Network

	$\lambda\,(1/year)$	$\mu\,(1/year)$
Cable line	0,0122/km	292
Switch	0,006	2190

Table 2
Quality of Functioning of Adjacent Feeders

The value of the quality of functioning indicator	F-157	F-185
E	0,8	0.7

- state 1 - all lines and switching equipment are functioning;
- state 2 - one circuit on the two-circuit section of the TS 543 - TS 456 line has failed, taking into account the possible states of the remaining elements of the distribution network;
- state 3 - the second circuit on the two-circuit section of the TS 543 - TS 456 line has failed, taking into account the possible
- states of the remaining elements of the distribution network;
- state 4 - one circuit on the two-circuit section of the TS 456 - TS 455 line has failed, taking into account the possible states of the remaining elements of the distribution network;
- state 5 - the second circuit on the two-circuit section of the TS 456 - TS 455 line has failed, taking into account the possible states of the remaining elements of the distribution network;
- state 6 – one circuit on the two-circuit section of the TS 455 – TS 457 line has failed, taking into account the possible states of the remaining elements of the distribution network;
- state 7 – the second circuit on the two-circuit section of the TS 455 – TS 457 line has failed, taking into account the possible states of the remaining elements of the distribution network;
- states 8 – 10 – all feeder consumers are disconnected from power supply as a result of relay protection tripping together with the failure of one of the lines of the sections TS 543 – TS 456, TS 456 – TS 455, TS 455 – TS 457, respectively;
- state 11 – all feeder consumers are disconnected from power supply as a result of relay protection tripping together with the failure of the section F-165 – TS 543.

A system of Kolmogorov equations is compiled based on the graph (Figure6). For the remaining power supply options for consumers, state graphs and systems of equations are compiled based on them, using similar rules and assumptions.

The initial data for calculating the quality of functioning are given in Table 1 and Table 2.

The result of solving the system of Kolmogorov equations is given in Table 3.

When calculating the quality of operation of the power supply options from adjacent feeders, the resulting value is determined as the product of the quality of operation of the corresponding feeder and the fragment of the distribution network supplying consumers of the feeder F-165. For example, for power supply from F-157, the resulting value of the quality of operation will be determined as follows:

$$E_1 = E_{543-456-455-457} \cdot E_{F-157}. \tag{18}$$

The results of the calculations are shown in Figure.7.

After analyzing the calculation results, it is possible to arrange the power supply centers according to the connection priority: F-165 and F-157. Additional analysis is required for the remaining options. For example, for TS457, electricity is received from F-185(2), and for TS456 - from F-185(1).

Let's check whether the existing network (Figure.1) will provide the quality of operation at the level of 0.89. In this case, there is no need to determine the value of the reduced costs.

Let's calculate according to the expression $E'_{setpoint} = 1 - \prod_{k=1}^{4}\left(1 - E^{'}_{k}\right)$. According to Figure 7 for all substations of the network under consideration, the specified quality of operation is ensured.

Table 3
Probabilities of System States According to the Graph in Figure 6

P1	0,999822
P2	9,15E-10
P3	9,15E-10
P4	9,15E-10
P5	9,15E-10
P6	9,15E-10
P7	9,15E-10
P8	4,45E-05
P9	4,45E-05
P10	4,45E-05
P11	4,45E-05

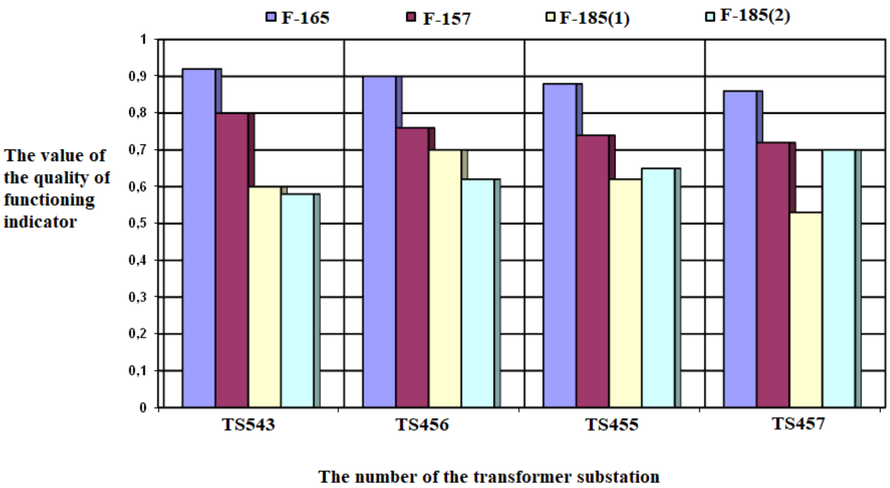

Figure 7: Results of the analysis of the quality of operation of a 10 kV electrical network depending on the consumer power supply scheme.

4. Conclusions

1. The development of renewable energy sources should lead to an improvement in the quality of electricity supply. However, failure to take into account the state of electrical networks to which RES are connected and the volumes of consumption may lead to the issuance of technical conditions for the connection of sources of overestimated power. Under such conditions, reconstruction of electrical networks is advisable.
   Limited financial resources require solving the problem of choosing priority areas of capital investments during the reconstruction of electrical networks, in particular during the development of renewable energy sources.
2. The work improves the method of determining the optimal strategy for the development of an electrical model, which is based on building a dynamic model that takes into account time and spatial relationships. Spatial relationships take into account changes in the network configuration, and temporal relationships - the time difference between the implemented measures during the development (reconstruction) of the electrical network. The proposed approach allows applying the principle of optimality of dynamic programming and implementing a relatively simple algorithm for finding the optimal strategy for the development of the electric network.
3. The choice of the optimal strategy for the development of electric networks is carried out according

to the criterion of the quality of functioning. Optimality conditions were obtained, which are the ratio of the increments in the cost of measures performed at a certain step and the quality of functioning in the state to which the performed measures led. This allowed reducing the number of states considered when determining the optimal development strategy.

4. The examples of calculations given in the work show the feasibility of using a comprehensive assessment of the options for solving both design and operational problems.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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